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On a Null Controllability Region of Nonlinear Systems

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It is known that if the linearized models of nonlinear systems are controllable then the original nonlinear systems are also controllable near the origin. In this paper, we discuss the size of the null controllability region of nonlinear systems by functional analysis methods under the condition that the linearized system is controllable at the origin. Without adding any extra assumptions, we derive the quantitative estimation of this size. Thus our result is applicable to large classes of nonlinear systems and always gives us an estimation of null controllability region. In practical sense, to know the size of the controllability region is very useful to synthesize the systems and moreover it is also important to know what parameters seriously inference to the size The result also contains some suécient conditions of complete controllability, since complete controllability is equivalent to the case that the estimated null controllability region coincides with the whole state space.

Key Words: Controllability, Nonlinear Systems, Null Controllability Region

1. Introduction

In terms of nonlinear system controllability, there are various researches based on many mathematical tools such as implicit function theorems [1)2)], Liapunov methods [3)], diãerential geometry[4)] and so on. Lee and Markus derived one of the most important results in this åeld.[1)2)] The result is that if a linearized system is controllable at an equilibrium point then there exists a local controllable area of the original nonlinear system around the equilibrium point. Kalman also indicated in the discussion[2)] that this fact was also true in case of the linearized system was time varying. The result gives us a proper reason to use linearized systems in stead of the original nonlinear systems. But the result does not refer the region where we can use the linearized systems properly in place of the original nonlinear systems. In this paper, we try to estimate the size of this area by using functional analysis methods. The functional analysis methods including 'åxed point theorem' were frequently used in the researches of nonlinear controllability in the 1970s. The typical researches were done by Davison[5)], Lukes[6)] and Yamamoto[7)]. Davison derived a suécient controllability condition for a system described as $\dot{x} = A(t;x)x + B(t;x)u$. Lukes and Yamamoto also derived suécient conditions of the system as $\dot{x} = A(t)x + B(t)u + f(x;u;t)$. They used 'åxed point theorem' to derive their results, but in order to do so, they assumed very strict conditions. Namely Davison assumed $|A(t;x)| < M$, $|B(t;x)| < N$ for any x. Lukes and Yamamoto assumed that $|f(x;u;t)| < L$ is satisåed for any x and u. We do not assume anything except the linearized system is controllable. Based on this simple frame, we derive a quantitative estimation of the null controllability region. Since our approach does not require special assumptions, the result is applicable to wide range nonlinear systems. The result is also able to derive some complete controllability conditions because the complete controllability is equivalent to the case that the null controllability region coincides with the whole state space, R^n. This result includes a complete controllability condition derived by Yamamoto[7)]. The result can also estimate the size of null controllability region of linear systems with input magnitude constraints.

2. Problem Description and Deånitions

Let us consider the following nonlinear system.

$$\frac{dx}{dt} = f(x;u) \qquad (S)$$

Where, x is n-dimensional state vector and u is m-dimensional control vector. f is a function of x and u, and it is årst continuously diãerentiable with respect to x and u. The function satisåes $f(0;0) = 0$. Naturally, we assume the existence and uniqueness of the solution of this system's diãerential equation. [(*)]

The problem concerned here is to determine the area from where the initial state can be driven to the origin; it is so called as 'null controllability problem.'

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(*) The system satisåes Local Lipsichtz condition, then the existence and the uniqueness are locally guaranteed. In this paper, we only treat the problem within a speciåed region $x \in X_{*[0;T^*]}$ and the region $u \in \Omega_{*[0;T^*]}$, where Lipsichtz condition always holds. Thus we can assume the existence and the uniqueness.

Definition (controllability). We call a set $U_\delta = \{x \mid |x| \le \delta\}$ as controllable region when there exists a control function $u(t)$, $[0 \le t \le T]$ for a finite T, that can drive any state of starting from U_δ to the origin as $x(T) = 0$ If this region is not null i.e., $U_\delta \ne \phi$, then the system is controllable, and the region coincides with the whole space i.e., $U_\delta = R^n$, then the system is completely controllable.

The definition used here is so called "Null Controllability." The general concept of controllability is a little bit different from the defined controllability, however we can easily expand the result to the case of general controllability.

