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New Direct Closed-Loop Identification Method for Unstable Systems and Its Application to Magnetic Suspension System

Lianming SUN*, Yasuhiro MIYAKE**,
Hiromitsu OHMORI*** and Akira SANO***

Closed-loop identification is essentially needed in control of an unstable plant stabilized by a feedback controller. A new direct closed-loop identification approach is proposed based on an output inter-sampling scheme, in which by taking faster sampling of the output than the input of the system the restrictive identifiability condition is removed. In a case of unstable system, since the noise model becomes non-minimum phase system, the prediction error method is not available. In order to solve these problems we clarify that the output inter-sampling yields the SIMO structure of the plant model and give a new identification method in which the denominator polynomial can be identified in an open-loop manner and the numerator polynomial can also be obtained by a modified instrumental variable method. The proposed algorithm is validated in an experimental study using a magnetic suspension system.

Key Words: system identification, closed-loop identification, unstable system, multi-rate sampling, magnetic suspension system, identifiability.

1. Introduction

Unstable plants such as magnetic suspension systems are always stabilized by feedback controller so it is of great interest to identify model parameters of the plants operating in closed-loop. Conventional approaches to closed-loop identification can be categorized into 3 groups [1),2)]: (a) direct method, (b) indirect method and (c) joint input-output method. The direct methods use the control input and plant output directly to identify the plant model as if it were operating in open loop, while the indirect methods obtain the estimation of the closed-loop system regarding the externally exciting test signal as input first then determine the plant model from the estimate and the known controller. On the other hand, the joint input-output methods regard the control input and plant output as the outputs of a multivariable innovation system. Some approaches of these three categories could deal with closed-loop identification problem of unstable plants, nevertheless most of them are indirect ones [3)∼7)], and except the controller-switching approach [8)] few direct algorithm is reported. Obviously the indirect and joint input-output methods require that the controller be linear to calculate the transfer function of unknown plant model. Although it is a very simple approach that identifies the plant model only from the control input and plant output no matter whether the controller is nonlinear, the effective direct method for unstable plant has not been developed yet. In practical applications, when the plant is nonlinear, an appropriate nonlinear controller is often required for plant model linearization or estimation of plant physical parameters, so it is very attractive if the direct methods can work for unstable plant identification. Then, the main objective of this work is to develop an effective direct approach to identify unstable plant operating in closed-loop.

Compared with open loop identification, two serious problems must be considered in closed-loop identification. One problem is the bias caused by the correlation of control input and the output noise in closed-loop systems, the other one is the deficient data rank due to the feedback loop. They can be avoided by some specified identifiability conditions [9),10)], in which the orders of controller are higher than that of the transfer function of plant model. Nevertheless, the controller is usually designed using a reduced low order plant nominal model, which may conflict to the identifiability conditions. On the other hand, the prediction error method (PEM) or maximum likelihood method (ML) requires that the noise model should be a minimum phase model. Thus the conventional direct method that uses PEM or ML cannot be applied to unstable plant identification directly since the noise model be-

* Faculty of Environmental Engineering, The University of Kitakyushu, 1-1 Hibikino, Wakamatst-ku, Kitakyushu 808-0135, Japan
** Hachioji Semiconductor Test Division, Agilent Technologies Japan, Ltd., 9-1,Takakura-Cho,Hachioji-Shi,Tokyo, 192-8510 Japan
*** Faculty of Science and Technology, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan

comes to a non-minimum phase model. Furthermore the cancellation of zeros and unstable poles occurs in unstable plant identification, and such difficulties require some special techniques to be handled.

The identifiability of direct closed-loop identification approach based on output inter-sampling scheme has been investigated [11], however, the unstable plant case has not been discussed in detail yet. In this paper, the output inter-sampling based direct closed-loop identification approach is presented for unstable plant identification. The new approach is a two-stage algorithm. The first stage utilizes subspace-based methods to estimate the denominator polynomial of the unstable plant model by making use of the particular inter-sampled model structure. Then in the second stage the specified instrumental variable is generated to estimate the numerator polynomial even though the noise model is a non-minimum phase model. Furthermore for the magnetic suspension system a more convenient approach to numerator identification is given by making use of the physical model structure. The identifiability of the proposed identification approach is also investigated. Finally the effectiveness of the proposed approach for unstable plant identification is demonstrated through identification experiments on the magnetic suspension system.

2. Closed-Loop Identification Problem of Unstable Plant

Consider a continuous time unstable plant operating in closed-loop. The plant is stabilized by a discrete-time controller, which adds the control input to plant through a zero-order holder with holding period T. Then the identification problem of the closed-loop can be considered in discrete-time formulation, as illustrated in **Fig. 1**. Here $r(mT)$, $u(mT)$, $y(mT)$ are the samples of reference, plant input, i.e. control input, plant output at instant mT respectively, and are denoted as $r(m)$, $u(m)$, $y(m)$ for the simplicity of notation in the following discussion. In order to illustrated the effectiveness of the proposed method, the severe case where the reference signal $r(mT) = 0$ is considered. $w(mT)$ is a white output noise with zero-mean and variance σ_w^2. The plant model corresponding to the sampling interval T is denoted as T-model, which is given by

$$\frac{B(z^{-1})}{A(z^{-1})} = \frac{b_{\tau_T} z^{-\tau_T} + \cdots + b_{n_b} z^{-n_b}}{1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a}} \tag{1a}$$

$$\frac{D(z^{-1})}{C(z^{-1})} = \frac{d_0 + d_1 z^{-1} + \cdots + d_{n_d} z^{-n_d}}{1 + c_1 z^{-1} + \cdots + c_{n_c} z^{-n_c}} \tag{1b}$$

where $z^{-1} = e^{-sT}$, τ_T is the delay time and $\tau_T \geq 1$. The case in presence of input noise $s(m)$ can also be coped with in almost the same methodology, however we will consider the case where $s(m) = 0$ for the purpose of notation simplicity.

