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Generation of Digital Elevation Map of the Moon from Observation Data Interpolated by Fractal Modeling†

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Navigation, guidance, and control (GN&C) based on imagery information are key technologies for lunar landings. A digital elevation map (DEM) is needed to verify GN&C algorithms. We herein propose a new method for generating a DEM that resembles that made from past measurements and at the same time allows a magnified use for lower altitudes. In the proposed method, small rises and falls, which are lost in a coarse DEM generated from photographic measurements, are extrapolated by fractal modeling. Furthermore, small craters are added to match their statistical distributions from data collected during previous measurements. The proposed method is applied to a DEM of the moon. The obtained DEM shows that the small rises and falls and craters lost in the original DEM are restored, and the restoration appears quite similar to the observation photographs. A georama is produced by laser lithography, and photographs of the georama also appear similar to the observation photographs.

Key Words: moon, digital elevation map, fractal modeling

1. Introduction

Recently, moon exploration programs have been revived. Some moon exploration missions send unmanned spacecraft, which are landed softly and are used to make observations. Since we already have a certain amount of knowledge about the surface of the moon, in future missions, spacecraft will probably be landed at specified points.

When specifying a landing point, it is convenient to use the features of the lunar surface in the immediate neighborhood and match them to reference data such as previous photographs or images simulated from digital elevation maps (DEMs). This requires navigation, guidance, and control (GN&C) technology based on imagery information. The identification and tracking of landing points is technologically problematic because the appearance of the terrain changes as the spacecraft descends. Furthermore, since smaller craters and rocks not included in the prepared DEM may become visible during the final landing phase, the detection and avoidance of these obstacles is another problem.

In order to investigate these problems and verify GN&C algorithms, we must create a high-resolution, high-fidelity DEM for the generation of simulated images. If a DEM of good resolution (i.e., on the order of a few meters) is available, it is possible to generate simulated images for verification. However, the most commonly obtainable data for the lunar surface are photographs or coarse DEMs. The former is not adequate for simulating images for altitudes, camera angles, or lighting conditions that differ from the original altitudes, camera angles, or lighting conditions. Furthermore, the latter is not adequate for simulating images for an identical point from high altitudes to low altitudes because information about small rises and falls and small craters is lost. An alternative approach is to generate an imaginary moon surface based on statistical relations derived from past observations. However, this is not practical if there is no correspondence to the actual moon surface.

We propose a new method of generating a DEM for the verification of GN&C technologies based on imagery information. The derived DEM should resemble the actual observations and be detailed enough to be used for low-altitude simulations. The proposed method uses fractal modeling in order to complement small rises and falls lost in a coarse DEM derived from observations. The derived DEM adds smaller craters based on statistical relations related to their forms and distributions. For hardware experiments, the obtained DEM is used to generate a georama by laser lithography. Layers of synthetic resins are accumulated by tenths of 1-mm pitches, and the accuracy is ±0.1 % of the DEM. Accordingly, the similarity of the georama to the computational model is much higher than that of a clay model.

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The remainder of the present paper is organized as follows. In Section 2, conventional methods of generating DEMs are summarized and compared with the proposed method. Local terrain restoration and crater shape and distribution models are described in Sections 3 and 4, respectively. In Section 5, an experimental georama production is presented. Conclusions are presented in Section 6.

2. Generation of Planetary Surface Model

2.1 Conventional Methods

A planetary surface model is required to verify the navigation system of landers or rovers that uses imagery information of the planetary surface. Previous studies on the generation of planetary surface models can be categorized into four methods. Each method can generate simulated images of the surface and has a real model used in evaluating the results of image processing.

(a) Use measurement data [1]:
The procedure in Ref. [1] is described as an example. First, a georama is produced based on a DEM of Phobos derived from observations. Image data of Viking and Phobos 2 are referenced, and fine rilles and undulations are added to the georama. A laser displacement sensor is then used to create a contour map of the georama, which is used as the actual model. Finally, photographs of the georama are taken using a CCD camera for use as simulated images. This method is advantageous because the georama can be used to make a real model during its production. However, it is difficult to generate a real model from the georama, and the statistical relations of the surface are not applicable.

(b) Create imaginary terrain in the computer [2)~5)]:
The procedure in Ref. [4] is described as an example. First, a lunar surface model with large and small rises and falls is prepared. Second, randomly distributed craters are added to the model based on statistical relations such as crater shape and distribution, and the model then becomes the real model. Simulated images are made by computer graphics (CG) techniques such as rendering. This method is advantageous because the derived model satisfies statistical relations. However, it is not easy to create ditches or crevices by CG, and the obtained images may not look realistic.