In the sequel, we evaluate the size of the region U_δ. Before discussing the detail, we prepare something to be necessary. Let us consider the reverse-time system of the system(1) and expand it at the origin. We will get the following system

$$\begin{aligned} \frac{dx}{dt} &= -f(x,u) \\ &= Ax + Bu + g(x,u) \qquad (1) \\ & \quad A = -\frac{\partial f}{\partial x}(0,0), \quad B = -\frac{\partial f}{\partial u}(0,0) \end{aligned}$$

By considering the above system, we can treat the null controllability as a problem of finding a reachable region from the origin. To find the reachable region is easier than to find the null controllability region.

Markus et.al. [2)] proved that original nonlinear systems are controllable if their linearized systems are controllable, namely the pairs of (A, B) are controllable pairs. Thus, in the sequel, we assume the pair (A, B) in the system (1) is controllable pair and consider about the system(1) instead of the system(S)

Let $X_r[0,T]$ be is a Banach space $C^n[0,T]$, which consists of continuous functions $x : [0,T] \to R^n$ with a constraint that their norms are less than r. Similarly, we define $\Omega_\mu[0,T]$ in a Banach space $C^m[0,T]$, which consists of continuous functions $u : [0,T] \to R^m$ with a constraint that their norms are less than μ. The definition of the norm used hears is as follows. Let define the norm of conventional number space by the following manner.

$$|x| = \sum_{i=1} |x_i|, \quad |u| = \sum_{i=1} |u_i|$$

The norm of the element x in the Banach space $C^n[0,T]$ and the norm of the element u in the Banach space $C^m[0,T]$ are defined as follows.

$$\|x\|_{[0,T]} = \max_{t \in [0,T]} |x(t)|$$
$$\|u\|_{[0,T]} = \max_{t \in [0,T]} |u(t)|$$

Though we do not distinguish notations of number space from notations of Banach space, readers can easily distinguish them. We define the norm of the matrix $A \in R^{n \times n}$, $B \in R^{n \times m}$ as the induced norm from the above defined vector norm. We use the same notation for the matrix norm as the vector norm. Let $|A| = a$ and $|B| = b$. Since the pair (A, B) is a controllable pair, there exists an inverse of the following matrix. Let $w(T)$ be the norm of this inverse matrix. $w(T) = |W_T^{-1}|$

$$W_T = \int_0^T e^{A(T-\tau)} BB' e^{A'(T-\tau)} d\tau \qquad (2)$$

A' and B' are the transposes of the matrices A and B. η is a scalar function defined on R^2:

$$\eta(r, \mu) = \max_{\substack{|x| \le r \\ |u| \le \mu}} |g(x, u)| \qquad (3)$$

Let us consider a state $x_u(t)$, $0 \le t \le T$, that started from the origin and driven by a control $u \in \Omega_{\mu[0,T]}$. The norm of the state $\|x_u\|_{[0,T]}$ depends on T and μ, and it also has an upper bound. Let $\xi(\mu, T)$ be this upper bound. We can show the following lemma.

Lemma 1. If there exists positive solution r of the following equation (4), then $\xi(\mu, T) = r$

$$r = arT + b\mu T + \mu(r, \mu)T \qquad (4)$$

Proof. It is enough to prove that the positive solution r of (4) satisfies $\|x_u\|_{[0,T]} \le r$ We assume $\|x_u\|_{[0,T]} > r$ and derive the contradiction. If $\|x_u\|_{[0,T]} > r$ then there exists some $\hat{t} < T$ which satisfies $|x_u(\hat{t})| = r$ and $\|x_u\|_{[0,\hat{t}]} = r$
On the other hand, $x_u(\hat{t})$ can be written by (5) and its norm is evaluated by (6).

$$\begin{aligned} x_u(\hat{t}) = & \int_0^{\hat{t}} Ax_u(\tau)d\tau + \int_0^{\hat{t}} Bu(\tau)d\tau \\ & + \int_0^{\hat{t}} g(x_u, u)d\tau \qquad (5) \end{aligned}$$

$$\begin{aligned} |x_u(\hat{t})| \le & \; a \cdot \|x_u\|_{[0,\hat{t}]} \cdot \hat{t} + b \cdot \mu \cdot \hat{t} \\ & + \eta(\|x_u\|_{[0,\hat{t}]}, \mu) \cdot \hat{t} \qquad (6) \end{aligned}$$

By substituting $r = |x_u(\hat{t})|$ and $r = \|x_u\|_{[0,\hat{t}]}$ in the equation(6), we get (7).

$$r \le ar\hat{t} + b\mu\hat{t} + \eta(r, \mu)\hat{t} \qquad (7)$$

Since $\hat{t} < T$ and $0 < ar + b\mu + \eta(r, \mu)$, then we get (8).

$$(ar + b\mu + \eta(r, \mu))\hat{t} < (ar + b\mu + \eta(r, \mu))T \qquad (8)$$

From(7) and (8), we conclude $r < arT + b\mu T + \eta(r, \mu)T$. Namely $r \ne arT + b\mu T + \eta(r, \mu)T$, and this conclusion contradicts with our assumption. Thus, we have proved

that the positive solution r of (4) is an upper bound that satisfies $\|x_u\|_{[0;T]} \le r$.
(Q.E.D.)

