Trans. of the Society of Instrument and Control Engineers
Vol.E-2, No.1, 42/53 (2002)

Suppression Control Method of Torque Vibration for Brushless DC Motor Utilizing Repetitive Control with Fourier Transform

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Variable speed drive systems of brushless DC motor have been widely used for industry applications, home electric appliances, and so on, due to the progress of the power electronics, because of simple structure, easy maintenance, high efficiency, etc.. However, structural imperfectness of brushless DC motor and its control system produce torque ripple, and cause mechanical vibration by torque ripple and acoustic noise.

In this paper, authors propose a suppression control method of vibration for brushless DC motor utilizing feedforward compensation control, and a generation method of compensating signals for the feedforward control by repetitive control with Fourier transform utilizing a vibration signal acquired by an acceleration sensor attached to the motor frame.

Because the vibration signals detected by the acceleration sensor contain various complicated frequency components due to motor torque vibrations and mechanical resonant vibrations around the motor load system, we can not stabilize the repetitive control system and reduce the vibration by directly using the vibration signals from sensor itself. So we propose a generation method of the compensating signals of the repetitive controller considering the periodicity of motor torque ripples, and using Fourier transformer which can select particular frequency components. In order to realize on-line generation of feedforward compensating signals to reduce the vibration, auto-tuning method of the repetitive control parameters is also presented.

An experimental system to detect and reduce the motor torque vibration is constructed, and the effectiveness of the proposed method is confirmed by experimental results.

Key Words: Brushless DC motor, Torque ripple, Acceleration sensor, Repetitive control, Fourier transform

1. Introduction

Variable speed drive systems of brushless DC motors have been widely used for industry applications, home electric appliances, and so on, due to the progress of the power electronics, because of simple structure, easy maintenance, high efficiency, etc.. Moreover, because the brushless DC motor can realize the same high performance as DC motor, it is used in the fields where the high speed and high accuracy control is required. However, structural imperfectness of brushless DC motor and its control system produce torque ripple, which causes mechanical vibration and acoustic noise [1]-[3]. As the ripple torque suppression methods, the feedforward compensation method of the compensation current estimated from the spatial harmonics of reluctance [4], the method of using skewed rotor [5], [6] and so on are reported.

Also, it was well known that the torque ripple could be reduced by addition of special feedforward compensation voltage/current to the usual voltage/current for the control input [4]. However, how do we obtain the feedforward compensation signals or data? Of course, we can derive the compensation signals by analysis considering the structure of the motors, estimation from the e.m.f. waveshape, etc. approximately, but cannot obtain the accurate signals. Finally for the individual motor, we must make a fine adjustment of the compensation signal in order to suppress the torque ripple sufficiently.

In earlier papers, from the viewpoint of direct reduction of the vibration of the motor frame caused by torque

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ripple, we attached the acceleration sensor on the motor frame, and we have proposed some generation methods of compensating signals to reduce the torque ripple of the brushless DC motor, 3-phase HB-type stepping motor and the induction motor by repetitive control. [7]-[10] (Fig.1).

The purposes of this research is to construct the control system utilizing the feedforward compensation signals to reduce the torque ripple, and the generation method of compensating signals for the feedforward control by repetitive control. Furthermore, in this paper, the auto-tuning method of the repetitive control parameters is considered for the reasons as shown below:

(1) Even when the characteristics of mechanical systems are not known clearly, the repetitive control must be always performed stably.
(2) In the case of the operating point of the motor and the parameter of the controlled system change, on-line update of the compensation signal is necessary to reduce the vibration increased by those change.

In this paper, we propose a suppression control method for torque vibration of brushless DC motors utilizing the acceleration sensor, the Fourier Transformer and the repetitive controller. In the proposed system, only one frequency component of vibration signal from acceleration sensor is inputted to repetitive controller, the stability of the control system can be improved. Approximate analysis is performed to study the stability of the repetitive control system. In order to realize on-line generation of feedforward compensation signals to reduce the vibration, auto-tuning method of the repetitive control parameters is also presented. An experimental mechanical system to realize the mechanical vibration is constructed, and the effectiveness of the proposed method is confirmed by experimental results.

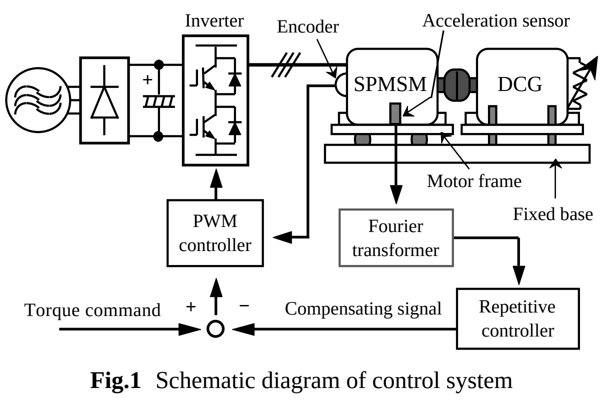


Fig.1 Schematic diagram of control system

2. Detection Method of Mechanical Vibration

2.1 Detection System of Mechanical Vibration

Fig.2 shows the detection system of the motor vibration. This system is the usual case, where the motor base and the load base are separated each other. Here, the motor base is supported by vibration proof rubbers and the load (DC generator) base is fixed by bolts.

The ripple torque is caused by the imperfectness of the motor, current sensors, driving source, etc. when the motor is rotating. The rotational speed of the brushless DC motor is fluctuated due to the ripple torque. At the same time, the ripple torque is applied to the motor frame by a reaction from the rotor and, as a result, the vibration of the motor frame occurs. Since the vibration is periodical in phase with the rotation of the motor, the repetitive control can be applied to the reduction of the vibration.

2.2 Extraction of Specific Component of Vibration by Fourier Transform

The periodical torque ripple occurs synchronously with the motor rotation. And also the repetitive controller has large loop gain (basically infinity) only for the fundamental repetitive frequency component and its harmonics. For the above reasons, the repetitive control system is effective to reduce the vibration due to periodical torque ripples.

