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Classes of Petri Nets That a Necessary and Sufficient Condition for Reachability is Obtainable

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The central issue of this paper is to find a class of Petri nets that a necessary and sufficient condition for reachability is obtainable. For this purpose, several new classes of Petri nets are defined by structural conditions related to directed circuits.

A necessary and sufficient condition for reachability is obtained for *trap circuit Petri nets* (TC nets), where a Petri net is called a TC net if the set of places on any directed circuit forms a trap, and for *deadlock circuit Petri nets* (DC nets), where a Petri net is called a DC net if the set of places on any directed circuit forms a deadlock. The class of TC nets is a subclass of *normal Petri nets*. For normal Petri nets, a sufficient condition for reachability is obtained.

Reachability for any conflict-free Petri net can be decided by finding a legal firing sequence for a finite number of minimal solutions of the state equation. This property also holds for larger classes of Petri nets. These are a class of non-decreasing circuit Petri nets (NDC nets), where a Petri net is called an NDC net if the number of tokens on any directed circuit is not decreased by any firing of transitions, and a class of non-increasing circuit Petri nets (NIC nets), where a Petri net is called an NIC net if the number of tokens on any directed circuit is non increased by any firing of transitions. The class of NDC nets and the class of NIC nets are subclasses of TC nets and DC nets, respectively.

Key Words: discrete event systems, Petri nets, reachability, deadlock, trap

1. Introduction

Petri nets are widely studied as a model of discrete event systems, and have various kinds of application areas such as computer hardware/software, communication protocol, sequential control, and knowledge representation [1]. Petri nets represent structure of systems. This enables us to characterize each system by its structure, and to analyze it using specific properties on the structure.

One of the central issues in the analysis of Petri nets is the reachability problem, the problem to decide whether a given goal marking is reachable from the initial marking. It was already shown that the reachability problem is decidable [2]. In addition, necessary and sufficient conditions for reachability were obtained for some subclasses of Petri nets such as marked graphs and conflict-free Petri nets [3),4]. On the other hand, classes of Petri nets with semilinear reachability sets were studied with respect to *persistency*, which is a property on behavior of Petri nets [5)~7]. All of these subclasses of Petri nets are defined by structural properties.

A Petri net is defined by local description on the connection between two kinds of nodes, places and transitions. Local structure composes macrostructure. As such macrostructure, we consider conditions related to the set of places on each directed circuit, and study the reachability problem for Petri nets with such structure.

First we define two new classes of Petri nets. One is the class of *trap circuit Petri nets* (TC nets), where a Petri net is called a TC net if the set of places on any directed circuit forms a trap, and the other is the class of *deadlock circuit Petri nets* (DC nets), where a Petri net is called a DC net if the set of places on any directed circuit forms a deadlock. The condition of being a TC net is a special case of being *a normal Petri net*, which was studied as a class of Petri nets having semilinear reachability sets. A sufficient condition for reachability can be obtained for normal Petri nets.

Nonnegative integer solutions of the state equation in a given Petri net play the important role in the analysis of reachability. In conflict-free Petri net, reachability can be decided by finding a legal firing sequence for a finite number of minimal solutions. We show that this property is also valid in larger classes of Petri nets, such as *non-decreasing circuit Petri nets* (NDC nets) and *non-*

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increasing cuircuit Petri nets (NIC nets).

Finally, we discuss relationship between these new classes and existing classes of Petri nets. These results on subclasses of Petri nets can be applied to analysis, design, and control of discrete event systems represented by Petri nets.

2. Preliminaries

Let $\mathbb{N}$ denote the set of nonnegative integers, and let $\mathbb{N}^k$ denote the set of k dimensional nonnegative integer vectors.

