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Multivariable Discrete Model Reference Adaptive Control Using an Autoregressive Model with Dead Time of the Plant and Its Application

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A new method for designing multivariable model reference adaptive control system is presented. In many adaptive control approaches, at least the following two assumptions regarding the plant have to be made. (1) The number of plant poles and zeros is known. (2) The plant is the minimum phase. The proposed method can avoid these restrictions by using the adaptive controller designed for an autoregressive model with dead time, based on Lyapunov's direct method. Next, in order to illustrate the effectiveness of this model for the both minimum phase and non-minimum phase plants, the results of computer simulation are given. Finally, the method is successfully applied to the real plant testing the performance of refrigerant compressors.

Key Words: adaptive control, multivariable system, discrete-time control, model reference control, autoregressive model, dead time, refrigerant compressors

1. Introduction

Thus far many different approaches to adaptive control system have been proposed [1),2)]. Generally, it has been said that the adaptive control problem is difficult to establish the systematic design theory. However, since the model reference adaptive control system (MRACS) can clearly be investigated based on the stability theory in these attempts, it attracts most of the current interest and has made steady progress in practical design method.

Usually, the adaptive control algorithm is best implemented with a digital computer, since the controller requires complicated computations especially for the multi variable case. From this point of view, the adaptive control system must be designed in the discrete-time form.

In recent years, many approaches for a discrete adaptive control system have been developed [3)-15)]. However, in the design of MRACS, it must be assumed at least two conditions; 1) the number of plant poles and zeros is known, and 2) the zeros of the plant lie inside the unit circle of the z-plane (it is called the minimum phase system). In practical cases, the exact number of plant poles and zeros is rarely known. Moreover, the plant may be a distributed parameter system, which often appears in process control. The assumption 2) restricts the application of such methods to plants controlled using a digital computer. It often occurs that the discrete-time form of the plant interconnected by a set of samplers and a zero-order hold device becomes a system which has the zeros outside the unit-circle of the z-plane (called as non-minimum phase system), depending on the dynamic behavior of the plant and the sampling period, even if the continuous-time linear plant is the minimum phase system. In the multivariable case, the assumption of minimum phase is more restrictive. We should note that a multivariable transfer function with interaction could have zeros outside the unit circle of the z-plane even if each transfer function in the system has no zeros outside of the unit circle of the z-plane. This leads to the serious problem in the application to the multivariable real processes.

For this problem, the conventional model reference adaptive control scheme has been extended to handle non-minimum phase systems by modification in terms of zero shifting[16)] and dynamic compensation of the reference model [17)]. However, in these schemes, it may be difficult to design for an unknown plant so that the augmented system becomes a minimum phase system, and also so that the adaptive control system has fast convergence. Especially in multivariable cases, the design procedure is complicated and not clear.

In order to circumvent the assumptions 1) and 2), we propose a new approach in which the discrete-time model reference adaptive control is carried out using a controller designed for a multivariable AR model with dead time of the plant. Thus far, a plant is assumed to be modeled by an ARMA model, and an adaptive controller is designed for the model. In this case, if the plant has

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unstable zeros, the adaptive control system becomes unstable, since the adaptive controller is designed for model matching and acts so as to cancel the zeros. This is fatal in the case of an ARMA modeling. Thus, the fixed idea that a plant is described by an ARMA model will be changed. In general, a model is defined to be an approximate description of a plant constructed in terms of the objective. Let us consider what model of a plant is useful for the objective of adaptive control. A model reference adaptive control acts always so as to obtain pole-zero cancellation. If an AR model with dead time is adopted as the plant model, the adaptive controller does not generate structurally poles to cancel zeros, since the model has no zeros. Therefore, the stable control is expected even for non-minimum phase systems. Although there is a difference in degree, any model is an approximation of a plant, we think it advisable to avoid the ill-suited model to the design of adaptive control. However, the fitness of the AR model with dead time for any plant may be questionable. Such fitness should be evaluated by the control performance of the adaptive controller designed for the model. In fact, this scheme could work fairly well for many different types of single-input, single-output plants [18]. Moreover, this method enables the design of the adaptive control system only by knowing approximately a step response of the plant to be controlled, without a priori knowledge about the transfer function, orders, and parameters etc. of the plant. This method can be readily applied to weakly nonlinear plants, slowly time-varying plants, and distributed parameter plants. From the practical point of view, the structure of the adaptive controller becomes simpler than that for an ARMA model, since the AR model with dead time has no zeros.