However, because the vibration signals detected by the acceleration sensor contain various frequency components and the mechanical system around the motor and load has complicated resonant characteristics, we can not stabilize the repetitive control system and reduce the vibration by directly using the vibration signals from sensor [8]. Then, we investigate the method which only one frequency component of the vibration signal is extracted from the vibration signals detected by the acceleration sensor, and the repetitive control is performed for every frequency component of the vibration.

Generally, since the period of the vibration may not coincide with that of the power supply but may be multiple of that, we set the period of the vibration to T_r (= N_r T, $T = f^{-1}$, f : motor driving frequency). For example, in the case of N_r =2, the frequency components of the vibration are *0.5f*, *1.0f*, *1.5f*, ... and *0.5f* is fundamental wave component of the vibration. If the vibration signal α_S detected by the acceleration sensor is expressed as equation (1), the Fourier coefficients a_n, b_n are given equation (2) and (3), respectively, and a certain component of the vibration $\alpha_{S\,nf}(t)$ whose frequency is nf is given by equation (4), where, $n = m / N_r$ (m=1, 2, 3, ...), $\omega = 2\pi f$. Therefore, we can extract the specific component of the vibration from the vibration signal which contains various frequency components by equations (2), (3), (4).

$$\alpha_S(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_n \cos(n\omega t) + \sum_{m=1}^{\infty} b_n \sin(n\omega t) \tag{1}$$

$$a_n = \frac{2}{T_r}\int_0^{T_r} \alpha_S(t)\cos(n\omega t) \tag{2}$$

$$b_n = \frac{2}{T_r}\int_0^{T_r} \alpha_S(t)\sin(n\omega t) \tag{3}$$

$$\alpha_{S\,nf}(t) = a_n \cos(n\omega t) + b_n \sin(n\omega t) \tag{4}$$

In this research, the above-mentioned calculations on real time are realized by the digital signal processor (denoted by DSP in the following). In this paper, $\alpha_{S\,nf}(t)$ is called *nf* frequency component.

Fig.3 shows waveforms of the detected acceleration signal α_S and *1.5f* and *5f* frequency components extracted by Fourier transformation when the driving frequency of the motor f is 12.5Hz. From Fig.3, we can confirm that the only one component of the vibration is extracted from the vibration signal which contains various frequency components.

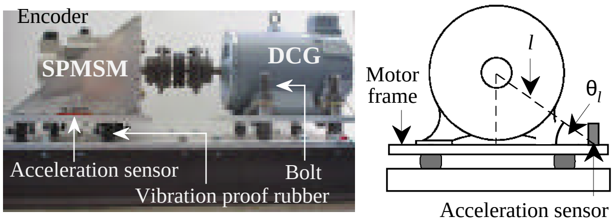


Fig.2 Detection system of motor frame vibration

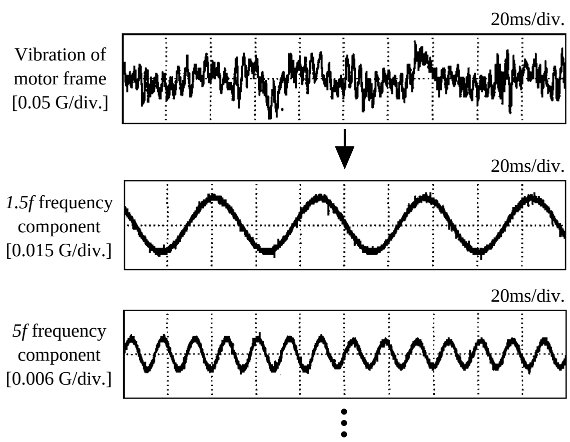


Fig.3 Extraction method of specific component of vibration by Fourier transform

3. Configuration of Control System

Fig.4 shows a block diagram of the proposed system. Here, d axis coincides with a direction of the magnetic flux produced by the field of SPMSM (Surface Permanent Magnet Synchronous Motor), and α_S is the mechanical vibration signal of the motor frame. The reduction control of the mechanical vibration is performed by the repetitive control system and/or the feedforward control system.

3.1 Dynamic Equations of SPMSM

Generally, in a d - q coordinate system, the dynamic equations of SPMSM, used as the brushless DC motor, are described equation (5). Here, the coordinates transformations from the (u, v, w) coordinate system to the (d, q) coordinate system are described as equation (7) by using the transformation matrix [c] expressed as equation (6).

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_a + pL_a & -\omega_{re}L_a \\ \omega_{re}L_a & R_a + pL_a \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{re}\Phi_f \end{bmatrix} \tag{5}$$

$$[c] = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos\theta_{re} & \sin\theta_{re} \\ -\sin\theta_{re} & \cos\theta_{re} \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \tag{6}$$

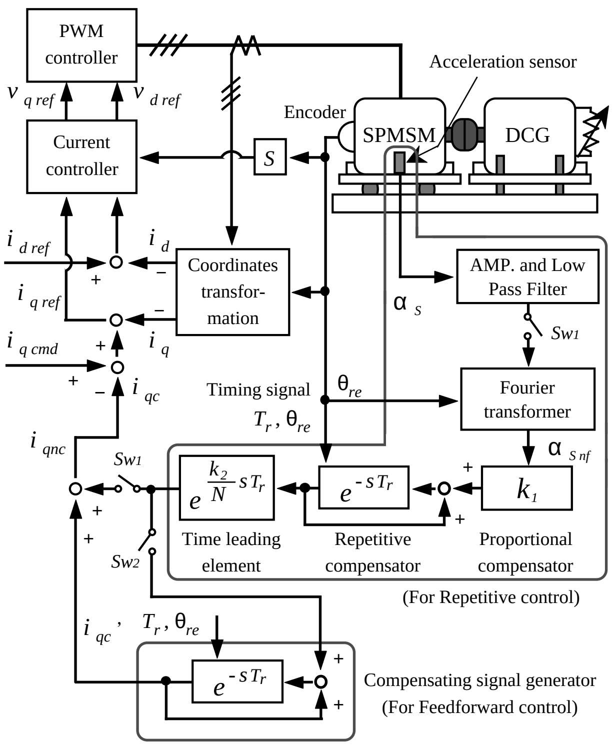


Fig.4 Configuration of control system

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} c \end{bmatrix} \begin{bmatrix} v_u \\ v_v \\ v_w \end{bmatrix}, \quad \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} c \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} \tag{7}$$

where i_d, i_q : d, q-axis components of armature current
v_d, v_q : d, q-axis components of armature voltage
R_a : armature winding resistance
L_a : armature winding self-inductance
ω_{re} : electrical angular speed of the rotor referred to the motor frame
Φ_f : maximum value of PM flux linkage
p : operator d/dt