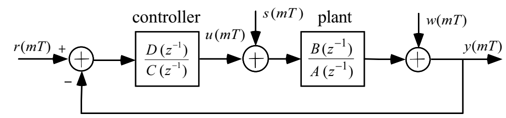


Fig. 1 Discrete-time closed-loop control system

Under these configurations, the following identifiability conditions

$$n_c > n_b - \tau_T, \quad \text{or} \quad n_d > n_a - \tau_T \tag{2}$$

are required for the full rankness of observation data matrix in the conventional equation error model or subspace based direct methods [9) 12)], which use the plant input and output signals $\{u(m),\ y(m)\}$. In practical application, however, the identifiability conditions are almost not satisfied due to the controller is usually designed by using a order-reduced nominal plant model so the controller order may be lower than that of the real plant model.

On the other hand, when using PEM for the unstable plant in Fig. 1, the plant output $y(m)$ becomes to

$$y(m) = \frac{D(z^{-1})B(z^{-1})}{C(z^{-1})A(z^{-1}) + D(z^{-1})B(z^{-1})} r(m) + \frac{C(z^{-1})A(z^{-1})}{C(z^{-1})A(z^{-1}) + D(z^{-1})B(z^{-1})} w(m) \tag{3}$$

then the noise model is a non-minimum phase model since it has the unstable poles of the plant model. Consequently it requires special technique even in indirect methods using PEM algorithm, and results in the computational complicity consequently. The problem of cancellation of zeros and unstable poles, which may result in the instability of the identification algorithm, also occurs in the conventional direct methods for unstable plant.

Therefore the main objective of this work is to overcome these severe problems of the closed-loop identification for unstable plants.

3. Implementation and Property of Output Inter-Sampling

The implementation of output inter-sampling is illustrated in **Fig. 2**, where the plant output is sampled at p times shorter interval $\Delta = T/p$ than the control period T. Here $p \geq 2$, p is denoted as inter-sampling rate, and the inter-sampled output signal is denoted as $y(k\Delta)$ or $y(k)$. When the output sampling is synchronized with the feedback controller, the plant samples are added to the controller at every $p\Delta$ like that in Fig. 1, then the output inter-sampling does not affect the controller design at all. In the output inter-sampling scheme, the control input is held for $k \in [mp, (m+1)p)$, then $u(k\Delta) = u(mT) = u(m)$ holds. Compared with the model in Fig. 1, the Δ-model is given by

$$\frac{B_\Delta(q^{-1})}{A_\Delta(q^{-1})} = \frac{b_{\Delta,\tau_\Delta} q^{-\tau_\Delta} + \cdots + b_{\Delta,n_{b\Delta}} q^{-n_{b\Delta}}}{1 + a_{\Delta,1} q^{-1} + \cdots + a_{\Delta,n_a} q^{-n_a}} \tag{4}$$

where $q^{-1} = e^{-\Delta s}$. τ_Δ is the delay time and $\tau_\Delta = p(\tau_T - 1) + 1$. The output noise in Δ-model becomes to $w(k\Delta)$, where $k\Delta$ is the sampling instant. Holding the samples of plant output for $p\Delta$ has the same effects on identification problem.

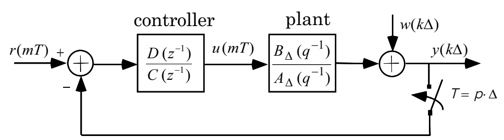


Fig. 2 Closed-loop system with output inter-sampling

As illustrated in [11], the relation between T-model and Δ-model is summarized in Theorem 1.

Theorem 1. Provide that the plant input is held for T. $A_\Delta(q^{-1})$ and $B_\Delta(q^{-1})$ are the denominator and numerator polynomials of the plant Δ-model, while $A(z^{-1})$ and $B(z^{-1})$ are those of the corresponding T-model. Then the parameters of $A(z^{-1})$ and $B(z^{-1})$ can be given by

$$A(z^{-1}) = \det(\boldsymbol{I} - \boldsymbol{\Lambda}^p z^{-1}) \tag{5a}$$

$$B(z^{-1}) = z^{-\frac{\tau_\Delta - 1}{p} - 1} \boldsymbol{\psi} \cdot \mathrm{adj}(\boldsymbol{I} - \boldsymbol{\Lambda}^p z^{-1}) \sum_{i=0}^{p-1} \boldsymbol{\Lambda}^i \boldsymbol{\varphi} \tag{5b}$$

where $\boldsymbol{\Lambda}$, $\boldsymbol{\varphi}$, $\boldsymbol{\psi}$ are the state space realizations of Δ-model.

Following Theorem 1, once $A_\Delta(q^{-1})$ and $B_\Delta(q^{-1})$ are identified, the estimates of $A(z^{-1})$ and $B(z^{-1})$ can be determined uniquely. It has also been demonstrated that the severe identifiability conditions are relieved greatly in the new direct closed loop identification approach based on Theorem 1 [11].

In this work another distinctive feature of the output inter-sampling will be utilized in the closed-loop identification for unstable plant. As illustrated in **Fig. 3**, the inter-sampled plant model can be described by a single-input multi-output model with common denominator polynomial and different numerator polynomials in the transfer function. Furthermore, the numerator polynomials can be given by Theorem 2.

Theorem 2. The SISO Δ-model, which is described in the formulation of sampling interval Δ, can be converted into a SIMO model structure described by sampling interval T, as illustrated in Fig. 3. The numerator polynomial $B_j(z^{-1})$ of the subsystem with output $y(mT + j\Delta)$ is given by

$$B_j(z^{-1}) = z^{-\frac{\tau_\Delta - 1}{p}} \boldsymbol{\psi} \cdot \mathrm{adj}(\boldsymbol{I} - \boldsymbol{\Lambda}^p z^{-1}) \cdot \left(\sum_{i=0}^{j-1} \boldsymbol{\Lambda}^i \boldsymbol{\varphi} + \sum_{i=j}^{p-1} \boldsymbol{\Lambda}^i \boldsymbol{\varphi} z^{-1} \right) \tag{6a}$$

$$B_0(z^{-1}) = B(z^{-1}), \quad j = 0 \tag{6b}$$

where $j = 0, \cdots, p-1$.