In the present study, the above method is extended to handle multi-input, multi-output plants and the experimental results of the proposed adaptive control system are presented. First, a stable adaptive control system is designed for an AR model with dead time. Next, in order to illustrate the effectiveness of this model, the results of computer simulation for both minimum phase and non-minimum phase multivariable plants are given. Finally, the proposed method is applied to a real plant. The real plant is a plant testing the performance of refrigerant compressors. The plant is the 3-inputs, 3-outputs system, which contains an unknown compressor, whose characteristics must be evaluated. The set point changes stepwise over a wide range of the operating condition. Since the operating condition of the plant varies very wide, the stable control is not always obtained using the conventional PID controller with fixed parameters. Moreover, since the plant is the thermal process, the very long testing time is required. Thus, an adaptive control is necessary to obtain a stable control behavior for a wide range of the operating condition and to reduce the testing time. On-line computer experiments have been carried out using the plant testing the performance of refrigerant compressors.

2. Statement of problem

Consider the case where the multi-inputs, multi-outputs, continuous-time plant is controlled using the adaptive algorithm implemented with a digital computer.

It is assumed that the plant interconnected by sets of samplers with the sampling period T and zero-order hold devices is adequately modeled by a multivariable AR model with dead time of the form:

$$\begin{bmatrix} \overline{A}_1(q^{-1}) & & 0 \\ & \ddots & \\ 0 & & \overline{A}_m(q^{-1}) \end{bmatrix} y(k) = \begin{bmatrix} q^{-l_1} b_{11} & \cdots & q^{-l_1} b_{1m} \\ \vdots & & \vdots \\ q^{-l_{m1}} b_{m1} & \cdots & q^{-l_{m1}} b_{mm} \end{bmatrix} u(k) + d' \tag{1}$$

where $y(k), u(k)$ are the m[th] order output vector and input vector, respectively. $\overline{A}_i(q^{-1})$, $i=1,\dots,m$ denotes the n th order scalar polynomial in the unit delay operator q^{-1}. l_i, $i=1,\dots,m$ represents .the equivalent dead time between the input $u(k)$ and i[th] output $y_i(k)$ †.

It can be seen that Eq. (1) consists of a set of multi-input, single-output systems having a common input vector.

In this study, the plant is basically modeled by discrete-time multivariable AR model with dead time. However, there exists the modeling error, parameter variation, constant disturbance and so on, we introduce the m[th] order residual vector d' into the model, which corresponds these errors.

The plant model Eq. (1) can also be represented as:

$$y(k) = \sum_{i=1}^{n} A'_i \, y(k-i) + \sum_{i=1}^{m} B'_i \, u(k-l_i) + d' \tag{2}$$

† In this study, it is assumed that $\overline{A}_i(q^{-1})$ has the common order n. This is only for simplifying the design procedure and the computation in implementation. If the orders of $\overline{A}_i(q^{-1})$ are independently known, we can easily modify the control scheme under the condition.
The assumption that the equivalent dead time between the input $u_j(k)$ ($j=1,\dots,m$) in the input vector $u(k)$ and i[th] output has the common value l_i, is related to the condition about the stable invariant zeros. If the equivalent dead time between the input $u_j(k)$ ($j=1,\dots,m$) and i[th] output has the independent value, the invariant zeros of the AR model with dead time cannot be obtained from Eq. (3) and may not exist inside the unit circle of the z-plane.

It is very difficult to assign the values of l_i and n independently and accurately. However, in many cases (in case of the application mentioned later), the adaptive controller using the AR model with dead time whose values of l_i and n are simply assigned based on the response of each output for the dominant input, gives fairly well control performance.
It seems that the rare cases require the accurate assignment of l_i and n .

where A'_i, $i=1,\ldots,m$ is the $m \times m$ coefficient matrix and B'_i, $i=1,\ldots,m$ is the $m \times m$ matrix with the element of i th row is b_{ij}, $j=1,\ldots,m$ and all elements except for i th row are zeros.

In this case, the invariant zeros of the system Eq. (2) are defined by the roots of the following equation.

$$\left|\sum_{i=1}^{m} B'_i z^{n-l_i}\right| = z^{mn-\sum_{i=1}^{m} l_i}\begin{vmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mm} \end{vmatrix} = 0 \tag{3}$$

From this equation, it can be seen that all zeros of the plant model lie at the origin of the z-plane.

Equation (2) can be represented by rewriting the multi-input, single-output system in the plant model corresponding each dead time, as follows.

$$\bar{y}(k+l) = \sum_{i=0}^{n-1} A_i y(k-i) + \sum_{i=0}^{l-1} B_i u(k-l_i) + d \tag{4}$$

where,

$$l \triangleq \max l_i \tag{5}$$

$$\bar{y}^T(k+l) \triangleq [y_1(k+l_1),\ldots,y_m(k+l_m)] \tag{6}$$

where A_i, $i=0,\ldots,n-1$, B_i, $i=0,\ldots,l-1$ are the unknown constant matrices depending A'_i and l_i, A'_i, B'_i and l_i, respectively. The overall equivalent dead time d is determined from A'_i, d' and l_i.

In the design of the adaptive control system, we assume that the equivalent dead time l_i, $i=1,\ldots,m$ and the order of the plant model n can be determined from a priori knowledge about the plant which can be estimated using the step response of the plant. In Eq. (4), B_1 is assumed to be non-singular. This condition is equivalent to the necessity and sufficient condition that the plant model can be decoupled by a state feedback [20], and the sufficient condition that the plant input $u(k)$ can always be calculated.