Next, we consider the case that the equation (4) has a positive solution. Let D be a set of parameters ρ and T, which let the equation (4) have a positive solution r. In this case, T defined as $T = r/(ar + b\rho + \varphi(r;\rho))$ is always an element of D for any given r and ρ, thus the parameter set is not empty, $D \neq \emptyset$.

3. Main Result and Its Proof

Let us consider a mapping $F_{x_f} : (\Omega_{r[0;T]} \times \Gamma_{\rho[0;T]}) \to (\Omega_{r[0;T]} \times \Gamma_{\rho[0;T]})$:

$$\forall x \in \Omega_{r[0;T]}, \forall u \in \Gamma_{\rho[0;T]}$$
$$F_{x_f}(x;u) = (x_{\tilde u};\tilde u) \quad (9)$$
$$\tilde u(t) = B' e^{A'(t-\tau)} W_T^{-1} \left\{ x_f - \int_0^T e^{A(T-\tau)} g(x;u)\,d\tau \right\}$$

If this mapping F_{x_f} has a fixed point $(x^*;u^*)$, then this fixed point $(x^*;u^*)$ satisfies the following equation (10) and (11).

$$x^*(t) = \int_0^t e^{A(t-\tau)} B u^*(\tau)\,d\tau + \int_0^t e^{A(t-\tau)} g(x^*;u^*)\,d\tau \quad (10)$$

$$u^*(t) = B' e^{A'(T-\tau)} W_T^{-1} \cdot \left\{ x_f - \int_0^T e^{A(T-\tau)} g(x^*;u^*)\,d\tau \right\} \quad (11)$$

From (10) and (11), we can conclude $x^*(0) = 0; x^*(T) = x_f$. Namely, x_f is a reachable point from the origin, in other words, x_f is in the null controllability region of system (S). Therefore, the problem to find U_s is equivalent to a problem to find a set of x_f where the mapping F_{x_f} has a fixed point. By considering this new problem, we can derive the following main theorem.

Theorem 1 (main). If the linearized system of (S) is controllable at the origin, then the null controllability region U_s includes the following region $\tilde U_s$.

$$\tilde U_s := \left\{ x \,\middle|\, |x| < \sup_{\substack{0<\rho<\infty \\ 0<T<\infty}} \left(\frac{\rho}{b e^{2aT}\omega(T)} - T e^{aT} \varphi(\psi(\rho;T);\rho) \right) \right\}$$

(proof) Let ρ^* and T^* be values that gives supremum of the above condition and r^* be as $\psi(\rho^*;T^*) = r^*$. Now we prove the mapping F_{x_f} has a fixed point in the region $\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]}$ for any $x_f \in \tilde U_s$. At first, we show F_{x_f} has the following three properties:

(P1) $F_{x_f}(\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]}) \subset (\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]})$
(P2) F_{x_f} is continuous.
(P3) $F_{x_f}(\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]})$ is relatively compact.

Evidence of (P1):

For some x and u included in the sets as $x \in \Omega_{r^*[0;T^*]}$, $u \in \Gamma_{\rho^*[0;T^*]}$, let $x_{\tilde u}$ and $\tilde u$ be their mapped values as $F_{x_f}(x;u) = (x_{\tilde u};\tilde u)$. Then $\tilde u$ can be described as follows.

$$\tilde u = B' e^{A'(T^*-t)} W_{T^*}^{-1} \left\{ x_f - \int_0^{T^*} e^{A(T^*-\tau)} g(x;u)\,d\tau \right\} \quad (12)$$

The norm of $\tilde u$ is evaluated by the following inequality.

$$\|\tilde u\|_{[0;T^*]} \le b e^{aT^*} \omega(T^*) \Big\{ |x_f| + T^* e^{aT^*} \varphi(r^*;\rho^*) \Big\} \quad (13)$$

By considering $x_f \in \tilde U_s$, we can derive (14) from (13)

$$\|\tilde u\|_{[0;T^*]} \le \rho^* \quad (14)$$

Since $\psi(\rho^*;T^*) = r^*$, $\|x\|_{[0;T]} \le r^*$. Then, we can conclude $x_{\tilde u} \in \Omega_{r^*[0;T^*]}$. Namely, we know the mapping F_{x_f} has the property (P1).