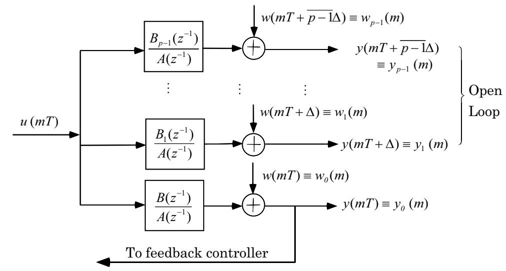


Fig. 3 SIMO expression of output inter-sampled plant model

4. Closed-Loop Identification Algorithm for Unstable Plant

4.1 Basic Idea of Identification Algorithm

Following Theorem 2, the transfer function of subsystem, whose output is $y(mT + j\Delta)$, is described as follows.

$$y_j(m) = \frac{B_j(z^{-1})}{A(z^{-1})} u(m) + w_j(m) \tag{7}$$

where $y_j(m) = y(mT + j\Delta)$, $w_j(m) = w(mT + j\Delta)$. Next we will use the observation data $\{u(m), y_j(m);\ j = 0, 1,\ \ldots,\ p-1; m = 1, 2,\ \ldots\ M\}$ within period MT for identification. Here the noise $w_j(m)$ is a white i.i.d noise with zero-mean and finite 4-th order moment, and is independent of the reference $r(m)$.

Note that the inter-sampled signal $y_j(m)$ is the noise $w_j(m)$ corrupted output of subsystem $B_j(z^{-1})/A(z^{-1})$ in Fig. 3. On the other hand, from the assumptions on noise, $w_j(\cdot)$ $(j = 1, \cdots, p-1)$ is independent of $w_0(\cdot)$ and $r(m)$. Furthermore, only the output of last subsystem is added into the controller, whereas the rest outputs for $j = 1, \cdots, p-1$ are just for identification other than for control. Then it leads to that noise $w_j(m)$ $(j = 1, \cdots, p-1)$ does be independent of $u(m)$, consequently, the single-input $(p-1)$-output model can be considered as if it were operating in open loop, so its identification can also be performed in open loop form. Moreover, the feature that the subsystems of SIMO model have common denominator polynomial yields that identification of the single-input $(p-1)$-output model can be performed using subspace methods, then the denominator polynomial $A(z^{-1})$ is estimated, and the remaining problem is how to estimate the numerator $B(z^{-1})$, i.e. the enumerator polynomial $B_0(z^{-1})$ of the last subsystem. As illustrated later, the noise model in error equation becomes to a non-minimum phase model, which we will use a modified instrumental variable method to deal with. That is the two-stage algorithm of the proposed new closed-loop identification approach.

4.2 Closed-Loop Identification Algorithm

Next the closed-loop identification algorithm is presented for unstable plant.

Step 1 Estimation of $A(z^{-1})$

The single-input $(p-1)$-output model excluding the last subsystem involved in feedback controller in Fig. 3, can be identified through MOESP algorithm [13] or N4SID [14], by making use of the property that the SIMO model holds common denominator polynomial. The corresponding estimates are denoted as $\hat{A}(z^{-1})$ and $\hat{B}_j(z^{-1})$, for $j = 1, \cdots, p-1$ respectively.

Step 2 Estimation of $B(z^{-1})$

Step 2-a Separation of unstable part and stable part from $\hat{A}(z^{-1})$

Factorizing the denominator estimate $\hat{A}(z^{-1})$ yields

$$\hat{A}(z^{-1}) = \hat{A}_s(z^{-1})\hat{A}_u(z^{-1}) \tag{8}$$

where the stable part $\hat{A}_s(z^{-1})$ have the roots all inside the unit circle, whereas the unstable part $\hat{A}_u(z^{-1})$ have the roots outside the unit circle. Denote the unstable part as

$$\hat{A}_u(z^{-1}) = 1 + \hat{\alpha}_1 z^{-1} + \cdots + \hat{\alpha}_{n_{us}} z^{-n_{us}} \tag{9}$$

where n_{us} is the order of $\hat{A}_u(z^{-1})$. Neglecting the estimation error, the subsystem for $j=0$ in (7) becomes to

$$y_0(m) = \frac{B_0(z^{-1})}{\hat{A}_s(z^{-1})\hat{A}_u(z^{-1})} u(m) + w_0(m) \tag{10}$$

Step 2-b Expression of relation between filtered input and output signals

Since $A_s(z^{-1})$ is stable, filtering the plant input signal $u(m)$ leads to

$$u_f(m) = \frac{u(m)}{\hat{A}_s(z^{-1})}$$

Then (10) becomes

$$y_0(m) = \frac{B_0(z^{-1})}{\hat{A}_u(z^{-1})} u_f(m) + w_0(m) \tag{11}$$

Moreover, by multiplying $\hat{A}_u(z^{-1})$ the above relation becomes to

$$y_{0f}(m) = B_0(z^{-1})u_f(m) + \hat{A}_u(z^{-1})w_0(m) \tag{12}$$

where $y_{0f}(m) = \hat{A}_u(z^{-1})y_0(m)$.

Step 2-c Generating instrumental variable

The noise model $\hat{A}_u(z^{-1})w_0(m)$ in (12) is a non-minimum model due to $\hat{A}_u(z^{-1})$ is unstable, so PEM is not applicable for this case. Here we use an IV method by generating a special instrumental variable. Provide that the observation data length is large enough, then the instrumental variable is generated through a non-causal filter [15] as follows.

1) First the time index of the filtered signal $u_f(m)$ is inversed as follows.

$$\bar{u}_f(m) = u_f(M-m)$$

2) Next the signal $\bar{u}_f(m)$ is filtered by a causal filter to generate a new signal $\bar{v}(m)$.

$$\bar{v}(m) = \frac{\bar{u}_f(m)}{\hat{\alpha}_{n_{us}} + \hat{\alpha}_{n_{us}-1}z^{-1} + \cdots + z^{-n_{us}}}$$

where the denominator parameters are the order-reversed parameters of $\hat{A}_u(z^{-1})$.