We can derive the control algorithm with less redundant parameters and less computational time, in which the dead time for each output is independently taken into account. However, for simplicity, we only show the control algorithm based on the maximum dead time in the plant model.

Let the reference model with decoupled structure for the plant Eq. (2) be described by:

$$y_i^*(k) = \sum_{j=1}^{n^*} a_{ij}^* y_i^*(k-j) + \sum_{j=1}^{n^*} b_{ij}^* r_i(k+1-l_i^*-j) \tag{7}$$

where, the coefficients a_{ij}^*, b_{ij}^* ($i=1,\ldots,m$; $j=1,\ldots,n^*$) are prescribed by the designer so that the model may yield a stable and desired response. $r_i(k)$ ($i=1,\ldots,m$) is the input to the i th reference model. l_i^* ($i=1,\ldots,m$) should be selected $l_i^* \geq l_i$ ($i=1,\ldots,m$) so as to avoid using the future values of the signals. For simplicity, we assign $l_i^* = l_i$ in the design of the adaptive control system.

Define the following signals about the reference model corresponding to Eq. (4).

$$\left.\begin{aligned} y^{*T}(k) &\triangleq [y_1^*(k),\ldots,y_m^*(k)] \\ \bar{y}^{*T}(k+l) &\triangleq [y_1^*(k+l_1),\ldots,y_m^*(k+l_m)] \end{aligned}\right\} \tag{8}$$

3. Design of control system

Define the output error and the parameter error as follows.

$$e(k) \triangleq y^*(k) - y(k) \tag{9}$$

$$\bar{e}(k) \triangleq \bar{y}^*(k) - \bar{y}(k) \tag{10}$$

$$D_i \triangleq \Theta_i - A_i,\ i=1,\ldots,n-1 \tag{11}$$

where Θ_i, $i=0,\ldots,n-1$ is the arbitrary constant matrix. Using the above definition and from Eqs. (2) and (8), we obtain the following error equation.

$$\begin{aligned} \bar{e}(k+l) = {} & \bar{y}^*(k+l) + \sum_{i=0}^{n-1} D_i y(k-i) \\ & - \sum_{i=0}^{n-1} \Theta_i y(k-i) - \sum_{i=0}^{l-1} B_i u(k-l_i) - d \end{aligned} \tag{12}$$

In order to perform PID control action with respect to the output error $\bar{e}(k)$, we add $a\bar{e}(k+l-1) + b\bar{e}(k+l-2)$ to both sides of Eq. (12) (The coefficients a and b are chosen so that the polynomial $z^2 + az + b$ z is stable. Note that these coefficients can be assigned for each error independently but we simply assign the common values.) and introduce the new variables as follows.

$$\begin{aligned} \bar{e}_f(k) &= \bar{e}(k) + a\bar{e}(k-1) + b\bar{e}(k-2) \\ \bar{y}_f^*(k) &= \bar{y}_f^*(k) + a\bar{y}_f^*(k-1) + b\bar{y}_f^*(k-2) \\ y_f(k) &= y(k) + ay(k-1) + by(k-2) \\ u_f(k) &= u(k) + au(k-1) + bu(k-2) \end{aligned} \tag{13}$$

Using the above variables, Eq. (12) can be written as follows.

$$\begin{aligned} \bar{e}_f(k+l) = {} & \bar{y}_f^*(k+l) + \sum_{i=0}^{n-1} D_i y_f(k-i) \\ & - \sum_{i=0}^{n-1} \Theta_i y_f(k-i) - \sum_{i=0}^{l-1} B_i u_f(k-l_i) \bar{y}_f^*(k+l) \\ & - (1+a+b)d \end{aligned} \tag{14}$$

Next, we shall determine the control input $u(k)$ as follows.

$$u(k) = u_f(k) - au(k-1) - bu(k-2) \tag{15}$$

where,

$$\begin{aligned} u_f(k) = \hat{B}_0^{-1}(k)\Bigg[& \bar{y}_f^*(k+l) - \sum_{i=0}^{n-1}\{\Theta_i - \hat{D}_i(k)\}y_f(k-i) \\ & - \sum_{i=0}^{n-1}\Theta_i y_f(k-i) - \sum_{i=0}^{l-1}\hat{B}_i(k)u_f(k-l_i) + g(k)\Bigg] \end{aligned} \tag{16}$$

where $\hat{B}_i(k)$, $i = 0,\ldots,l-1$; $\hat{D}_i(k)$, $i = 0,\ldots,n-1$ are the adaptive gain matrices. Both the adaptive gain matrices and the adaptive gain vector $g(k)$ are updated by the following adaptive law.