Also, the generated torque τ_M of the brushless DC motor is expressed as follows:

$$\tau_M = \tau_{Mi} + \tau_{rip1} \tag{8}$$

$$\tau_{Mi} = n_p \Phi_f i_q \tag{9}$$

where τ_{Mi} : torque component proportional to i_q
τ_{rip1} : torque ripple component caused by the imperfectness of the motor
n_p : number of pole-pairs

3.2 Motor Equations of the Mechanical System

The mechanical system shown in Fig.2 is approximated by one resonance system in order to express the fundamental characteristics of the repetitive control system. In this case, motion equation of the rotor and the motor frame are expressed as follows:

$$\tau_{ML} \equiv \tau_M - \tau_L = J_{ML} p \omega_{re} \tag{10}$$

$$\omega_F = p\theta_F, \quad \alpha_F = p\omega_F, \quad \alpha_S = (l\cos\theta_l)\alpha_F \tag{11}$$

$$\tau_M = (J_F p^2 + D_F p + K_F)\theta_F \tag{12}$$

$$\omega_{re} = (\omega_{rm} + \omega_F) n_p \tag{13}$$

where τ_L : load torque (constant torque here)
ω_F, α_F : mechanical angular speed and acceleration of the motor frame, respectively
θ_F : mechanical rotational angle of the motor frame
l, θ_l : refer to Fig.2
J_F : total inertia moment of the motor frame
K_F, D_F : spring constant and viscosity coefficient of the mechanical system to support the motor frame, respectively
ω_{rm} : mechanical angular speed of the rotor referred to the motor frame

3.3 Current Control System

From equation (5), the motor currents (i_d, i_q) are under influence of the electromotive force $\omega_{re}\Phi_f$ and the reactance $\omega_{re}L_d$, $\omega_{re}L_q$. In order to control i_d and i_q exactly by removing these influences, we adopt the non-interference control [11]. The voltage command of the inverter is described as follows:

$$\begin{bmatrix} v_{uref} \\ v_{vref} \\ v_{wref} \end{bmatrix} = \begin{bmatrix} c \end{bmatrix}^T \begin{bmatrix} v_{dref} \\ v_{qref} \end{bmatrix} \tag{14}$$

$$v_{dref} = k_I (i_{dref} - i_d) - \omega_{re} L_a i_q \tag{15}$$

$$v_{qref} = k_I (i_{qref} - i_q) + \omega_{re} (\Phi_f + L_a i_d) \tag{16}$$

where k_I : proportional gain of the current controller

Here, the PWM inverter, which drives the SPMSM, is assumed also an ideal voltage source ($v_d = v_{dref}$, $v_q = v_{qref}$). Furthermore, if k_I is large, $i_d = i_{dref}$ and $i_q = i_{qref}$, and therefore τ_{Mi} is expressed as follows:

$$\tau_{Mi} = n_p \Phi_f i_{qref} + \tau_{rip2} \tag{17}$$

where τ_{rip2} : torque ripple caused by the imperfectness of the current controller

Finally, the generated torque of the brushless DC motor τ_M is expressed by:

$$\tau_M = n_p \Phi_f i_{qref} + \tau_{rip} \tag{18}$$

$$\tau_{rip} = \tau_{rip1} + \tau_{rip2} \tag{19}$$

3.4 Repetitive Control System and Compensation Signal Generator

Since the periodical vibration of the brushless DC motor in phase with the rotation of the motor is dealt with in this paper, the repetitive control can be applied to the reduction of the periodical vibration. In the repetitive controller in Fig.4, the compensation signal i_{qc} for the torque ripple τ_{rip} is produced from the vibration signal detected by the acceleration sensor attached to the motor frame during the repetitive control.

In the continuous system, the repetitive compensator is the compensator which adds an input from the sensor to its output which has only the cycle T_r time-lag behind the input and it has large loop gain (basically infinity) only for the fundamental repetitive frequency component and its harmonics. Therefore, it can reduce the error (input signal) to zero, and it has the memory characteristics to those components. From the above-mentioned memory characteristics, even when the input is set to zero (S_{w1} in Fig.4 : OFF), the compensation signal acquired during S_{w1} ON can be outputted continuously. Using these characteristics, the repetitive compensator (implemented with memories) is also used as the compensation signal generator, which stores the acquired compensation signal, and outputs it as the feedforward compensation signal as shown in Fig.4.

LPF is a low pass filter to cut-off the higher frequency sensor noise. The Fourier transformer is a kind of filter to cut-off all higher and lower order harmonics except a particular vibration frequency, the parameter k_1 is a proportional compensator for adjusting control loop gain, and the time leading element (the parameter: k_2) is that to compensate the phase lag in the loop. The parameter k_1 and k_2 are determined so that the repetitive control system becomes stable for the individual vibration frequency component.

3.5 Algorithm of Repetitive Control and Feedforward Compensation Control

The repetitive compensator is the compensator which adds the input signal to the output one which delays the input one by only one cycle T_r. In this research, in order to realize the repetitive compensator by DSP, we have to design it by the discrete system. Fig.5 shows the fundamental operation of the repetitive compensator and the time leading element in the discrete system. As shown in Fig.5, the repetitive compensator consists of N memories, and in the case the control period is T', and the relation of $T_r = N\,T'$ is derived. As this control system, in the case the period of the input signal (ripple torque) T_r synchronizes with the rotation speed of the brushless DC motor, the number of N also change with change of T_r, in the system fixed the control period T' inconveniently. Then, T' is allowed to change and the repetitive compensator with N memories in one period T_r is realized by synchronizing the control period T' with the rotation angle of the rotor θ_{re}. The input signal to the repetitive compensator (= the output signal from the proportional compensator) is averaged within the period of T' and added to the data in the corresponding memory. The output signal from the repetitive compensator is obtained by reading data form the memory before input signal is added to the data in the memory, and holding it for T'. On the other hand, the time leading operation of $(k_2/N)T_r$ is realized by reading data from the memory which leads