3) Then the instrumental variable $v(m)$ is obtained by inversing the time index of $\bar{v}(m)$.

$$v(m) = \bar{v}(M-m)$$

Step 2-d IV based estimation of $B_0(z^{-1})$

Now the numerator polynomial of $B_0(z^{-1})$ can be estimated as follows.

$$\hat{\boldsymbol{\theta}}_b = [\,\hat{b}_{\tau_b} \quad \cdots \quad \hat{b}_{n_b}\,]^T = \left(\boldsymbol{V}^T\boldsymbol{\Phi}\right)^{-1}\boldsymbol{V}^T\boldsymbol{y}_{0f} \tag{13}$$

where

$$\begin{aligned}
\boldsymbol{V} &= [\,\boldsymbol{v}(1) \quad \cdots \quad \boldsymbol{v}(M)\,]^T \\
\boldsymbol{\Phi} &= [\,\boldsymbol{\phi}(1) \quad \cdots \quad \boldsymbol{\phi}(M)\,]^T \\
\boldsymbol{y}_{0f} &= [\,y_{0f}(1) \quad \cdots \quad y_{0f}(M)\,]^T \\
\boldsymbol{v}(m) &= [\,v(m-\tau_b) \quad \cdots \quad v(m-n_b)\,]^T \\
\boldsymbol{\phi}(m) &= [\,u_f(m-\tau_b) \quad \cdots \quad u_f(m-n_b)\,]^T
\end{aligned} \tag{14}$$

Remark: An alternative instrumental variable $v(m)$ can also be given by $v(m) = u_f(m-n_1)$ in Step 2-c, where $n_1 = \max(0, 1+n_{us}-\tau_b)$. Then Step 2-d can be performed in the same way as (13).

4.3 Identification Algorithm Using Priori Information of Physical Structure

In practical applications, the numerator of transfer function for some continuous time plant is just a constant η. If such information has been deduced from the physical property of the plant, the estimation of $B_0(z^{-1})$ in Step 2 can be implemented in an alternative simple way. Ordinarily the numerator order of the discrete-time model transfer function discretized via a zero-order holder is the same as the denominator order, even though the numerator of the original continuous time model is only a constant. When its parameters are estimated independently regardless their interactive relations, the estimation accuracy is often deteriorated greatly. Here we will consider a new simple but effective method to estimate the static gain of numerator using Theorem 2.

(1) The poles of the estimated model are denoted as $\bar{\hat{x}}_i$. Since the control input is held by a zero-order holder, the poles of the continuous time plant model $\hat{x}_i$ can be given by

$$\bar{\hat{x}}_i = \exp(-\hat{x}_i T), \quad i = 1, \ \ldots\ , n_a$$

Denote a continuous time model $\hat{G}(s)$ whose numerator is 1 and its poles are $\hat{x}_i$, then the continuous time plant model could be given by $\eta\hat{G}(s)$.

(2) Discretizing $\eta\hat{G}(s)$ through a zero-order holder yields the following discrete-time transfer function

$$\frac{\eta(\hat{\lambda}_1 z^{-1} + \hat{\lambda}_2 z^{-2} + \cdots + \hat{\lambda}_{n_a} z^{-n_a})}{1 + \hat{a}_1 z^{-1} + \hat{a}_2 z^{-2} + \cdots + \hat{a}_{n_a} z^{-n_a}}$$

where η is remained unknown yet.

(3) Following (6a) in Theorem 2, we have that

$$B_0(1) = \cdots = B_{p-1}(1)$$

then substituting the relation of η in (15) yields that

$$(p-1)\eta(\hat{\lambda}_1 + \hat{\lambda}_2 + \cdots + \hat{\lambda}_{n_a}) = \hat{B}_1(1) + \cdots + \hat{B}_{p-1}(1) \quad (15)$$

Thus the estimate $\hat{B}_0(z^{-1})$ can be given by

$$\hat{B}_0(z^{-1}) = \eta(\hat{\lambda}_1 z^{-1} + \hat{\lambda}_2 z^{-2} + \cdots + \hat{\lambda}_{n_a} z^{-n_a})$$

4.4 Identifiability Study

Consider the identifiability of the closed-loop new identification algorithm for unstable plant proposed in this paper. From Fig. 1 and Fig. 3, the input-output relations from $w_0(m)$ to $u(m)$ and from $w_j(m)$ to $y_j(m)$ are

$$u(m) = \frac{H(z^{-1})}{\Gamma(z^{-1})}(r(m) - w_0(m))$$
$$y_0(m) = \frac{F(z^{-1})}{\Gamma(z^{-1})}w_0(m) + \frac{F_0(z^{-1})}{\Gamma(z^{-1})}r(m) \quad (16)$$
$$y_j(m) = \frac{F_j(z^{-1})}{\Gamma(z^{-1})}(r(m) - w_0(m)) + w_j(m)$$

where $j = 1, \cdots, p-1$, and

$$\Gamma(z^{-1}) = CA + DB = 1 + \gamma_1 z^{-1} + \cdots + \gamma_{n_\gamma} z^{-n_\gamma}$$
$$H(z^{-1}) = DA = h_{\tau_d} z^{-\tau_d} + \cdots + h_{n_h} z^{-n_h}$$
$$F(z^{-1}) = CA = 1 + g_1 z^{-1} + \cdots + g_{n_f} z^{-n_f}$$
$$F_j(z^{-1}) = DB_j = s_{j,\tau_d+\tau_b} z^{-(\tau_d+\tau_b)} + \cdots + s_{j,n_{F_j}} z^{-n_{F_j}}$$

Recall the assumptions of the problem. Under the case where the reference signal is not a persistently exciting (PE) signal, for example $r(m) = 0$, the assumption that the output noise $w(k\Delta)$ is a white i.i.d noise implies that $w_j(m)$ is mutually independent for different j and m, and it ensures the PE property in the proposed algorithm. From (16), the following results can be deduced.