Substituting Eq. (16) into Eq. (14), we have

$$\bar{e}_f(k+l) = \Gamma(k)d(k) + m(k) \tag{17}$$

where

$$\begin{aligned} \Gamma(k) \underline{\underline{\Delta}} [& D_0 - \hat{D}_0(k), \ldots, D_{n-1} - \hat{D}_{n-1}(k), \\ & \hat{B}_0(k) - B_0, \hat{B}_1(k) - B_1, \ldots, \hat{B}_{l-1}(k) - B_{l-1}] \end{aligned} \tag{18}$$

$$\begin{aligned} d^T(k) \underline{\underline{\Delta}} [& y_f^T(k), \ldots, y_f^T(k+1-n), \\ & u_f^T(k), \ldots, u_f^T(k+1-l)] \end{aligned} \tag{19}$$

$$m(k) \underline{\underline{\Delta}} -(1 + a + b)d - g(k) \tag{20}$$

In this case, using the following adaptive law, the stability of the overall control system is assured when the plant is modeled by the AR model with dead time.

$$\Gamma(k) = \Gamma(k-l) - e(k)\bar{e}_f(k)d^T(k-l) \tag{21}$$

$$m(k) = m(k-l) - e(k)\bar{e}_f(k) \tag{22}$$

where

$$\left.\begin{aligned} e(k) = r(k) / [d^T(k-l)d(k-l) + h(k)] \\ 0 < r(k) < 2 \\ h(k) \geq 1 (if\ d \equiv 0\, then\, g(k) \equiv 0\, and\, h(k) \geq 0) \end{aligned}\right\} \tag{23}$$

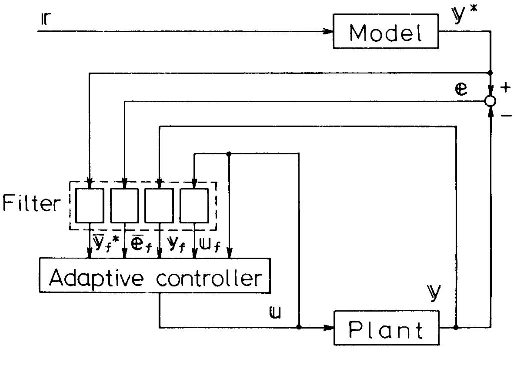


Fig. 1 Schematic diagram of the model reference adaptive control system

A schematic block diagram of the adaptive control system is shown in **Fig. 1**.

Consider the following Lyapunov function of Γ and m for the system described by Eqs. (17)-(23).

$$V(k) = tr[\Gamma^T(k-1)\Gamma(k-1)] + m^T(k-1)m(k-1) \tag{24}$$

Then, from Eqs. (17), (21)-(23), we have the l th difference $\Delta V(k)$ as follows.

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k+1-l) \\ &= -\frac{r(k)}{d^T(k-l)d(k-l) + h(k)} \\ &\quad \cdot \left[2 - r(k)\frac{d^T(k-l)d(k-l) + 1}{d^T(k-l)d(k-l) + h(k)} \right] \bar{e}_f^T(k)\bar{e}_f(k) \end{aligned} \tag{25}$$

If there is no divergence within finite time, $\Delta V(k)$ becomes negative semi definite. Since $\Delta V(k) \to 0$ as $k \to \infty$, it converges that $V(k) = V(k-l)$, $\Gamma(k) = \Gamma(k-l)$ and $m(k) = m(k-l)$, respectively. Moreover, if the signals in the vector $d(k)$ is sufficiently rich, then $V(k)$ becomes zero and it follows that $\Gamma(k) = 0$ and $m(k) = 0$. It also means that $\Gamma(k)$ and $m(k)$ are bounded. On the other hand, the fact that $\Delta V(k) \to 0$ as $k \to \infty$ leads to the following property.

$\bar{e}_f^T(k)\bar{e}_f(k) / [d^T(k-l)d(k-l) + h(k)] \to 0$

Since the AR model with dead time has no unstable zeros, then $\bar{e}_f \to 0$ and u_f becomes bounded when the reference model is stable and its input is bounded.

Moreover, if the coefficients a and b are chosen so that the polynomial $z^2 + az + b$ z is stable, it can be proven that $e(k) \to 0$ as $k \to \infty$ and $u(k)$ is bounded for all k [7),13),18)].

4. Numerical examples

To investigate the effectiveness of the adaptive control system mentioned above, computer simulations are performed for two different types of plants.

[Example 1]

The plant is a minimum phase system and the reference model is chosen as a decoupled second order systems with $n_i^* = 2$ $(i = 1,2)$ as follows:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{0.25}{s^2 + 1.9s + 0.25} & \dfrac{-0.05}{s^2 + 2s + 0.25} \\ \dfrac{-0.5}{s^2 + 3s + 1} & \dfrac{1}{s^2 + 3.2s + 1} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

$$Y_1^*(s) = e^{-0.4s} / (s^2 + 1.4s + 1) \cdot R_1(s)$$

$$Y_2^*(s) = 4e^{-0.4s} / (s^2 + 1.4s + 1) \cdot R_2(s)$$

Discretizing the above systems with the sampling period T=0.2, and design the adaptive control system based on the plant model Eq. (4) with $n=10$ and $l_i=3$ ($i=1,2$).