Evidence of (P2):

The region we are considering is $\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]}$. Since $F_{x_f}(\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]}) \subset \Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]}$ it is enough to consider the continuity inside the region: $\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]}$.

(1) The continuity of $\tilde u$ in respect of x and u is clearly known from the continuity of $g(x;u)$ in respect of x and u.

(2) Since $(x;u) \in \Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]}$, then the existence of $x_{\tilde u}$ is guaranteed. The assumption that the function f is partially 1-time continuously differentiable in respect of x and $\tilde u$ can show its continuity in respect of the parameter $\tilde u$. Moreover, since $\tilde u$ is continuous in respect of x and u, $x_{\tilde u}$ is continuous in respect of x and u inside the region $\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]}$. Thus, we can conclude (P2).

Evidence of (P3):

The property (P3) can be shown by using the theorem of Arzera[8]. Arzera's theorem indicates that if $F_{x_f}(\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]})$ is equicontinuous and uniformly bounded then it is relative compact. It is clear that $F_{x_f}(\Omega_{r^*[0;T^*]} \times \Gamma_{\rho^*[0;T^*]})$ is uniformly bounded.

The equicontinuity is shown by the following way. For any $(x; u)$ 2 F_{xf} (ü$_{rÉ[0;TÉ]}$ Ç ä$_{ñÉ[0;TÉ]}$); the following inequalities hold.

$$\begin{array}{l} \left\| \frac{d}{dt}u \right\|_{[0;T]} \;î\; añ^{É} \\ \left\| \frac{d}{dt}u \right\|_{[0;T]} \;î\; ar^{É} + bñ^{É} + g(r^{É}; u^{É}) \end{array} \quad (15)$$

Since the time derivatives are uniformly bounded, then we can conclude the equicontinuity. Namely, (P3) holds. ü$_{rÉ[0;TÉ]}$ Ç ä$_{ñÉ[0;TÉ]}$ is a closed convex set in Banach space $C^{n+m}{}_{[0;TÉ]}$. Thus with the properties (P1),(P2) and (P3) and Schauder's åxed point theorem can show that the map F_{xf} has a åxed point inside ü$_{rÉ[0;TÉ]}$ Ç ä$_{ñÉ[0;TÉ]}$. Therefore we can prove the main theorem.

(Q.E.D.)

The calculation of the term ë(ò(ñ; T); ñ) included in the main theorem may be diécult in a sense, however if we can get a solution r of (4) for some (ñ; T) 2 D, we can use this r in place of ò(ñ; T): Let r(ñ; T) be the solution of (4) for some (ñ; T) 2 D. Then the condition in the main theorem is rewritten as follows.

$$U_é \;õ\; \hat{U}_é \;ë\; x \;\Big|\; jxj < \sup_{(ñ;T)2D} \frac{ñ}{be^{aT}\,!(T)} \;Äe^{aT}(1 Ä aT)r(ñ; T) Ä b\,ñT \quad (16)$$

4. Several Corollaries

4.1 Linear Systems

In case of linear systems, $\hat{U}_é$ can be rewritten as follows by using ë(r; ñ) = 0

$$U_é \;õ\; x \;\Big|\; jxj < \sup_{\substack{0<ñ<1 \\ 0<T<1}} \frac{ñ}{be^{2aT}\,!(T)}$$

If we can set ñ as an arbitrarily large value, then $\hat{U}_é$ can also be arbitrary value, namely the system is completely controllable. If there exists some restriction on ñ, then the null controllability region is also restricted.

Corollary 1. In case of linear systems, the null controllability region with the admissible control ä$_{ñ[0;T]}$ is evaluated as follows:

$$U_é \;õ\; x \;\Big|\; jxj < \sup_{0<ñ<1} \frac{ñ}{be^{2aT}\,!(T)} \quad (17)$$

4.2 In Case of jg(x; u)j î L

Let us consider a case that the nonlinear term of the system(1) has an uniform upper bound L, which is independent of x or u. In this case, we can set ë(ò(ñ; T); ñ) = L and do the same discussion as in section 4.1. The result is summarized in the next corollary. This result is same as the result in the reference 7).