(1) $u(m)$ holds PE property.

(2) $u(m)$ is independent of $w_j(m+i)$ for $1 \leq j < p$, and i is an integer.

(3) $w_j(m)$ is a white noise.

Except the extremely low-passing ones, the feedback controller band for unstable plant is wide enough and the PE property of $w(k\Delta)$ ensures that of $u(m)$. Therefore the single-input $(p-1)$-output model can be identified unbiasly using subspace methods no matter what structure the controller has.

Moreover, when substituting the estimated denominator $\hat{A}(z^{-1})$ into T-model, only the numerator parameters remained unknown so the conventional identifiability condition (2) is satisfied obviously because there is not an unknown denominator in the identification model (12).

Now we illustrate the effectiveness of the proposed IV method. The expansion of $z^{-n_{us}}/\hat{A}_u(z^{-1})$ about z, which is a non-causal sequence, is given by

$$\frac{z^{-n_{us}}}{\hat{A}_u(z^{-1})} = \beta_0 + \beta_1 z + \beta_2 z^2 + \cdots = \Omega(z) \quad (17)$$

On the other hand, the coefficients of $\Omega(z)$ are just the impulse response of the following model

$$1/(\hat{\alpha}_{n_{us}} + \cdots + \hat{\alpha}_1 z^{-n_{us}+1} + z^{-n_{us}})$$

By inversing its time index, $v(m)$ can be written as follows.

$$v(m) = \Omega(z)u_f(m) \quad (18)$$

Then $v(m)$ can be written in

$$v(m) = \Omega(z)u_f(m) = \frac{\Omega(z)A_u(z^{-1})A_s(z^{-1})D(z^{-1})}{\hat{A}_s(z^{-1})\Gamma(z^{-1})}(r(m) - w_0(m)) \underset{M\to\infty}{\approx} \frac{D(z^{-1})}{\Gamma(z^{-1})}(r(m - n_{us}) - w_0(m - n_{us})) \quad (19)$$

When $i \geq 0$, $v(m)$ and $\hat{A}_u(z^{-1})w_0(m+i)$ are independent, then the following property

$$E\left\{\boldsymbol{v}(m)\left(\hat{A}_u(z^{-1})w_0(m)\right)\right\} = \boldsymbol{0} \quad (20)$$

holds. Furthermore, following Theorem 4.1 in Reference [16],

$$E\left\{\boldsymbol{v}(m)\boldsymbol{\phi}^T(m)\right\} \quad (21)$$

has full rank with probability 1, then the consistency of the estimate $\hat{B}_0(z^{-1})$ obtained by IV method is also guaranteed.

On the other hand, the IV estimation error can be evaluated by

$$\sqrt{M}(\boldsymbol{\theta}_b - \hat{\boldsymbol{\theta}}_b) \sim \mathcal{N}(0, \boldsymbol{P})$$

where

$$\boldsymbol{P} = \left(\boldsymbol{V}^T\boldsymbol{\Phi}/M\right)^{-1}\boldsymbol{S}\left(\boldsymbol{\Phi}^T\boldsymbol{V}/M\right)^{-1}$$
$$\boldsymbol{S} = \sigma_w^2 E\left\{\left(\sum_{i=0}^{\infty}\boldsymbol{v}(m+i)\alpha_i\right)\left(\sum_{i=0}^{\infty}\alpha_i\boldsymbol{v}^T(m+i)\right)\right\}$$

If the roots of $A_u(z^{-1})$ are close to the unit circle, the singular values of matrix $\boldsymbol{V}^T\boldsymbol{\Phi}/M$ are larger than those when $u_f(m-n_1)$ is taken as instrumental variable. Then $\mathrm{tr}\{\boldsymbol{P}\}$ becomes smaller so better accuracy is obtained.

5. Closed-Loop Identification Experiment on Magnetic Suspension System

5.1 Experiment Device and Conditions

We use the magnetic suspension system, which is illustrated in **Fig. 4**, as an unstable plant to demonstrate the effectiveness of the proposed new direct closed-loop identification algorithm. Let $M[kg]$ be the mass quantity of steel ball, $Y + y(t)[m]$ the gap distance between the steel and electromagnet, $R[\Omega]$ the resistance. Denote

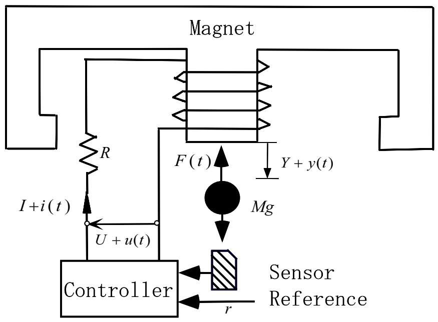


Fig. 4 Magnetic suspension system

$U+u(t)[V]$ and $I+i(t)[A]$ as the circuit voltage and current respectively, where U and I are the stationary ones when the steel is at the stationary position Y, while $u(t)$, $i(t)$ and $y(t)$ are the corresponding variation from the stationary position. The objective of the stabilization controller is to keep the position of the steel ball at reference $r[m]$. The position of steel ball and the deviation from the reference are measured through a laser senor. The control voltage $U+u(t)$ is generated through a PID controller as shown later.