A Petri net is a quadruple $M = (P, T, A, m^0)$, where $P = \{p_1, p_2, \cdots, p_v\}$ is a finite set of *places*, $T = \{t_1, t_2, \cdots, t_w\}$ is a finite set of *transitions*, $A : P \times T \cup T \times P \to \{0, 1\}$ is a function representing arcs between places and transitions, and $m^0 \in \mathbb{N}^v$ is *the initial marking.* We call the triple $C = (P, T, A)$ *a Petri net structure.* Note that Petri nets considered in this paper are assumed to be single-arc.

When $A(t_j, p_i) = 1$, t_j is called *an input transition* of p_i, and p_i is called *an output place* of t_j. When $A(p_i, t_j) = 1$, p_i is called *an input place* of t_j, and t_j is called *an output transition* of p_i.

We define two matrices $B^+ = [b^+_{ij}]$ and $B^- = [b^-_{ij}]$, where $b^+_{ij} = A(t_j, p_i)$ and $b^-_{ij} = A(p_i, t_j)$. B^+ is called *the input incidence matrix* and B^- is called *the output incidence matrix.* Let $B = B^+ - B^-$.

For $s \in P \cup T$, let

$$^\bullet s = \{r \mid r \in P \cup T \wedge A(r, s) = 1\},$$

$$s^\bullet = \{r \mid r \in P \cup T \wedge A(s, r) = 1\}.$$

We say that a transition t_j is *enabled* in a marking m if

$$m - B^- e_j \geq \mathbf{0} \tag{1}$$

where e_j denotes the unit vector with value 1 in the j-th component. By *firing* of transition t_j, the marking changes to the following m':

$$m' = m + Be_j \tag{2}$$

A finite sequence of transitions of arbitrary length is called *a firing sequence.* Let T^* denote the set of all firing sequences including the empty firing sequence. Given a firing sequence σ, let $\psi(\sigma)$ denote *the firing count vector* of σ, which is a w dimensional nonnegative integer vector and its j-th component indicates the number of occurrences of transition t_j in the sequence.

A firing sequence $\sigma = s_1 s_2 \cdots s_r,\ s_i \in T\,(i = 1, 2, \cdots, r)$ is called *legal* in marking m, denoted by $m \xrightarrow{\sigma}$, if each transition $s_i\,(i = 1, \cdots, r)$ can fire from m in this order. Firing of σ changes the marking m to m' defined by

$$m' = m + B\psi(\sigma) \tag{3}$$

We denote this situation by $m \xrightarrow{\sigma} m'$. Let m, m' be two markings. Then m' is called *reachable* from m if there exists a firing sequence σ such that $m \xrightarrow{\sigma} m'$. We shall simply write $m \to m'$ to denote only the reachability of m' from m. We say that a vector $x \in \mathbb{N}^w$ is *feasible* in m if there exists a firing sequence σ such that $\psi(\sigma) = x$ and $m \xrightarrow{\sigma} m'$.

Using (3), we immediately have the following lemma.

Lemma 1. Let $M = (C, m^0)$ be a Petri net. If $m^0 \to m^T$, then there exists $x \in \mathbb{N}^w$ such that $m^T = m^0 + Bx$.

Given a Petri net $M = (C, m^0)$, the set of markings reachable from the initial marking m^0, $\mathcal{R}(M) = \{m \mid m^0 \to m\}$, is called *the reachability set* of M.

Let $x = [x_i] \in \mathbb{N}^w$ be a firing count vector. We define the subnet M_x induced by x is defined as the restriction of M to T_x and P_x, where T_x is the set of transition t_i with $x_i > 0$ and P_x is the set of places adjacent to transitions in T_x.

A sequence $c = c_1 c_2 \cdots c_k$ of places and transitions is called *a directed path* if $A(c_i, c_{i+1}) = 1\,(i = 1, 2, \cdots, k-1)$, and is called *a directed circuit* if $c_1 = c_k$ in addition.