In this case, the reference model Eq. (7) with $n_i^*=2$, $l_i^*=3$ ($i=1,2$). The reference inputs are given by step changes $r_1(k)=1.0$ and $r_2(k)=2.0$ respectively.

Figure 2 shows the simulation result when the plant is modeled by Eq. (4) with $d=0$.

The design parameters are:

$r=1.0$, $a'=b'=1.0$, $h=0.0$, $\hat{B}_0(0)=diag(b_{11}^*, b_{22}^*)$, $\Theta_i=diag(a^*_{1i+1}, a^*_{2i+1})$ ($i=0,1$), $\Theta_i=0$ ($i=2,...,9$), and $g(k)\equiv 0$ ($d=0$).

where, $a=-(a'+b')$, $b=b'/(1+a+b)$.

The initial values of the adaptive system are set at zero except for $\hat{B}_0(0)$.

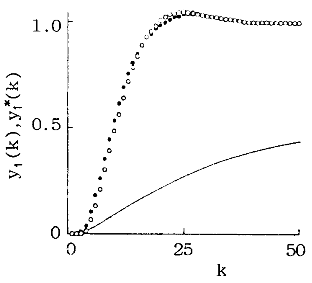


Fig. 2 Simulation results for the minimum phase plant

In Fig. 2, ● denotes the plant output, ○ the output of the reference model and the solid line indicates the step response of the plant for $u^T(k)=[1.0, 2.0]$. It can be seen that the plant outputs are close to the model outputs, and it is confirmed that the plants can be well controlled by the AR model with dead time. In the cases where $g(k)\neq 0$ ($d\neq 0$) is adopted, more stable control results can be obtained.

[Example 2]

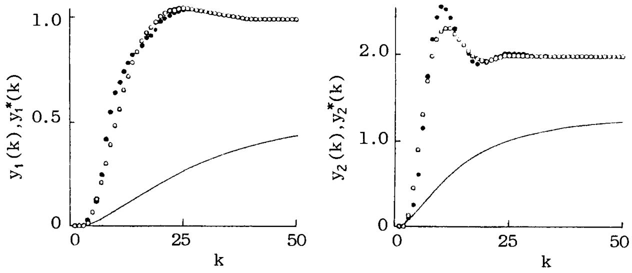
The plant is a non-minimum phase system with an unstable invariant zero in the right half area of the s-plane. The transfer function matrix is as follows.

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{0.25}{s^2+1.9s+0.25} & \dfrac{-0.05(0.3s+1)}{s^2+2s+0.25} \\ \dfrac{-0.7}{s^2+3s+1} & \dfrac{1}{s^2+3.2s+1} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

In this case, the discrete time form of the plant with the sampling period T=0.2 has one invariant zero outside the unit circle of the z-plane. In the design of the adaptive control system, the above plant is modeled by the AR model with dead time, with $n=10$, $l_1=3$ and $l_2=2$ in Eq. (4). The reference model has the same structure and parameters as the first example except for $l_1^*=3$ $l_2^*=2$ in Eq. (7).

Figure 3 shows the simulation result when the non-minimum phase plant is modeled by Eq. (4). In this case, $a'=b'=0.0$, $h=2.0$ are assigned. The other design parameters of the adaptive systems are the same as those of the first example. From this figure, it can be seen that the proposed method is effective for the non-minimum phase system.

Fig. 3 Simulation results for the non-minimum phase



plant

The plant model parameters n and l_i should be assigned carefully so that a desirable response of the adaptive system may be obtained. The assignation of l_i should be made more carefully than that of n. l_i is not the pure time delay. For stable plants, the standard value of l_i is assigned the discrete time at which the tangent at the inflection point of the step response of the plant intersects the time-axis plus one sampling period. On the other hand, the value of n does not have much effect on the performance of the adaptive system. However, generally speaking, the larger the n is assigned, a more stable response is obtained.

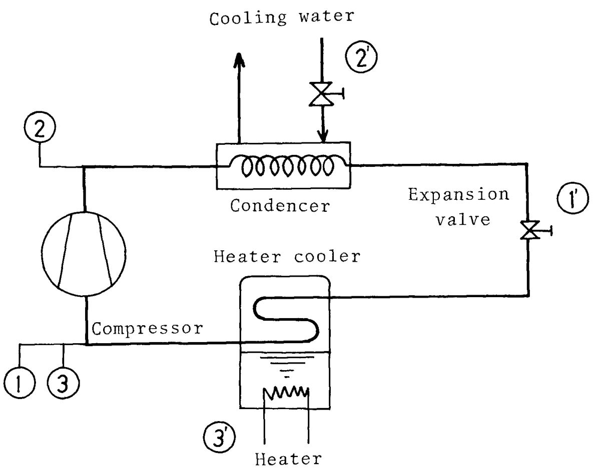
If the design parameters a and b are chosen as appropriate values, the oscillations in inputs $u_i(k)$ can be reduced and also the response can be improved

Finding the optimum adjustment of the design parameters for any plant is one of the most practical problems left for future study.