The behavior of the physical mechanic and electric elements are described by

$$M\frac{d^2(Y+y(t))}{dt^2} = Mg - F(t) \tag{22a}$$

$$U+u(t) = R(I+i(t)) + \frac{d}{dt}L(t)(I+i(t)) \tag{22b}$$

where $L(t)$ is the inductance of the coil and is approximated by

$$L(t) = \frac{Q}{Y+y(t)} + L_\infty \tag{23}$$

Q and L_∞ are the constants depending on the coil, magnetic core and the steel ball. Substituting the electromagnetic force given by

$$F(t) = 0.5(I+i(t))^2\frac{\partial L(t)}{\partial y(t)} \tag{24}$$

into ($22a$) and ($22b$) leads to

$$M\frac{d^2y(t)}{dt^2} = Mg - 0.5Q\left(\frac{I+i(t)}{Y+y(t)}\right)^2 \tag{25a}$$

$$u(t) = Ri(t) - \frac{Q(I+i(t))}{(Y+y(t))^2} + \left(L_\infty + \frac{Q}{Y+y(t)}\right)\frac{di(t)}{dt} \tag{25b}$$

On the other hand, at the stationary point we have that

$$0 = Mg - 0.5Q\left(I/Y\right)^2, \quad U = RI \tag{26}$$

($25a$) and ($25b$) are the nonlinear differential equations about $i(t)$ and $y(t)$, however, when the steel ball moves within a small region around the stationary position, they can be approximated by linear equations. Then the linearized transfer function from the variation of control voltage $u(t)$ to gap variation $y(t)$ is written in 27.

$$G(s) = \frac{y(s)}{u(s)} = \frac{\eta}{(Ms^2-\alpha)(Ls+R)+\eta\gamma} \tag{27}$$

where $\alpha = 2QI^2/Y^3$, $\eta = 2QI/Y^2$, $\gamma = LI/Y$. Obviously there is an unstable pole $s = \sqrt{\alpha/M}$ in the linearized model of the magnetic suspension system, and the feedback controller is necessary for stabilization.

In the practical experiment device, the nominal measurements of physical parameters are $M = 0.54$kg, $R = 11.233\Omega$, respectively. The feedback controller is a PID controller, whose control interval is $T = 0.0024[s]$. The effectiveness of the proposed approach is illustrated through changing the inter-sampling rate p, for comparison with the conventional method where $p = 1$.

5.2 Experiment Results

Let the stationary position be $Y = 6$mm, the nominal discrete-time model for position Y, i.e. the T-model is given by

$$\frac{B(z^{-1})}{A(z^{-1})} \approx \frac{10^{-6}(-0.0411z^{-1} - 0.1631z^{-2} - 0.0404z^{-3})}{1 - 2.9793z^{-1} + 2.9488z^{-2} - 0.9698z^{-3}} \tag{28}$$

where the 3 poles are 1.1058 (unstable), 0.9666, 0.9071respectively. (28) is the nominal model obtained from the physical equations by substituting their corresponding coefficient values, and will be used as a comparative reference for estimation evaluation.

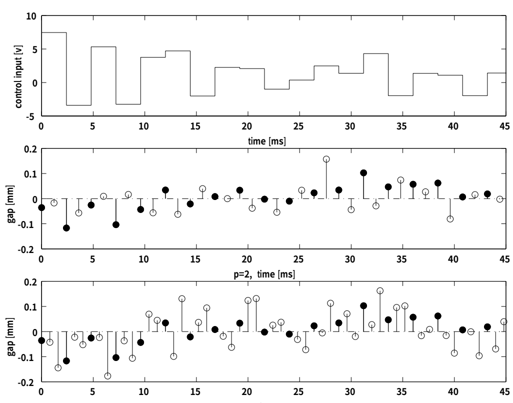


Fig. 5 Control input and inter-sampled output data. From the top: $u(m)$, $y_j(m)$ ($p=2$), $y_j(m)$ ($p=3$). •: Outputs at mT, ∘: Outputs at $mT+j\Delta$ for $1 \le j \le p-1$

As for the stabilization of the unstable system, a simple PID controller is adopted in the experiment, whose transfer function is

$$\frac{D(z^{-1})}{C(z^{-1})} = \frac{-10^5(0.5973 - 1.1897z^{-1} + 0.5833z^{-2})}{1-z^{-1}}$$

Obviously the identifiability condition (2) is not satisfied, and the full rankness of the input-output data matrix does not hold, as shown later.

In the nominal model (27) the numerator is just a constant then the identification method given in Section 4.3

is applicable by using the results of Theorem 2. When the steel ball is well controlled to the reference level, little noise enters into the system so the assumptions on the PE property of noise are not satisfied, and any method will fail to work for this case. Here a white i.i.d noise is added into the inter-sampled data in the experiment, where the variance of the noise is within a specified range. Compared with the conventional indirect methods using the values of the external signals, the proposed new approach does not use any measurement of the noise, which is the distinct feature of the direct methods. The noise used in the experiment is a white Gaussian noise with zero-mean and standard deviation 0.1mm, a small value compared with the reference level. **Fig. 5** shows an example of the input and output data. The observation data collected for 18s are used for identification. Further, the sampling rate p is changed from $p = 2$ to $p = 6$, and the identification results are compared with the conventional one where $p = 1$. The experiments are performed for 10 times at every sampling rate p respectively.

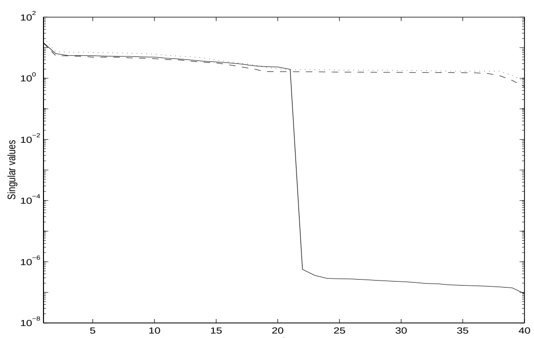


Fig. 6 Sigular values of $\mathbf{\Phi}$. Solid line: $p = 1$, dashed line: $p = 2$, dotted line: $p = 3$

Case 1 $p = 1$ (Conventional method)

As discussed previously, the identifiability condition (2) is not satisfied under this case, then the data matrix used in subspace method in Step 1 given by

$$\mathbf{\Phi} = \begin{bmatrix} y_j(m_1-1) & \cdots & y_j(m_1+M-2) \\ \vdots & & \vdots \\ y_j(m_1-L) & \cdots & y_j(m_1-L+M-1) \\ u(m_1-1) & \cdots & u(m_1+M-2) \\ \vdots & & \vdots \\ u(m_1-L) & \cdots & u(m_1-L+M-1) \end{bmatrix}^T$$

is rank deficient, and it is verified easily by calculating the singular value of $\mathbf{\Phi}$. The singular values σ_i for $j = p-1$, $L = 20$, $M = 2500$ is plotted in **Fig. 6**. When $p = 1$ (conventional method), $\{\sigma_i\}$ falls extremely from $i = 22$ then the condition number becomes very large so the conventional direct method is ill-conditioned and fails to identify the plant model. As illustrated in **Fig. 7**, the estimated poles for 10 times are almost inside or close to the unit circle, and the unstable pole is not identified well.