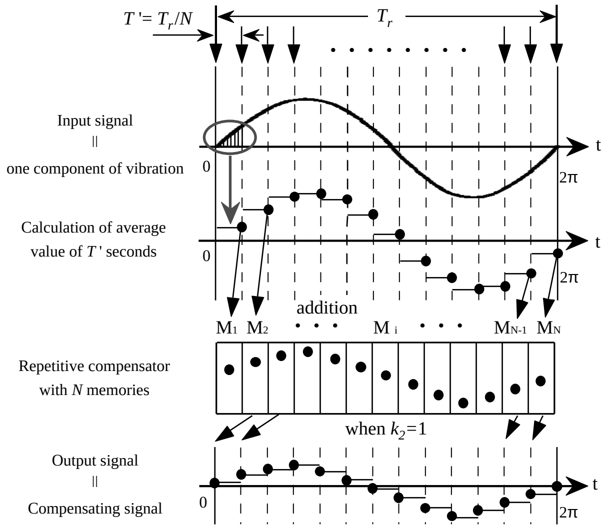


Fig.5 Fundamental operation of repetitive controller

by k_2 rather than the memory added the input signal.

In the proposed system, as the previous section explained, the repetitive compensator (implemented with memories) is also used as the compensation signal generator, which stores the learned compensation signal, and outputs it as feedforward compensation signal.

When the switch S_{w1} in Fig.4 closes, the repetitive control is carried out and at the same time the signal for the torque ripple compensation is renewed by the input signal and restored in the controller. When the selected frequency component of the vibration is reduced negligibly small, the switch S_{w1} opens and the signal input to the repetitive controller stops. After that instant, the open-loop feedforward control is carried out repetitively using the stored signal in the repetitive controller as a feedforward compensation signal. It should be noted that the repetitive control with the Fourier transformer could be carried out for one frequency component, even including the previously acquired compensation signals for the other frequency components.

After finishing compensation signal acquisition, this compensation signal are stored in the other memories when the switch S_{w1} opens and S_{w2} closes.

4. Stability of Repetitive Control System

The gain of the repetitive controller becomes infinity at multiple of the repetitive frequency. Therefore the repetitive control system may become unstable, i.e., the vibration diverges. Here, we examine the transient characteristics of the repetitive control system and describe how to determine the parameters k_1 and k_2.

4.1 System Equations and Block Diagrams

From the torque equations (8) and (18) and the motion equations of the mechanical system (11) and (12), a block diagram of repetitive control system as shown in Fig.6. Here, the ripple torque ΔT_{rip} is regarded as a disturbance, and the Fourier transformer is not included. The transfer functions shown in Fig.6 are expressed as follows:

$$G_1(s) = \frac{\Delta T_{Mi}}{\Delta I_{qref}} = n_p \Phi_f \tag{20}$$

$$G_2(s) = \frac{\Delta\Omega_F}{\Delta T_M} = \frac{s}{J_F s^2 + D_F s + K_F} \tag{21}$$

$$G_3(s) = \frac{\Delta A_S}{\Delta A_F} = \frac{l\cos\theta_l}{9.8} \tag{22}$$

4.2 System Stability

With respect to ΔA_S, the block diagram shown in Fig.6 can be transformed to that shown in Fig.7. The transfer functions shown in Fig.7 are expressed as follows:

$$P(s) = G_1(s)G_2(s)G_3(s)G_{LPF}(s) \tag{23}$$

$$C(s) = k_1 \; e^{\frac{k_2}{N}sT_r} \tag{24}$$

$$G(s) = G_2(s)\,G_3(s)\,s \tag{25}$$

where $P(s)$: transfer function of motor and load (from Δi_{qref} to ΔA_S')

$C(s)$: transfer function of proportional compensator and time leading element

$G(s)$: transfer function from ΔT_{rip} to ΔA_S'

$G_{LPF}(s)$: transfer function of a noise filter (low pass filter)

The stability of the repetitive control system is determined by the frequency response of the transfer function 1-$C(s)P(s)$ in Fig.7. According to the small gain theory [12], the following condition must be satisfied in order to stabilize the system.

$$\left|1 - C(j\omega)P(j\omega)\right| < 1 \qquad \omega \tag{26}$$

Equation (26) means that all points on the Nyquist loci of 1-$C(j\omega)P(j\omega)$ must stay inside the unit circle for stable operation, but impossible. On the other hand, because the frequency region of the vibration signal is limited by the Fourier transformer before inputted to the repetitive controller in the proposed system, the stability criterion of the system can be expressed by

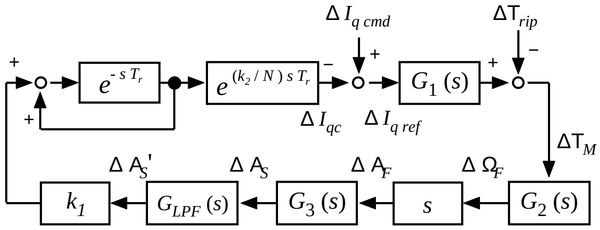


Fig.6 Block diagram of linearized system

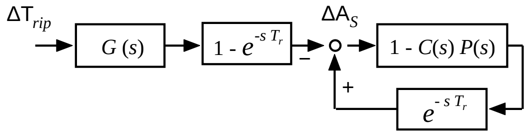


Fig.7 Transformed equivalent block diagram

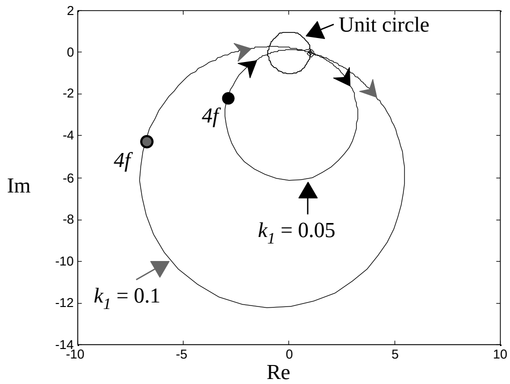
$$\left|1 - C(j\omega_n)P(j\omega_n)\right| < 1 \tag{27}$$