A Petri net M is called *weakly persistent* if the following holds for any marking $m \in \mathcal{R}(M)$ and for any firing sequences $\sigma, \sigma' \in T^*$: if $m \xrightarrow{\sigma} \wedge\ m \xrightarrow{\sigma'} \wedge\ \psi(\sigma') \leq \psi(\sigma)$, then there exists a firing sequence $\sigma'' \in T^*$ such that $m \xrightarrow{\sigma' \cdot \sigma''} \wedge\ \psi(\sigma' \cdot \sigma'') = \psi(\sigma)$.

3. Deadlocks and Traps

For a set $Q \subseteq P$ of places, let

$$I(Q) = \{t \mid t \in T \wedge {}^\bullet t \cap Q = \emptyset \wedge t^\bullet \cap Q \neq \emptyset\},$$

$$O(Q) = \{t \mid t \in T \wedge {}^\bullet t \cap Q \neq \emptyset \wedge t^\bullet \cap Q = \emptyset\}.$$

$I(Q)$ is the set of input transitions of Q from the outside, and $O(Q)$ is the set of output transitions of Q to the outside.

Q is called *a deadlock*(1) when $I(Q) = \emptyset$. A place without any input transitions constitutes a deadlock by itself, and is called *a single-place deadlock.* A deadlock without any single-place deadlocks is called *a circuit deadlock.*

Q is called *a trap* when $O(Q) = \emptyset$. A place without any output transitions constitutes a trap by itself, and is called *a single-place trap.* A trap without any single-place traps is called *a circuit trap.*

A deadlock (trap) is called *token-free* if any place in it

(1) A deadlock is often called a siphon in literature.

has no tokens. We obtain the following result on deadlocks and traps.

Lemma 2. Let $M = (C, m^0)$ be a Petri net. Suppose that there exists a firing sequence $\sigma \in T^*$ such that $m^0 \xrightarrow{\sigma} m^T$, and let $x = \psi(\sigma)$. Then the following holds:

(i) M_x has no token-free deadlocks in m^0.

(ii) M_x has no token-free traps in m^T.

Proof. In the firing of σ, every transition in M_x fires at least once by the definition. If there exists a token-free deadlock in m^0, then every transition having an output place in the deadlock also has at least one input place in the deadlock, and therefore it is not enabled. That is, any transition having an input place in the deadlock cannot be enabled forever, and we obtain (i).

If there exists a trap with at least one token, then the number of tokens in the trap is not decreased to 0 by any firing of transitions. Therefore, if a trap has no tokens after firing of a firing sequence, then every transition having an input place in the trap has not fired, i.e., it is not in the firing sequence, and we obtain (ii). ∎

In what follows, we shall abbreviate a deadlock without any tokens as TFD, and a trap without any tokens as TFT.

The following result is known [10].

Lemma 3. If no transitions are enabled in a marking m, then there exists at least one TFD in m.

Lemma 4. Let $M = (C, m^0)$ be a Petri net, let m be a marking, and let $x \in \mathbb{N}^w$. if $m + Bx \geq \mathbf{0}$, then M_x has no single-place TFD's in m.

Proof. If M_x has a single-place TFD, then the number of tokens in the place need to be negative in marking $m + Bx$. This contradicts $m + Bx \geq \mathbf{0}$. ∎

Lemma 5. Let $M = (C, m^0)$ be a Petri net and let $x \in \mathbb{N}^w$. If the following (i) and (ii) hold, then $m^0 + Bx \in \mathcal{R}(M)$.

(i) $m^0 + Bx \geq \mathbf{0}$;

(ii) For any reachable marking $m \in \mathcal{R}(M)$: if there exists $y \in \mathbb{N}^w$ such that $m = m^0 + By$ and $y \leq x$, then M_{x-y} has no circuit TFD's in m.