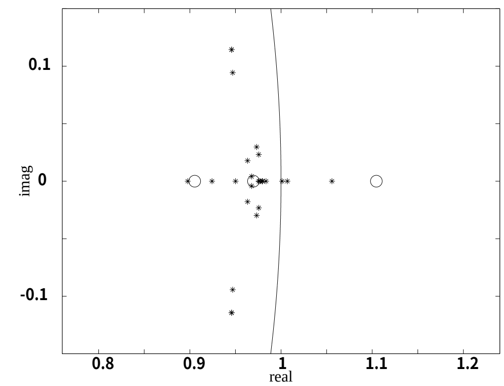


Fig. 7 Estimated poles ($p = 1$). "*": estimate, "o": theoretical nominal value

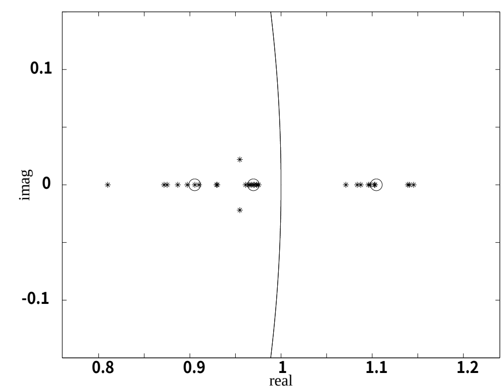


Fig. 8 Estimated poles ($p = 2$). "*": estimate, "o": theoretical nominal value

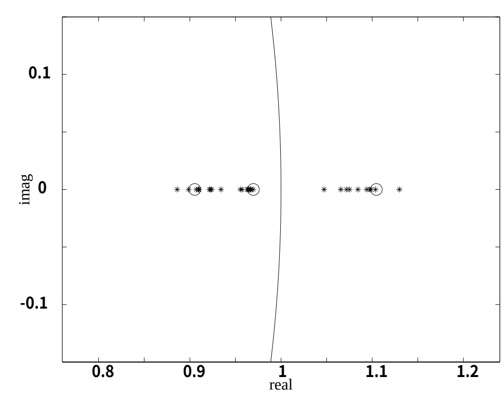


Fig. 9 Estimated poles ($p = 3$). "*": estimate, "o": theoretical nominal value

Case 2 $p = 2$ and $p = 3$ (Proposed approach)

The full rankness of the inter-sampled data matrix is confirmed in Fig. 6 and the estimated poles in Step 1 for

Table 1 Estimated parameters(results of 10 experiments)

	nominal	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$
a_1	-2.9793	-2.9130 $\pm$0.0461	-2.9714 $\pm$0.0250	-2.9630 $\pm$0.0205	-2.9520 $\pm$0.0127	-2.9568 $\pm$0.0131	-2.9672 $\pm$0.0082
a_2	2.9488	2.8310 $\pm$0.0888	2.9310 $\pm$0.0521	2.9184 $\pm$0.0385	2.8991 $\pm$0.0240	2.9077 $\pm$0.0254	2.9274 $\pm$0.0160
a_3	-0.9698	-0.9179 $\pm$0.0429	-0.9599 $\pm$0.0274	-0.9556 $\pm$0.0181	-0.9473 $\pm$0.0113	-0.9511 $\pm$0.0124	-0.9605 $\pm$0.0080
$b_1[\times 10^{-6}]$	-0.0411	0.1660 $\pm$0.4858	-0.0532 $\pm$0.0215	-0.0418 $\pm$0.0116	-0.0325 $\pm$0.0052	-0.0356 $\pm$0.0064	-0.0391 $\pm$0.0067
$b_2[\times 10^{-6}]$	-0.1631	-0.2165 $\pm$0.8399	-0.2104 $\pm$0.0840	-0.1655 $\pm$0.0461	-0.1282 $\pm$0.0207	-0.1408 $\pm$0.0252	-0.1547 $\pm$0.0264
$b_3[\times 10^{-6}]$	-0.0404	0.1126 $\pm$0.3276	-0.0519 $\pm$0.0201	-0.0409 $\pm$0.0115	-0.0316 $\pm$0.0051	-0.0347 $\pm$0.0063	-0.0383 $\pm$0.0065

Table 2 Estimated parameters for various reference r

	$r=3$mm		$r=4$mm		$r=6$mm	
	nominal	estimate	nominal	estimate	nominal	estimate
a_1	-2.9843	-2.9674 $\pm$0.0160	-2.9826	-2.9712 $\pm$0.0180	-2.9795	-2.9630 $\pm$0.0205
a_2	2.9549	2.9229 $\pm$0.0304	2.9530	2.9298 $\pm$0.0354	2.9488	2.9184 $\pm$0.0385
a_3	-0.9710	-0.9559 $\pm$0.0145	-0.9707	-0.9590 $\pm$0.0354	-0.9698	-0.9556 $\pm$0.0181
$b_1[\times 10^{-6}]$	-0.05881	-0.0516 $\pm$0.0079	-0.0537	-0.0587 $\pm$0.0090	-0.0411	-0.0418 $\pm$0.0116
$b_2[\times 10^{-6}]$	-0.2336	-0.2042 $\pm$0.0315	-0.2134	-0.2325 $\pm$0.0355	-0.1631	-0.1655 $\pm$0.0461
$b_3[\times 10^{-6}]$	-0.0579	-0.0505 $\pm$0.0079	-0.0529	-0.0575 $\pm$0.0088	-0.0404	-0.0409 $\pm$0.0115