5. Application to a real plant

The control objective is the plant testing the performance of refrigerant compressors. In the actual testing method, the performance of the refrigerant compressor, which is indicated by an input power of the compressor and a refrigerating capacity, is evaluated at several sets of the operating condition.

Figure 4 shows the schematic diagram of the plant. The plant is composed of the four main elements, that is, compressor, condenser, expansion valve and heater-cooler (evaporator), in a cyclic manner.



1: Evaporating pressure y_1

2: Condensing pressure y_2

3: Temperature of refrigerant at compressor suction inlet y_3

1':Opening of expansion valve u_1

2':Flow rate of cooling water u_2

3':Electrical input of the heater u_3

Fig. 4 Schematic diagram of the plant testing a refrigerant compressor

The controlled variables are the evaporating pressure y_1, the condensing pressure y_2 and the temperature of refrigerant at compressor suction inlet y_3. The control inputs corresponding the outputs, are the opening of expansion valve u_1, the flow rate of cooling water u_2 and the electrical input to the heater u_3.

Corresponding to the testing conditions of refrigerant compressor, the set-point of plant outputs y_1, y_2 and y_3 are changed in a step manner, and the performance of the refrigerant compressor for each testing condition is evaluated when all the plant variables become steady.

In general, since the plant testing the performance of refrigerant compressor contains thermal processes, the time required for the test is very long. In addition, since the plant contains a compressor with unknown characteristics varying over a wide range of the operating condition, the stable control is not always obtained using the PID controller as shown later. The plant, from the control point of view, is the 3-inputs, 3-outputs continuous-time distributed parameter system which has nonlinear and unknown characteristics†.

Thus, the adaptive technique is needed and we adopt the proposed model reference adaptive control algorithm to the plant. In order to obtain the stable control behavior for a wide range of the operating condition, to reduce the testing time, and to improve the testing method, the model reference adaptive control algorithm proposed here is adopted.

The reference model is designed for the desired response time, stability and decoupling properties. The reference inputs are changed for the testing conditions in a step manner.

First, **Fig. 5** shows the typical control behavior of the temperature of refrigerant at the compressor suction inlet y_3 when a PID controller is used for each loop of three controlled variables. The control behavior using a PID controller with fixed parameters which are tuned for the first step change of the testing condition, changes to worse according to the step changes of the testing condition and eventually becomes unstable as shown in Fig. 5.

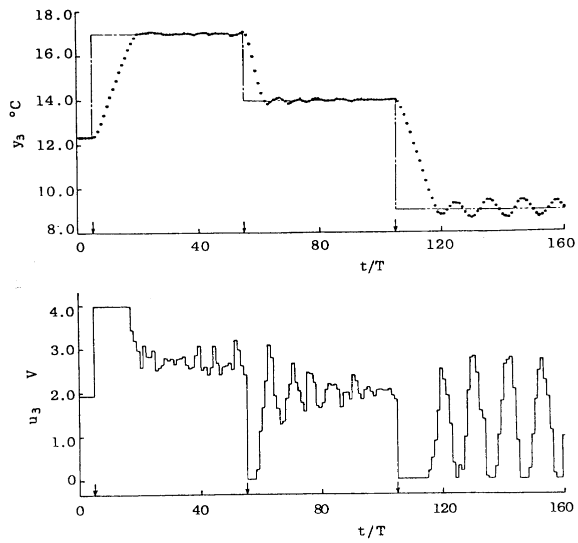


Fig. 5 Control behaviors of y_3 and u_3 when a PID control is used

Next, **Fig. 6** shows the simulation result, calculated using the proposed MRAC algorithm for a mathematical model of the plant. The plant model of the refrigerant compressor had been determined by fitting multivariable autoregressive model[19]. This ARMA model has 30 state variables. For this ARMA model of the plant, the control is performed using an adaptive controller designed based on an AR plant model with $n=2$, and the dead time $l_1=1$, $l_2=1$, $l_3=2$. This plant model is a non-minimum phase system, which has two zeros ($0.602 \pm j1.037$) outside of the unit circle of the z-plane.

† The controlled plant can be approximately modeled by the reduced order linear lumped parameter time invariant system as shown in **Fig. 4'.**

The response of the each path depends on the operating condition. The average response times are about 1 minute for $u_1 \to y_1$, 2 to 3 minutes for $u_2 \to y_2$, 15 to 20 minutes for $u_3 \to y_3$ which. has the integral characteristic. Refer to 19) for the detailed specification of the main component of the plant.