Proof. From (i) and Lemma 4, M_x has no single-place TFD's in m^0. Moreover, no circuit TFD's exist by (ii). By Lemma 3, M_x has at least one enabled transition in m^0. Let m' be the marking after firing of one of such transitions, and let x' be the remaining firing count vector. Then $m' + Bx' \geq \mathbf{0}$ holds. By the same argument, $M_{x'}$ has at least one enabled transition. Repeating this process until the remaining firing count becomes $\mathbf{0}$, we will reach to the marking $m^0 + Bx$. ∎

In the above proof, the transition to be fired at each step is selected arbitrarily from enabled transitions such that the corresponding component of the remaining firing count vector is positive. From this fact, we have the following lemma.

Lemma 6. Let $M = (C, m^0)$ be a Petri net. If the condition (ii) of Lemma 5 holds for any $x \in \mathbb{N}^w$ such that $m^0 + Bx \in \mathcal{R}(M)$, then M is weakly persistent.

4. A Necessary and Sufficient Condition for Reachability in Trap Circuit Petri Nets

In this section, we first define a class of Petri nets in which no TFD's are newly generated by any firing of transitions, and obtain a necessary and sufficient condition for reachability in the class.

A Petri net is said to be *a trap circuit Petri net* (TC net) if the set of places on each directed circuit is a trap.

Lemma 7. Let $M = (C, m^0)$ be a TC net and let $x \in N^w$. Suppose that M_x has no circuit TFD's in m^0. Then the following holds for any $m \in \mathcal{R}(M)$: if there exists $y \in N^w$ such that $m = m^0 + By$ and $y \leq x$, then M_{x-y} has no circuit TFD's in m.

Proof. Let Q be any circuit deadlock of M_{x-y}. Since M_{x-y} is a subnet of M_x, Q is contained in M_x. Since M is a TC net, Q is a circuit trap in M.

(i) The case that $I(Q) = \emptyset$ in M_x: Since Q is a circuit deadlock in M_x, Q has token(s) in m^0 by the assumption. Since Q is a circuit trap in M, the number of tokens in Q cannot become 0 by any firing of transitions, and Q still has token(s) in m.

(ii) The case that $I(Q) \neq \emptyset$ in M_x: Since $I(Q) = \emptyset$ in M_{x-y}, firing of transitions in $I(Q)$ puts some token(s) on places in Q. Since Q is a circuit trap in M, Q still has token(s) in m. ∎

Theorem 1. Let $M = (C, m^0)$ be a TC net and let m^T be a marking. Then $m^0 \to m^T$ if and only if there exists $x \in \mathbb{N}^w$ such that

(i) $m^T = m^0 + Bx$;

(ii) M_x has no circuit TFD's in m^0.

Proof. Necessity: (i) is obtained from Lemma 1, and (ii) from Lemma 2.

Sufficiency: From (ii) and Lemma 7, we obtain the condition (ii) of Lemma 5. Hence, we have $m^0 \to m^T$ by Lemma 5. ∎

We also have the following result by Theorem 1, Lemma 6, and Lemma 7.

Theorem 2. Any TC net is weakly persistent for arbitrary initial marking.

5. A Necessary and Sufficient Condition for Reachability in Dealock Circuit Petri Nets

For a Petri net structure $C = (P, T, A)$, let $C^{-1} = (P, T, A^{-1})$ is called *the inverse* of C, where $A^{-1} : P \times T \cup T \times P \to \{0, 1\}$ is a function such that for each $r, s \in P \cup T$: $A^{-1}(r, s) = A(s, r)$. We immediately have the following.

Lemma 8. Let $M = (C, m^0)$ be a Petri net. Then $m^0 \stackrel{\sigma}{\to} m^T$ in M if and only if $m^T \stackrel{\sigma^{-1}}{\to} m^0$ in $M^{-1} = (C^{-1}, m^T)$, where σ^{-1} is the sequence obtained by arranging each transition of σ in the reverse order.