(c) Use natural terrain [6,7]:
The procedure in Ref. [6] is described as an example. A rock having shape and texture similar to those of an asteroid is measured by a 3D digitizer to obtain a DEM, which is used as the real model. Photographs of the model are taken in order to be used as simulated images. This method is advantageous because crevices and fine textures are included in the simulated images. However, this method has the same disadvantages as method (a).

(d) Use observation photographs [3,8,9]:
The procedure in Ref. [8] is described as an example. Photographs taken during the Apollo missions are used as the real model. Craters are detected by image processing, and humans compare the obtained results with the original photographs for verification. The advantage of this method is its simplicity because the real model consists of images. However, this method cannot be applied to different lighting conditions, and there is no 3D reference to verify 3D information extracting methods such as shape from shading.

2.2 Proposed Method

A new method is proposed to solve the abovementioned difficulties. First, fractal modeling is used in order to complement small rises and falls that are lost in the coarse DEM derived from observations ("local terrain restoration"). Second, smaller craters are added considering the statistical relations of their forms and distributions ("adding craters"). Simulated images are then generated by rendering. For hardware experiments, the obtained DEM is made into a georama by laser lithography, and photographs of the georama are then used as simulated images.

This method combines methods (a) and (b) and has broad applicability owing to the real model in the computer. In addition, laser lithography provides this method with a high-fidelity georama. Furthermore, since a coarse DEM is used as a basis, the global terrain looks realistic.

Figure 1 summarizes the main features of the conventional and proposed methods based on the reference data, the real model, and camera image simulation.

3. Local Terrain Restoration

3.1 Fundamental Approach

Landscapes, such as coastlines or mountain undulations, look similar regardless of their scaling, and so are said to have self-similarity [10]. Terrain having self-similarity can be modeled with fractal curves or fractal curved surfaces. On the other hand, DEMs derived from past observations tend to have insufficient resolu-

Fig. 1 Conventional and proposed methods for the generation of planetary surface models.

Fig. 2 Fundamental flow of local terrain restoration.

tion in the height dimension and lose information related to smaller craters or undulations.

Therefore, we propose the use of measured data for global terrain and the restoration of small rises and falls with fractal curved surfaces. **Figure 2** shows the fundamental flow.

3.2 Fractional Brownian Motion [10)]

Fractional Brownian motion (fBm) is one of the most effective mathematical models for representing natural terrain having self-similarity. **Figure 3** shows an example of one-dimensional fBm. One-dimensional fBm $X(t)$ is a one-valued function of one parameter t. The symbols H $(0 < H < 1)$ and D represent the scaling and fractal dimension, respectively. The plot becomes steep as H approaches 0 and becomes gentle as H approaches 1.

A typical change in X, $\triangle X \equiv X(t_2) - X(t_1)$, with respect to a change in t, $\triangle t = t_2 - t_1$, is specified by using a scaling law with H as

$$\triangle X \propto \triangle t^H \tag{1}$$

In the case of fBm with Euclidean space dimension E, it holds that $D = E+1-H$. When H is approximately 1, D is close to E. As H approaches 0, the difference between D and E becomes larger. Correspondingly, in Fig. 3, the change in the ordinate is much more complex than that in the abscissa as H becomes smaller.

Based on Eq.(1), the spectral density $S(f)$ of fBm also obeys a power law

$$S(f) \propto \frac{1}{f^\beta}, \tag{2}$$

where f is a frequency and β satisfies the relation $\beta = 2H + E$.

3.3 Fourier Filtering Method [10)]

The Fourier filtering method is introduced here in order to generate a DEM of fractal dimension D. This method makes use of the fact that the spectral density of fBm is proportional to $1/f^\beta$. The coefficients of the discrete Fourier transform (DFT) of the DEM are set so as to satisfy this relation, and the inverse discrete Fourier transform (IDFT) is then applied to obtain the DEM.

In two-dimensional space, spectral density S generally depends on two frequency variables, u and v, which correspond to the x and y directions, respectively. The statistical characteristic of S, however, are isotropical on the

Fig. 3 One-dimensional fBm example.

xy plane. As a result, S depends on $(u^2+v^2)^{1/2}$. If the surface of S is cut along a line on the xy plane, S of fBm is expected to obey the one-dimensional scaling law, as follows:

$$S(u,v)dudv = \frac{S_0}{(u^2+v^2)^{H+1}}dudv, \tag{3}$$

where S_0 is a constant.