Corollary 2. If the nonlinear term of the system(1) satisåes jg(x; u)j î L, where L is constant, then the system is completely controllable with unbounded control input u. In case that the control input should be included in an admissible control set ä$_{ñ[0;T]}$, the null controllability region is described as follows.

$$U_é \;õ\; x \;\Big|\; jxj < \sup_{0<ñ<1} \left(\frac{ñ}{be^{2aT}\,!(T)} \;Ä\; Te^{aT}L \right)$$

(18)

The diãerence between the above evaluation and the evaluation for linear systems is just a term $Te^{aT}L$, which is derived by nonlinear eãect. If the magnitude of the admissible control is so small, the above evaluation may have no means.

4.3 In Case of jg(x; u)j < kjuj

If jg(x; u)j î kjuj, then we can derive the following corollary.

Corollary 3. If the nonlinear term g(x; u) of the system(1) satisåes jg(x; u)j î kjuj, where k is a constant that satisåes 1=k > $\inf_{0<T<1}$ $(Te^{3aT}b\,!(T))$, then the system(1) is completely controllable with unbounded control input u. If the control input is restricted in a admissible control ä$_{ñ[0;T]}$, the null controllability region is evaluated as:

$$U_é \;õ\; x \;\Big|\; jxj < \sup_{0<ñ<1} ñ \left(\frac{1}{be^{2aT}\,!(T)} \;Ä\; Te^{aT}k \right) \quad (19)$$

4.4 Bilinear Systems

Let us consider a case that the system(1) is a bilinear system, namely the nonlinear term can be described as $g(x; u) = \sum_{i=1} C_i x u_i$, where u_i is the i-th element of u. Let $jC_ij = c_i$ and $\sum_{i=1} c_i = c$. We can derive the following corollary.

Corollary 4. The null controllability region of the following bilinear system

$$\underline{x} = Ax + Bu + \sum_{i=1} C_i x c_i$$

can be evaluated by:

$$U_s \;õ\; x \;\Big|\; jxj < \sup_{\substack{0<ñ<1 \\ 0<T<1}} \left(\frac{ñ}{be^{2aT}\,!(T)} \;Ä\; ñ^2 bcT^2 \exp(a + cñ)T \right) \quad (20)$$

The above corollary is easily derived from the facts that ò(ñ; T) = bñT exp(ã + cñT) and ë(r; ñ) = cr ñ.

4.5 In Case of $g(x; u) = x^0Qx$

Let us consider a case that the nonlinear term $g(x; u)$ of the system(1) is quadratic, namely $g(x; u) = x^0Qx$. Since we can set as ë(r; ñ) = r^2q and $q = jQj$, ò(ñ; T) is written as follows by using (4).

$$ò(ñ; T) = r(ñ; T) = \frac{1 Ä aT Ä \sqrt{(1 Ä aT)^2 Ä 4bñT^2q}}{2Tq} \quad (21)$$

From the relation(16), we can conclude the next corollary.

Corollary 5. The null controllability region $U_é$ of the nonlinear system,

$$\underline{x} = Ax + Bu + x^0Qx$$

is evaluated as follows.

$$U_s \; õ \; ö \; x \; â \; jxj < \sup_{(ñ;T)2D} î \frac{ñ}{be^{aT} ! (T)} + e^{aT} bñT \; Ä \; e^{aT}(1 Ä aT)$$

$$Å \; \frac{1 Ä aT Ä \sqrt{(1 Ä aT)^2 Ä 4bñT^2q}}{2Tq} \; ö \; õ \; ïõ \quad (22)$$

where, $q = jQj$ and $D = f(ñ; T)j1 Ä aT > 2T \sqrt{bñq} > 0g$

5. Conclusion

In this paper, we have evaluated size of the null controllability region of general nonlinear systems without adding any conditions except the linearized systems at the origin are controllable. We have also derived several corollaries for speciåed nonlinear systems such as bilinear systems, quadratic systems and so on. There have been many researches using functional analysis and åxed-point theorem, but almost all of them try to derive qualitative properties by adding strict conditions to the target nonlinear systems. On the other hand, this paper derives the quantitative properties without adding any speciåc restriction. This result is applicable to many calsses of nonlinear systems and it may derive various quantitative results as well as qualitative results.

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