A Petri net is said to be *a deadlock circuit Petri net* (DC net) if the set of places on any directed circuit is a deadlock. It is easy to see that any DC net is the inverse of a TC net. From Theorem 1 and Lemma 8, we obtain a necessary and sufficient condition for reachability in DC nets.

Theorem 3. Let $M = (C, m^0)$ be a DC net and let m^T be a marking. Then $m^0 \to m^T$ if and only if there exists $x \in \mathbb{N}^w$ such that

(i) $m^T = m^0 + Bx$;

(ii) M_x has no circuit TFT's in m^T.

A DC net is not necessarily weakly persistent, but from Theorem 3 we have the following algorithm to obtain a firing sequence to m^T.

Algorithm:

Given a firing count vector $x = [x_i] \in \mathbb{N}^w$ such that $m^T = m^0 + Bx$:

Step 1. Find a transition t_j that satisfies all of the following three conditions, and fire it.

(i) $x_j > 0$:

(ii) t_j is enabled;

(iii) Let x' be the remaining firing count vector after a firing of t_j. Then $M_{x'}$ has no TFT's in m^T.

Step 2. If $x = \mathbf{0}$, then halt. Otherwise, let $x := x - e_j$ and go to step 1.

6. A Sufficient Condition for Reachability in Normal Petri Nets

A Petri net is said to be *a normal Petri net* if the set of places on any directed circuit contains a circuit trap(2).

Let M be a normal Petri net. We say that a marking m is called *sufficient* for M if the following holds. Let Q be the set of places on any directed circuit. If $I(Q) = \emptyset$ or $O(Q) \neq \emptyset$, then Q contains a trap having token(s) in m.

(2) This definition of normal Petri nets is equivalent to that in [7], but is in a different form.

Such a set Q of places is a deadlock of M or may be a deadlock of a subnet of M.

Lemma 9. Let $M = (C, m^0)$ be a normal Petri net such that m^0 is sufficient for M_x, and let $x \in \mathbb{N}^w$. Then the following holds for any reachable marking $m \in \mathcal{R}(M)$: if there exists $y \in \mathbb{N}^w$ such that $m = m^0 + By$ and $y \leq x$, then M_{x-y} has no circuit TFD's.

Proof. Let Q be any circuit deadlock in M_{x-y}. Since M_{x-y} is a subnet of M_x, Q is contained in M_x. There are two cases:

(i) The case that $I(Q) = \emptyset$ and $O(Q) \neq \emptyset$ in M_x: In the subnet M_x, Q contains a trap with token(s) in the initial marking m^0. Therefore, Q has token(s) also in m.

(ii) The case that $I(Q) \neq \emptyset$ and $O(Q) = \emptyset$ in M_x: Since $I(Q) = \emptyset$ in M_{x-y}, firing of transitions in $I(Q)$ put token(s) on places in Q before the marking reached to m. Since $O(Q) = \emptyset$ (i.e., Q is a trap) in M_x, Q has token(s) also in m. ∎

Theorem 4. Let $M = (C, m^0)$ be a normal Petri net. Then $m^0 \to m^T$ if there exits $x \in \mathbb{N}^w$ such that

(i) $m^T = m^0 + Bx$;

(ii) m^0 is sufficient for M_x.

Proof. From (ii) and Lemma 9, we have the condition (ii) of Lemma 5. Therefore, we obtain $m^0 \to m^T$ by Lemma 5. ∎

Any TC net is a normal Petri net, and any marking in a TC net is sufficient for the net as a normal Petri net.

Using Lemma 6 and Lemm 9, we have the following theorem.

Theorem 5. A normal Petri net is weakly persistent if the initial marking is sufficient.

7. Feasibility of a Minimal Solution of State Equation

Let $M = (C, m^0)$ be a Petri net and let m^T be a marking. Then $m^T - m^0 = Bx$ is called *the state equation* with respect to the reachability. By (3), we know that the firing count vector of any firing sequence legal in the initial marking is a nonnegative integer solution of the state equation. Since we need only nonnegative integer solutions in checking the rechability, we shall simply call a solution to denote a nonnegative integer solution of the state equation.