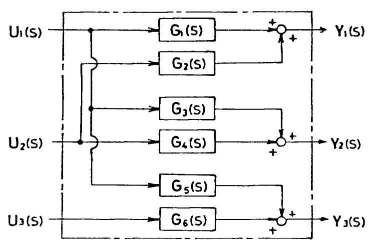


Fig. 4' Block diagram of the controlled system

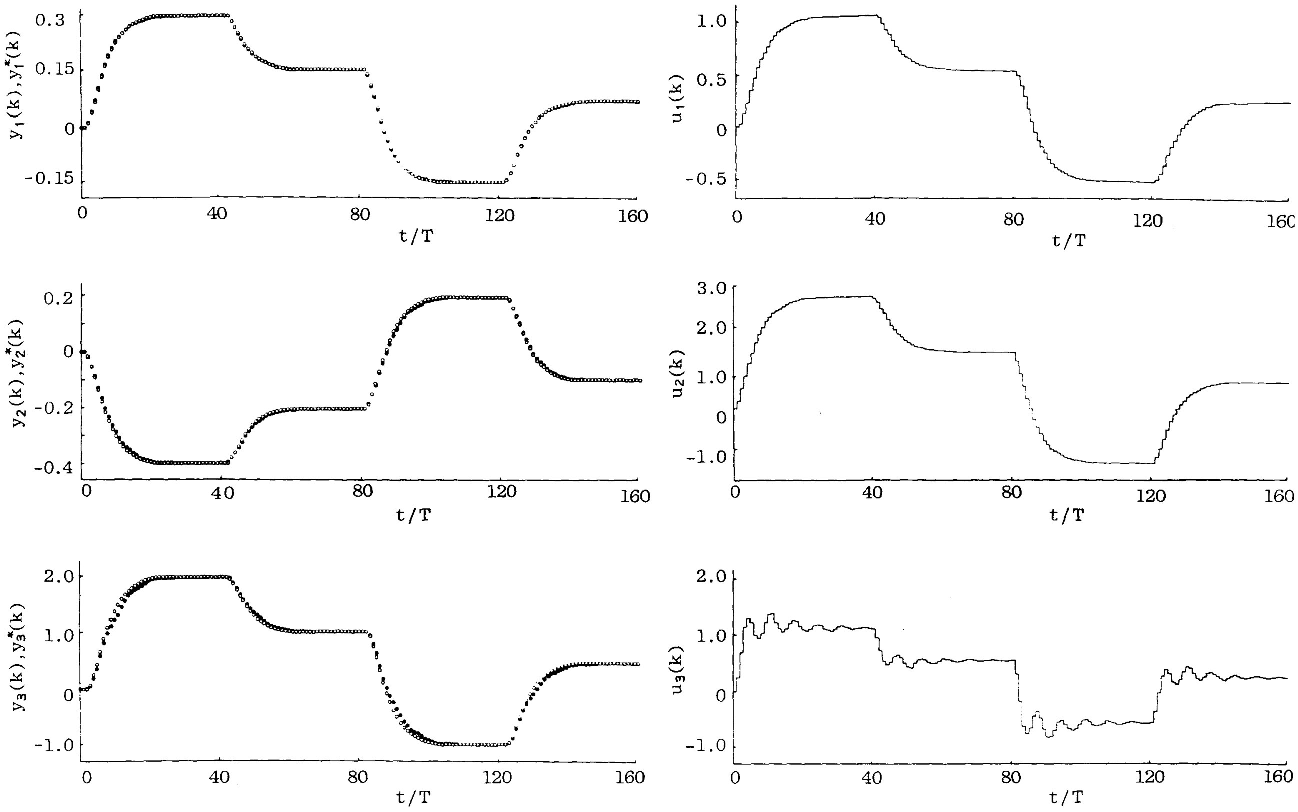


Fig. 6 Simulation results of adaptive control in a refrigerant compressor test

The decoupled continuous-time transfer function of the reference model is composed of three second order lag system with dead time as:

$$Y_i^*(s) = 0.000025e^{-st_i} \big/ (s^2 + 0.01s + 0.000025),$$

$$i = 1,2,3,\ t_1 = 0,\ t_2 = 0,\ t_3 = 60$$

The discrete-time form of the reference model is obtained by discretizing the above system interconnected by a set of samplers with the sampling period T=60sec.

Then, $n_i^* = 2$ ($i = 1,2,3$), $l_1^* = 1$, $l_2^* = 1$, $l_3^* = 2$, respectively.

The design parameters are selected as follows:

$$r = 1.0,\ a' = 5.0,\ b' = 15.0,\ h = 3.0,$$

$$\hat{B}_0(0) = diag(0.28, -0.07, 0.3),$$

$$\Theta_0 = diag(0.005, 0.75, 2.0),$$

$$\Theta_1 = diag(0.0, -0.14, -1.0)$$

From the simulation results Fig.6, the effectiveness of the proposed method is confirmed irrespective of the reduced modeling of the high order multivariable system.