where ω_n: angular frequency of vibration extracted by the Fourier transformer

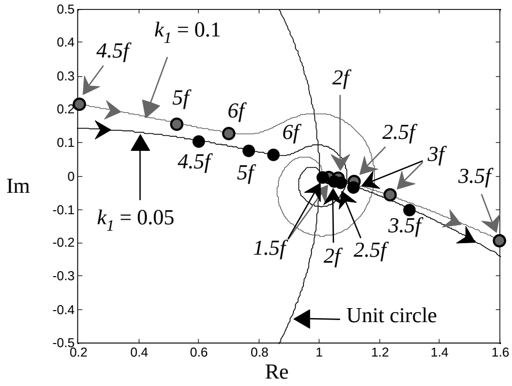
Fig.8 shows the Nyquist loci of 1-$C(j\omega)P(j\omega)$ and the unit circle in the case without the time leading element (k_2=0). Here, the marked point on the loci labeled *nf* is called Nyquist point of ω_n (=2π*nf*). System parameters for analysis are shown in Table 1. When the frequency is increased from zero to infinite, the Nyquist loci starts from a point *1 + j0*, and it returns to the same point due to the high-pass nature of the acceleration sensor and the low-pass one of the mechanical system after drawing a large circle corresponding to the vibration mode (near the natural frequency of the mechanical system : resonance point). In the proposed system, if only Nyquist point of ω_n extracted by the Fourier transformer stays inside the unit circle, the stability of the repetitive control system is guaranteed.

Most Nyquist points (operating points) other than the resonance point exist near the point *1 + j0*. We can see in Fig.8 that if the value of k_1 is large, a loci of vector will be expanded centering on the point *1 + j0*. In the case operating points exist inside the unit circle, so the operating points can be then apart from the circumference of the unit circle, convergence speed of the vibration can be made quick (for example, *4.5f*, *5f*, *6f* in Fig.8). On the other hand, in the case operating points exist outside the unit circle, the operating points are apart from the unit circle greatly and divergence speed of the vibration will become quick (for example, *3.5f*, *4f* in Fig.8). Therefore, it is advisable to set small value on k_1, unless it causes inconvenience for the convergency of the vibration.

As mentioned above, in the case operating points exist outside the unit circle, stabilization of the repetitive con-

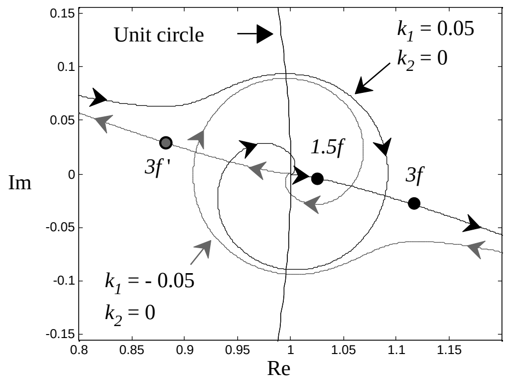


(a) Overall Nyquist loci (k_2 = 0)

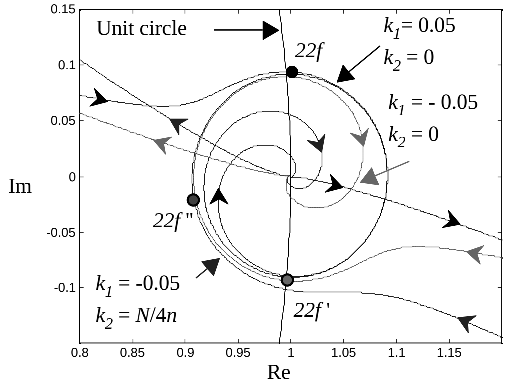


(b) Expanded loci of (a) around (1+j 0) (k_2 = 0)

Fig.8 Nyquist loci of transfer function (1-$C(s)P(s)$) : f =14Hz (Without time leading compensation)



(a) When operating point exists out of unit circle (case 1)



(b) When operating point exists out of unit circle (case 2)

Fig.9 Nyquist loci of transfer function (1-$C(s)P(s)$) : f =14Hz (With time leading compensation)

Table 1 System parameters

SPMSM	
Rated voltage : 80[V]	Rated current : 7.2[A]
Rated torque : 1.27[N m]	
R_a : 0.8[Ω]	L_a : 0.92mH
Φ_f : 0.051[Vsec/rad]	n_p : 2[pole pairs]
Load : DC Generator	
Rated voltage : 145[V]	Rated current : 11.9[A]
Rated torque : 8.2[N m]	
Motor + Load	
J_F =8.06×10^{-2}[kg m^2/rad]	
D_F =4.10×10^{-1}[N m sec/rad]	
K_F =9.84×10^{3}[N m/rad]	
f_0 =55[Hz] (Natural frequency of mechanical system)	
LPF : 8th order Butterworth low pass filter	
Cut off frequency : 1[kHz]	

trol system using only the proportional compensator is impossible. Then, we think that the operating point can be moved into the unit circle by the time leading compensation (k_2) or reverse of the sign of k_1 as shown in Fig.9. *3f* and *22f*, *3f* ' and *22f* " in Fig.9 denote the operating points without and with the phase control, respectively. In the case the operating point is at the place denoted by *3f* as shown in Fig.9(a), it can be moved into the unit circle by reversing the phase (by setting the sign of k_1 is negative) (*3f* → *3f* '). But, in the case the operating point is at the place by *22f* as shown in Fig.9(b), it cannot be moved into the unit circle by only reversing the sign of k_1 (*22f* → *22f* '). In such a case, the operating point can be moved into the unit circle by compensating only 90 degrees phase (by using the time leading compensator k_2) (*22f* ' → *22f* "). Here, the value of k_2 is a natural number corresponding to about 90 degrees phase of *nf* frequency component.