The two-dimensional IDFT is formulated as follows:

$$X(x,y) = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} a_{k,l}e^{2\pi i(kx+ly)}, \tag{4}$$

where wave numbers k and l are from 0 to $N-1$ and normalized spatial variables x and y are from $0/N$ to $(N-1)/N$. Coefficients $a_{k,l}$ must satisfy the following scaling law:

$$E\left(|a_{k,l}|^2\right) \propto \frac{1}{(k^2+l^2)^{H+1}}, \tag{5}$$

where E denotes the expectation operator. Consequently, these coefficients are set as

$$a_{kl} = re^{i\theta} \tag{6}$$

where i is an imaginary unit, θ is a uniform random number in the domain $[0, 2\pi)$, and r is a random number that satisfies the following Rayleigh distribution:

$$E[r^2] = \frac{1}{(k^2+l^2)^{H+1}}. \tag{7}$$

Note that coefficients $a_{k,l}$ must satisfy the conjugate condition because function X is a real function.

3.4 Terrain Restoration by Fractal Modeling

The procedure used to realize the restoration flow in Fig. 2 is shown below (**Fig. 4**):

(a) Apply DFT to the DEM derived from observations and obtain coefficients $a_{k,l}$.

(b) Omit terms of higher orders in $a_{k,l}$. If the wave number of $a_{k,l}$, which is given as $\sqrt{k^2+l^2}$, exceeds the threshold value r_{max}, it is judged to have higher order terms and $a_{k,l}$ is set as zero. Symbol r_{max} is decided based on DEM characteristics such as its resolution. Four identical sectors of radius r_{max} appear on each corner of the table of $a_{k,l}$ to satisfy the conjugate condition.

(c) Replace higher order terms of $a_{k,l}$ with the coefficients satisfying the scaling law (5), which are computed from Eq. (6).

(d) Apply IDFT to $a_{k,l}$. The obtained DEM will appear similar to the actual surface of the moon and at the same time will have small rises and falls.

4. Adding Craters

4.1 Crater Distribution Model

The following relation holds for cumulative size distributions (n, total number per unit area 1 [km^2] larger than a given diameter, d [km]).

$$\log_{10} n = -\alpha \log_{10} d + B, \tag{8}$$

where constants α and B depend on both the age and geological features of the area. Here, α is set to 2.0, and B is set to -1.42. These are typical intermediate values of a relatively young sea, such as Rima Hadley, or an old sea, such as Mare Tranquillitatis [11].

If the area of DEM is given as A [km^2], the number of craters having a diameter larger than d [km] is given by Eq. (8) as $An = A \times 10^{-1.42}d^{-2}$. It is straightforward to assume that craters having diameters of $An = 1, 2, 3, \cdots, k$ are the first, second, third, $\cdots$, kth largest craters in the area. Based on this assumption, the diameter of the kth largest crater is

$$d_k = \sqrt{\frac{10^{-1.42}A}{n}}. \tag{9}$$

4.2 Crater Shape Model

All fresh moon craters having a diameter of less than or equal to 10 [km] are thought to be bowl-shaped. The statistical relations of their diameter, depth, and rim height have been published [12)~14)]. In the present study, we consider a bowl-shaped crater with a diameter of approximately 1.7 [km] in the DEM as a standard for the crater shape model. First, the center of the crater is estimated from the contour line. Second, for each lattice point in the DEM, the height h of the crater is plotted with respect to the distance from center r (**Fig. 5**(a)). Finally, the plot is divided into three parts (central, rim, and outer parts), and each part is fitted with an appropriate curve parameterized by a non-dimensional distance r/r_1 (Fig. 5(b)).

Fig. 4 DEM restoration by fractal modeling.

$$\begin{cases} \text{concave radius} & r_0 = 0.844 r_1 \\ \text{crater depth} & h_0 = 0.367 r_1 \\ \text{rim height} & h_1 = 0.038 r_1 \end{cases} \quad (10)$$

The exponent β that represents the rim shape is set to 2.6 according to Schröter's law [14]. The remaining coefficients α, a_1, a_2, and a_3 are determined so that the height and gradient at the connecting points are the same for both curves and the squared sum of differences between the DEM height and curve height at lattice points is minimized.