Figure 7 shows the control behavior obtained by on-line adaptive control. The reference model and the design parameters of the control system are the same as those of the simulation study.

Since the plant is a nonlinear and distributed parameter system with unknown parameters and stochastic disturbance affects the system, the control behavior is not the same as those of the simulation study. However, it can be seen that the adaptive control algorithm proposed here can be successfully applied.

The adaptive control algorithm is performed by a minicomputer (Melcom 70/30, 16 bits word length). Both the program and the data storage need about 30 kW memory space. The computation time takes less than one second from the data input of the controlled variables to data output of the control variables. The resolution of A/D and D/A converters is 12 bits. The algorithm is corded by FORTRAN with 64 bits floating arithmetic.

6. Conclusion

We have proposed the model reference adaptive controller for multi-input, multi-output plant using the AR model with dead time of the plant. The effectiveness of the proposed algorithm for different types of plants, e.g., minimum and non-minimum phase plants, was confirmed from the simulation studies and the experimental results for a real process.

This report may be the first case in which the multivariable adaptive control technique is successfully applied. From the practical point of view, however, there are still many problems to investigate. Finding the optimum adjustment of the design parameters for any plant, the selection of the adaptive law and the consideration of the measurement noise are the practical problems left for future study.

The authors hope that the proposed adaptive control technique will be applied for many types of real processes as a one of the key technique and will be improved to more feasible method by feedbacking the experimental results and the problems in the operation.

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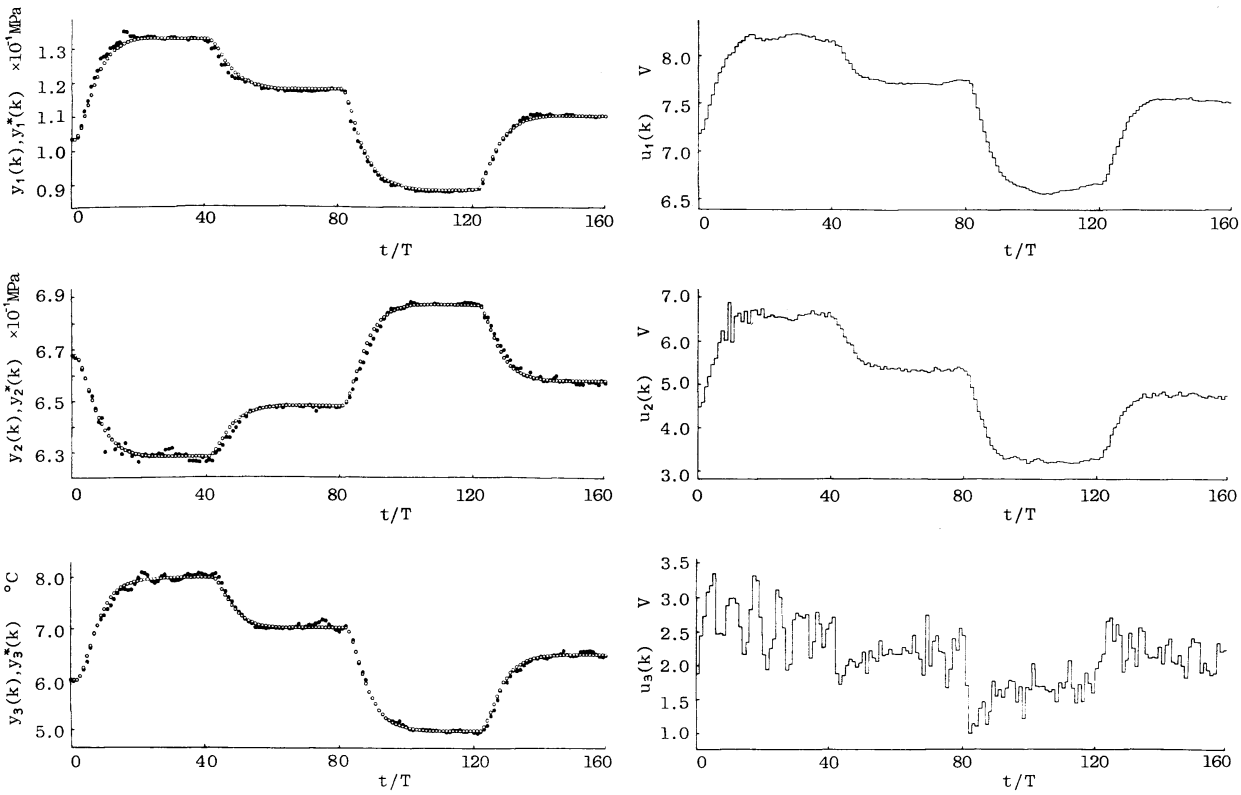

Fig. 7 Experimental results of adaptive control in a refrigerant compressor test

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