From the above results, for the selected frequency component of the vibration, which we want to reduce, we can always adjust the control parameters k_1 and k_2 so that the selected component stays inside the unit circle. As a result, the control system can be always stabilized. Here, although we examined the stability of the system based on the control model shown in Fig.6, theoretically, we think that the same method can also apply to any motors and mechanical system. However, in the case the operating point exist near the resonance point of the mechanical system, which shows sharp resonance characteristic, like *4f* frequency component shown in Fig.8 (a), so the gain of the transfer function 1-$C(s)P(s)$ is very high, it is necessary to set the value of k_1 quite small. Furthermore, when the operating point moves considerably on the locus in Fig.8(a) for the slight change of the vibration frequency by change of the rotational speed of the motor, it is difficult to guarantee the stability of the repetitive control system by adjustment of the above-mentioned phase compensation. In this case, it is desirable to perform the feedforward compensation control using the data obtained by the repetitive control around the resonance point. But, it is rare case that the vibration frequency coincides with one of the steep resonance point.

4.3 Auto-tuning Method of Control Parameters

As mentioned in the previous section, the stability condition of the repetitive control system is to satisfy the inequation (27), and if the operating point of ω_n exists in the unit circle, the stability of the system will be guaranteed. However, since the vibration signal caused by the ripple torque of the brushless DC motor include many components caused by the imperfectness of the motor, the repetitive control system may become unstable with the control parameters obtained by the approximate system modeling. Also, from the analysis results in the previous section, it is confirmed that a set of the control parameters which makes the repetitive control system stable certainly exists among the four kinds of parameters. Therefore, here, we propose the auto-tuning method of the repetitive control parameters (k_1, k_2) based on the approximate analysis. Fig.10 shows a flow chart of the auto-tuning algorithm of the parameters.

where A_n : amplitude of $\alpha_{S\,nf}$

Max, *Min* : threshold value to determine whether A_n diverges or converges

delay : time delay

N : number of memories of repetitive compensator (= number of partition of vibration signal)

$N/4n$: a natural number corresponding to about 90 degrees phase of nf frequency component of vibration

In the first place, the value of k is set so that all operating points of ω_n except sharp resonance point can be put inside the unit circle by appropriate selection of the values of k_1 and k_2 from {-k, k} and {0, $N/4n$}, respectively. If the switch S_{w1} in Fig.4 closes, the repetitive control is carried out using the initial parameters $k_1 = k$ and $k_2 = 0$. If the initial parameters are suitable for selected frequency component of the vibration $\alpha_{S\,nf}$, the vibration signal is reduced. On the other hand, if the initial parameters are not suitable for it, the vibration signal either tends to diverge or does not change so much. In the latter case, the parameters should be changed. Then, in the case the vibration signal tends to diverge and A_n exceeds the threshold value *Max* (i.e., $A_n \geq Max$), the learned data in a memory of the repetitive controller is cleared and the compensation signal is relearned. On the other hand, in the case the vibration signal does not change so much (i.e., $A_n \geq Min$), the phase of the learned compensation signal is changed 90 degrees. Thus, in the proposed auto-tuning method, the parameters of the repetitive control are changed according to the change of A_n.

It can also be confirmed from Fig.9 that, even when the characteristics of the motor and mechanical system are not enough grasped, it can be expected to put the operating point of ω_n in the unit circle, and to stabilize the repetitive control system according to the proposed

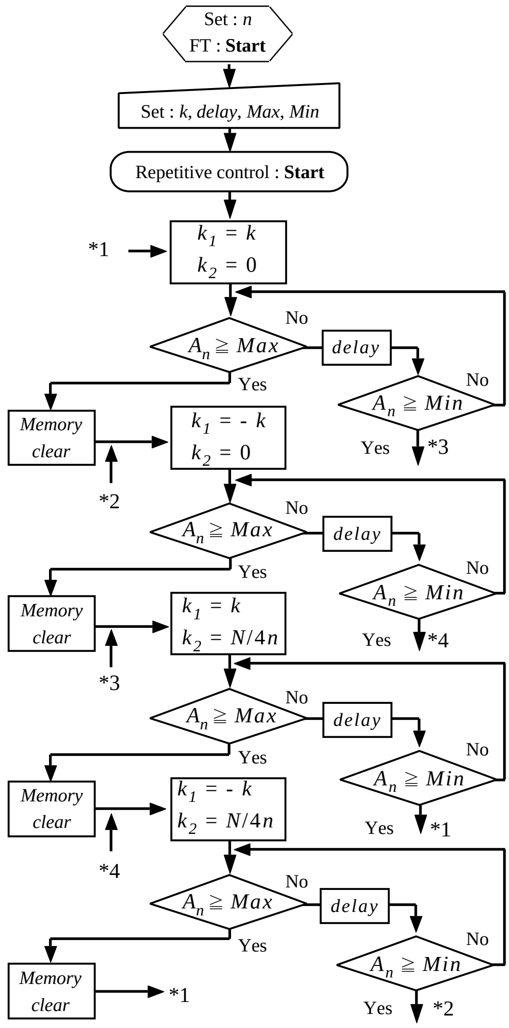


Fig.10 Flow chart of auto-tuning of parameters k_1, k_2

method In addition, in the case the operating point exist near the sharp resonance point of the mechanical system, the vibration signal diverge in any loop of the proposed algorithm are brought, therefore k_1 must be set small. However, in that case, k_1 reduces to zero and as a result it is almost same to not learning the compensation signal, the feedforward compensation control using the data obtained by the repetitive control around the resonance point must be carried out.

5. Experimental Results

In experiment, we use the mechanical system shown in Fig.2, the control system in Fig.11 and the system parameters in Table2. Here, T_c is the period of the inverter and T_s is that of the speed control. And we use the encoder, in order to detect the motor magnetic pole position information which is necessary for the Fourier transformation, the motor driving and the repetitive control.

Fig.12 shows the vibration waveform when the brushless DC motor drive, and its FFT spectrum. From Fig.12, the mechanical vibration (acceleration) contains various frequency components. It is considered to be generated by mutual interference among the motor structure, the harmonics currents caused by the imperfectness of the current controller, the mechanical characteristics of the experimental system and so on. Especially, *1.5f* and *3f* frequency components are considered to be amplified by the mechanical resonance phenomenon.