10 experiments are plotted in **Fig. 8** and **Fig. 9** respectively. The nominal poles, which are denoted as $\circ$, are all real ones and one of them is unstable. The clusters of estimated poles obtained by the proposed approach are almost distributed around their theoretical locations, and imply that the estimation results are appropriate in the new approach. Furthermore, the values of estimate mean and standard deviation of the estimated parameters for 10 times corresponding to the inter-sampling rate from $p=1$ to $p=6$ are summarized in **Table 1**. It can be seen that the estimates are very close to the nominal ones for $p \geq 2$, and the effectiveness of the proposed method is verified. Moreover, the same identification experiments are performed for the cases where the stationary positions are taken as $Y=3[mm]$, $Y=4[mm]$, $Y=6[mm]$, and the identification results are summarized in **Table 2**. It is illustrated that the estimates are also near to the nominal theoretical ones in every case, so the proposed direct identification algorithm can be applicable to unstable plant just using the plant input and output data. As shown above, once the parameters of unstable plant are identified directly, it is possible to re-design or update the controllers, and it provides a potential way to realize the iterative system identification and controller design.

6. Conclusions

The direct closed-loop identification approach for unstable plant is developed by inter-sampling the plant output at p times faster rate than that of the control input. Making use of the SIMO model structure obtained in the output inter-sampling scheme, it is demonstrated that the single-input $(p-1)$-output model, whose output noise is not fed back into the controller, is identifiable and the unstable denominator polynomial can be identified by subspace methods. Moreover, a new instrumental variable is generated to overcome the non-minimum phase problem for numerator estimation, and an estimation algorithm is also proposed by using the physical structure of the practical plant. The identifiability in the new approach is also studied and finally the effectiveness of the proposed new approach is demonstrated through some identification experiments on a magnetic suspension system.

References

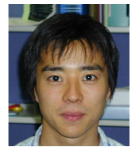
1) T. Söderström and P. Stoica: System Identification, Englewood Cliffs, NJ: Prentice-Hall (1989)
2) U. Forssell and L. Ljung: Closed-loop Identification Revisited, *Automatica*, **35**, 1215/1241 (1999)
3) F.R. Hansen, G.F. Franklin: On a Fractional Representa-

tion Approach to Closed-loop Experimental Design, *Proc. Am. Control Conf.*, Atlanta, USA, 1319/1320 (1988)
4) P.M.J. Van den Hof, R.J.P. Schrama, R.A. de Callafon and O. H. Bosga: Identification of Normalized Coprime Factors from Closed-loop Experimental Data, Euro. J. Control, **1**, 62/74 (1995)
5) W.S. Lee, B.D.O. Anderson, I.M.Y. Mareels and R.L. Kosut: On Some Key Issues in the Windsufer Approach to Adaptive Robust Control, *Automatica*, **31**-11, 1577/1594 (1995)
6) K. Nakashima and T. Fujii: Multivariable Control of Magnetic Levitation System Using Closed Loop Identification and H_∞ Control Theory, *Proc. IEEE Conf. Decision and Control*, Kobe, Japan, 3668/3673 (1966)
7) I.D. Landau and A. Karimi: Recursive Algorithms for Identification in Closed Loop: A Unified Approach and Evaluation, *Automatica*, **33**-8, 1499/1523 (1997)
8) L. Ljung, I. Gustavsson and T. Söderström: Identification of Linear Multivariable Systems Operating under Linear Feedback Control, IEEE Trans. on Automatic Control, **AC-19**-6, 836/840 (1974)
9) P.E. Wellstead and J. M. Edmunds: Least-squares Identification of Closed-loop Systems, Int. J. Control, **21**-2, 689/699 (1975)
10) T. Söderström, I. Gustavsson and L. Ljung: Identifiability Conditions for Linear Systems Operating in Closed Loop, Int. J. Control, **21**-2, 243/255 (1975)
11) L. Sun and A. Sano: Output Inter-sampling Approach to Closed-loop Identification, Trans. on SICE, **35**-8, 1069/1077 (1999)
12) L. Sun, Y. Miyake, H. Ohmori and A. Sano: Closed-Loop Identification of Unstable Systems Based on Output Inter-sampling, The 28th Symposium on Control Theory, 313/318 (1999)
13) M. Verhaegen and P. Dewilde: Subspace Model Identification Part I. The Output-Error State-Space Method Identification Class of Algorithms, *Int. J. Control*, **56**-5, 1187/1210 (1992)
14) P. van Overschee and B. De Moor: N4SID: Subspace Algorithms for the Identification of Combined Deterministic-Stochastic Systems, *Automatica*, **30**-1, 75/93 (1994)
15) U. Forssell and H. Hjalmarrson: Maximum Likelihood Estimation of Models With Unstable Dynamics and Nonminimum Phase Noise Zeros, in *14th IFAC World Congress*, Vol.H, 13/18 (1999)
16) T. Söderström and P. Stoica: Instrumental Variable Methods for System Identification, Lecture Notes in Control and Information Sciences, Springer (1983)

Lianming Sun (Member)

He received the Ph. D. degree from Keio University in 2000. He is currently a lecture in the Department of Information and Media Sciences, The Faculty of Environmental Engineering, The University of Kitakyushu. His research interests include system identification, blind identification and digital signal processing.

Yasuhiro Miyake (Member)



He received the M.E. degree from Department of Electrical Engineering, Keio University in 2001. He is a system engineer of Agilent Technologies Japan, Ltd currently. His research interests are instrument, system identification and parametric test design.

Hiromitsu Ohmori (Member)

He received the B.E., M.E. and Ph.D. degrees from Department of Electrical Engineering, Keio University in 1983, 1985 and 1988 respectively. He is currently a professor of Department of System Design Engineering, Keio University. His research interests include adaptive control, H_∞ control theory and its applications.

Akira Sano (Member)

He received the B.E., M.E. and Ph.D. degrees in Mathematical Engineering and Information Physics from the University of Tokyo, in 1966, 1968 and 1971 respectively. He is currently a professor of Department of System Design Engineering, Keio University. His research interests include control system design, adaptive control, system identification and digital signal processing.

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