$$\begin{cases} \text{(i) Central part: } (0 \le r < r_0) \\ \dfrac{h(r)}{r_1} = \dfrac{h_0}{r_1}\left(\dfrac{r}{r_1}\dfrac{r_1}{r_0}\right)^{\alpha} - \dfrac{h_0}{r_1} \\ \text{(ii) Rim part: } (r_0 \le r < r_2) \\ \dfrac{h(r)}{r_1} = a_1\left(\dfrac{r}{r_1} - 1\right)^3 + a_2\left(\dfrac{r}{r_1} - 1\right)^2 + \dfrac{h_1}{r_1} \\ \text{(iii) Outer part: } (r_2 \le r) \\ \dfrac{h(r)}{r_1} = \dfrac{h_1}{r_1}\left(\dfrac{r}{r_1} - a_3\right)^{-\beta} \end{cases} \quad (11)$$

$$\alpha = 1.31, \quad a_1 = 3.13, \quad a_2 = -1.09$$

$$a_3 = 2.55 \times 10^{-2}, \quad r_2 = 1.054 r_1, \quad \beta = 2.6$$

A crater shape of arbitrary radius r_1 can be computed from Eqs. (10) and (11).

Fig. 5 Crater shape model with three patched curves.

5. Experimental Georama Production

5.1 Georama Production Procedure

An example of DEM restoration based on the procedure described in Sections 3 and 4 is given below.

The DEM is 41.5 km in width and 30 km in length and includes Rima Hadley, which is not far from the Apollo 15 landing site.

(a) Extract an area of 30×30 km that includes both flat regions suitable for landing sites and undulatory regions to be avoided. Apply DFT to obtain coefficients $a_{k,l}$.

(b) Omit higher order terms of $a_{k,l}$. The threshold value r_{max} is set to 183, which is near the boundary where the spectral density falls apart from the statistical relation.

(c) Replace higher-order terms of $a_{k,l}$ with the coefficients computed using Eq. (6). Here, H is set to 0.8 based on the computation results of $H \sim 0.8, D \sim 2.2$ for the fractal dimension for several lunar DEMs.

(d) Apply IDFT.

(e) Add craters. Since the original DEM includes a few large craters, craters of diameter smaller than 870 [m] are added according to Eq. (9). We started at 870 [m] because the diameter of the smallest crater discernible from the DEM is approximately 15 times the horizontal resolution (58 [m]). The x and y positions of the crater are assumed to be uniformly distributed in the range $[0, \cdots, 30)$ [km]. Past observations have shown that some craters overlap. Therefore, no restrictions are placed on crater positions (such as disallowing overlapping).

5.2 Obtained Georama

Figure 6 shows simulated images generated from the original DEM and the restored DEM. In the same figure, a wide-area photograph and an enlarged photograph of Rima Hadley Central, observed by the Lunar Orbiter, are also shown as a reference. These images and photographs clearly show that the small rises and falls and craters lost in the original DEM are naturally restored by the proposed method.

A photograph of the georama with a scale of 1:50,000 produced by laser lithography is shown in **Fig. 7**. The photograph looks very realistic.

6. Conclusions

One of the key technologies in landing an unmanned

Fig. 6 Comparison of simulated images and observed photographs.

Fig. 7 Sample photograph of the georama.

spacecraft softly on the moon or on a planet is GN&C based on imagery information. For verification of this technology, a high-resolution DEM is a prerequisite for generating simulated images of the onboard camera under various conditions, including spacecraft altitude, position, CCD camera angle, and sun direction. Prior to a mission, although coarse DEMs may be available, high-resolution DEMs are not.

We have proposed a new method of generating a DEM that resembles actual observations and has sufficient resolution to be used for low-altitude simulations. The proposed method uses fractal modeling in order to complement small rises and falls that are lost in the coarse DEM derived from observations and adds smaller craters with proper statistical relations on their shapes and distributions. Therefore, the proposed method has the following features:

(a) Geographical features such as peaks and rims are visible because a coarse DEM derived from observations is used as its basis.

(b) The restoration of small rises and falls is easy and a natural appearance is provided because fractal modeling is applied.

(c) The risk of under- or over-estimation of crater effects is small because the shape and distribution of the craters are determined according to past observation data.

The proposed method was applied to a moon DEM. The obtained DEM shows that the small rises and falls and craters lost in the original DEM were restored. This restoration appears quite similar to the observation photographs. The georama of the restoration was produced by laser lithography, and photographs of the restoration appear similar to the observed photographs. Thus, high fidelity to the computational model was achieved.

In the future, we will include numerical simulations using the computational DEM and will perform hardware experiments by taking photographs of the georama from a CCD camera attached to a robot in order to develop and verify GN&C technology based on imagery information.

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