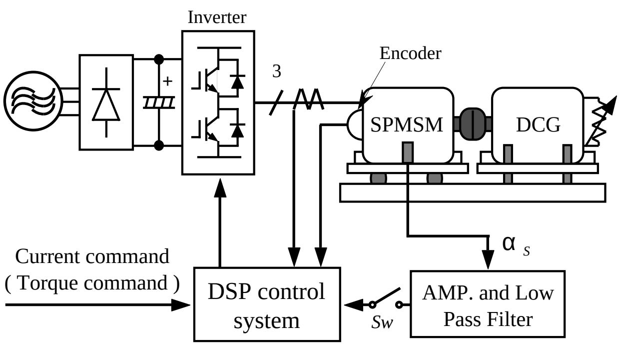


Fig.11 Configuration of experimental system

Table 2 Parameters for experimental system

INV. source : 120[V], T_c =100[μs]
Current controller
$i_{q\,cmd}$ = 4.6[A], $i_{d\,ref}$ = 0.0[A], f = 14[Hz], k_I = 2.4
Repetitive controller
Final values of parameters by auto-tuning algorithm

$1.5f : k_1 = k\ ,\ k_2 = N/4n$	$2f : k_1 = -k\ ,\ k_2 = 0$
$2.5f : k_1 = -k\ ,\ k_2 = 0$	$3f : k_1 = -k\ ,\ k_2 = 0$
$3.5f : k_1 = -k\ ,\ k_2 = N/4n$	$4f : k_1 = -k\ ,\ k_2 = N/4n$
$4.5f : k_1 = k\ ,\ k_2 = 0$	$5f : k_1 = k\ ,\ k_2 = 0$
$6f : k_1 = k\ ,\ k_2 = N/4n$	

Initial values
Memory : N =420 k = 0.05[A/G] *delay* : 15[sec]
Speed controller
Rotary encoder : 6000[pulses/rotate]
T_s =400[μs]
Resolution of A-D converter : 12[bit]
Input : -0.51 ~ +0.51[G] , ±1.25×10^{-4}[G/resolution]

Fig.13 shows an example of the auto-tuning process of the parameters for *3.5f* frequency component.

#1 : repetitive control (compensation signal learning) started ; $k_1 = k$, $k_2 = 0$
#1 - #2 : *delay* → vibration did not change so much → repetitive control stopped
#2 : changed of parameter to adjust 90 degrees phase shift of compensation signal → repetitive control restarted ; $k_1 = k$, $k_2 = N/4n$
#2 - #3 : vibration diverged (selected parameters at #2 were not suitable) → repetitive control stopped
#3 : *memory clear* → changed of parameters → repetitive control restarted ; $k_1 = -k$, $k_2 = N/4n$
#4 : vibration reduced (suitable parameters were selected → repetitive control continued

The auto-tuning algorithm of the control parameters for each vibration frequency components was carried out by experiment. In Table2, the final values of k_1 and k_2 determined by the auto-tuning algorithm are shown. From these results shown in Table 2, the parameters of *1.5f* frequency component differs from the value esti-

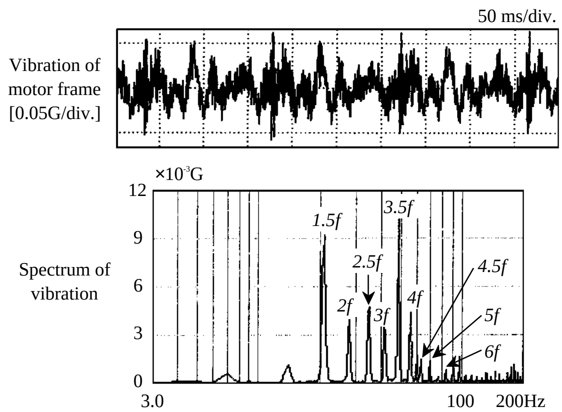


Fig.12 Vibration of motor frame

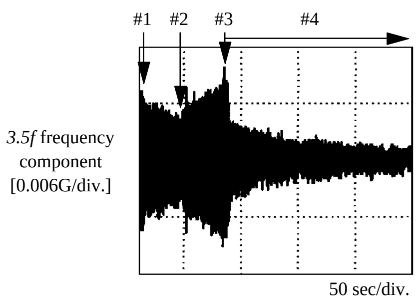
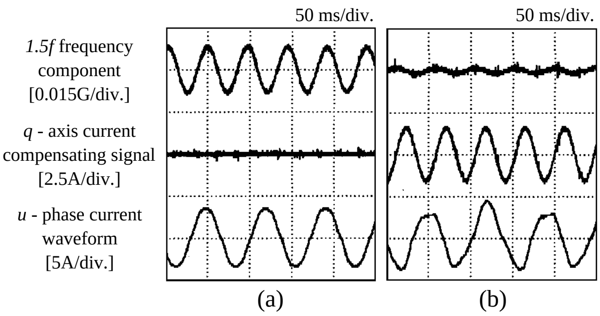


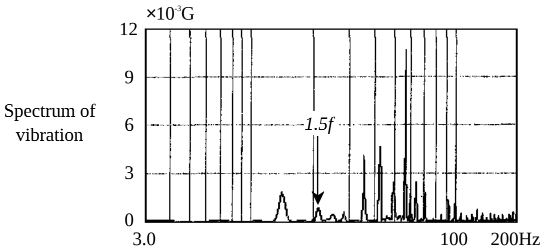
Fig.13 Auto-tuning process of repetitive control parameters

mated from Fig.9(a). Although it is considered that the resonance mode omitted by approximation of equation (12) has influenced near *1.5f* frequency component, and the characteristics of the mechanical system are not enough grasped in this experiment, we confirmed that stable vibration suppression control was realized by proposed auto-tuning algorithm of the control parameters. Fig.14 shows experimental results of the repetitive control for *1.5f* frequency component. Fig.14(a) and (b) show the case without and with the repetitive control, respectively. Fig.14(b) shows FFT spectrum of the motor frame vibration with the repetitive control. From Fig.14, only *1.5f* frequency component.

Next, we carried out the auto-tuning of the parameter and the repetitive control for *1.5f* to *6f* frequency components, respectively. And the compensation signals learned in each operations were compounded to obtain the compensation signal for feedforward compensation control in order to reduce from *1.5f* to *6f* frequency components simultaneously. Fig.15 shows experimental results of the feedforward compensation control using the compounded compensation signal by open-loop (in Fig.11 : S_w ; OFF) for the brushless DC motor on-load in steady state. Fig.15(a) shows the case without the repetitive control nor feedforward compensation control. Fig.15(b) shows the final result of the vibration suppression control where the repetitive control is performed for each of *1.5f, 2.5f, 3f, 3.5f, 4f, 4.5f, 5f, 6f* frequency components one by one with the acquired compensation data

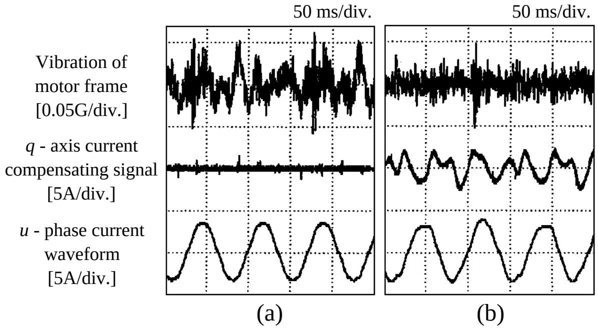


(a) Without vibration suppression control
(b) With vibration suppression control (*1.5f* only)

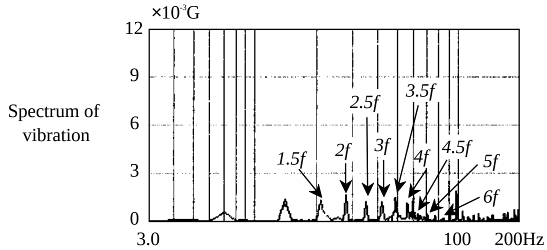


(c) FFT spectrum of vibration in the case of (b)

Fig.14 Experimental results for the repetitive control



(a) Without vibration suppression control
(b) With vibration suppression control (*1.5f*, *2f*, *2.5f*, *3f*, *3.5f*, *4f*, *4.5f*, *5f*, *6f* only)



(c) FFT spectrum of vibration with vibration suppression control

Fig.15 Experimental results for repetitive control and feedforward control

employed as the feedforward control. Fig.15(c) shows FFT spectrum of the motor frame vibration with the feedforward compensation control. From Fig.15, we can consider that the *1.5f, 2.5f, 3f, 3.5f, 4f, 4.5f, 5f, 6f* frequency components are considerably reduced.

Table 3 shows the vibration suppression control effect each frequency components of the vibration. The values in ( ) show the equivalent ripple torque $\tau_{n\ fe}$ converted from the acceleration $\alpha_{S\ nf}$ by equation (29), where T_n is amplitude of $\tau_{n\ fe}$, and expressed the amplitude T_n as a rate to the steady state torque T_0 in order to know the rough magnitude of the ripple torque. Here, natural (resonance) frequency of the mechanical system decided by J_F and K_F is set to f_0, in the domain of $f >> f_0$, so equation (25) turn into equation (28), the amplitude of the equivalent ripple torque T_n will be converted by equation (29).

$$G(s) = G_2(s)G_3(s)s \quad \frac{l\cos\theta_l}{9.8\ \ J_F} \qquad \because f \quad f_0 = \sqrt{\frac{K_F}{J_F}}/2\pi \tag{28}$$

$$T_n = \frac{9.8\ \ J_F}{l\cos\theta_l} A_n \tag{29}$$

In addition, the equivalent ripple torques may be seem-

Table 3 Experimental results

nf	without vibration suppression control		with vibration suppression control	
	A_n [×10^{-3}G]	T_n / T_0 ×100[%]	A_n [×10^{-3}G]	T_n / T_0 ×100[%]
1.5f	9.34	(11.9)	1.28	(1.63)
2f	3.98	(5.06)	1.58	(2.01)
2.5f	4.74	(6.03)	1.17	(1.49)
3f	3.52	(4.48)	1.17	(1.49)
3.5f	10.5	(13.4)	1.48	(1.88)
4f	4.44	(5.65)	1.07	(1.36)
4.5f	1.43	(1.82)	0.459	(0.57)
5f	1.33	(1.69)	0.408	(0.52)
6f	0.765	(0.97)	0.204	(0.26)

[steady state torque ：T_0 = 0.54N m] [rated torque : 1.27N m]
[T_n / A_n = 6.87 N m / G] by (28)

ingly larger than actual ones for mechanical resonance. From experimental results shown Fig.15 and Table 3, we can confirm usefulness of proposed method.

6. Conclusion

We proposed a suppression control method of the torque vibration of the brushless DC motors utilizing feedforward compensation control, and a generation method of compensation signals for the feedforward control by the repetitive control system with the Fourier transformer utilizing a vibration signal acquired by an acceleration sensor attached to the motor frame. Usefulness of proposed method was confirmed by the experimental results.

The results are summarized as follows:

(1) We showed that the stable vibration suppression control system was realized for various frequency components of the vibration by incorporating the Fourier transformer into the repetitive control system.

(2) We showed that stable repetitive control always could carry out, even when the characteristics of the mechanical system are not enough grasped, by applying the auto-tuning algorithm for the repetitive control parameters. Also, the control parameters can be changed automatically for change of the motor operating point or the controlled system by this proposed method, and the repetitive control system can behave stably.

(3) We showed that two or more vibration frequency components can be simultaneously suppressed by the feedforward compensation control using compensation signals generated by the repetitive control.

(4) Examples of application of the proposed control method is shown below:

(i) In the case that the changes in the motor parameters and mechanical characteristics are small, the compensation signals for the feedforward compensation control are learned or updated at the factory shipments or regular maintenance.

(ii) In the case that the characteristics of the motor and the mechanical system depend on the environment considerably, the acceleration sensor is incorporated into the motor control system and the feedforward compensation control using the compensation signals learned before is carried out monitoring the change of the vibration. However, when the compensation signal becomes inadequate and some frequency components of the vibration becomes large by the characteristic change etc., the repetitive control is carried out to update the compensation signal for the frequency components.

In this paper the proposed methods were examined in the case of low speed drive of the brushless DC motor. However, for the higher order frequency components of the vibration and those near the sharp resonance frequency of the mechanical system, the effectiveness of the proposed vibration suppression method was small. This problem will be investigated further.

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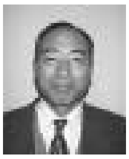

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Reprinted from Trans. of the SICE, Vol.36 No.5, 438/447 (2000)