Let x be a solution of the state equation $m^T - m^0 = Bx$ and let y be any nonnegative integer solution of a homogenous equation $\mathbf{0} = By$. Then any vector in the form $x + k \cdot y,\ k \in \mathbb{N}$ is also a solution of the state equation. Therefore, there exist infinitely many solutions in general. However, the following results was obtained for conflict-free Petri nets. This implies that the reachability can be decided by checking feasibility of a finite number of minimal solutions.

Lemma 10. Let $M = (C, m^0)$ be a conflict-free Petri net and let α be a solution of the state equation $m^T - m^0 = Bx$. If α is feasible, then any solution β such that $\beta \leq \alpha$ is also feasible.

TC nets do not necessarily have this property. For example, consider $m^0 = [0,0,0]^t$ and $m^T = [0,0,1]^t$ for the TC net in Fig. 1. The solutions of the state equation are $x = [1,1,1,0,0]^t + k[1,0,0,1,1]^t$, $k \in \mathbb{N}$. Solution $\alpha = [2,1,1,1,1]^t$ is feasible, because there exists a legal firing sequence $\sigma = t_5t_3t_1t_2t_1t_4$ such that $\psi(\sigma) = \alpha$, but solution $\beta = [1,1,1,0,0]^t$ is not. This is because M_α has no TFD's but M_β has a TFD in m^0.

Now we define a class of Petri net by using a stronger condition than that of TC nets. A Petri net is said to be *a non-decreasing circuit Petri net* (NDC net) if for each directed circuit, the number of tokens in the circuit is not decreased by any firing of transitions. It is easy to verify that any NDC net is a TC net.

Lemma 11. Let $M = (C, m^0)$ be an NDC net and let α, β be solutions of $m^T - m^0 = Bx$ such that $\beta \leq \alpha$. If M_α has no circuit TFD's in m^0, then M_β has no circuit TFD's.

Proof. Let Q be any circuit deadlock in M_β. If Q is a circuit deadlock of M_α, then Q is not a TFD of M_β. If Q is not a circuit deadlock of M_α, then there exists $t_j \in I(Q)$ in M_α such that $e_j \leq \alpha - \beta$. Considering $B(\alpha - \beta) = \mathbf{0}$, this contradict the property that that the number of tokens on any directed circuit does not decrease. ∎

Since any NDC net is a TC net, we have the following corollary by Theorem 1 and Lemma 11.

Corollary 1. Let $N = (C, m^0)$ be an NDC net and let m^T be a marking. Then $m^0 \to m^T$ if and only if there exists $x \in \mathbb{N}^w$ such that

(i) x is a minimal nonnegative integer solution of $m^T - m^0 = Bx$;

(ii) M_x has no circuit TFD's in m^0.

We can obtain a similar result on the inverse of NDC nets. A Petri net is said to be *a non-increasing circuit Petri net* (NIC net) if for each directed circuit, the number of tokens in the circuit is not increased by any firing of transitions. Any NIC net is a DC net.

By Corollary 1 and Lemma 8, we have the following.

Corollary 2. Let $N = (C, m^0)$ be an NIC net, and let m^T be a marking. Then $m^0 \to m^T$ if and only if there exists $x \in \mathbb{N}^w$ such that

(i) x is a minimal nonnegative integer solution of $m^T - m^0 = Bx$;

(ii) M_x has no circuit TFT's in m^T.

8. Discussion

In Fig. 2, we show relationship among new subclasses of Petri nets studied in this paper, together with existing classes of Petri nets, marked graphs, forward-conflict-free nets (FCF nets), and backward-conflict-free (BCF nets) nets.

The condition that (i) the state equation has a solution and (ii) M_x has no TFD's in the initial marking (M_x has no TFT's in the goal marking), is necessary and sufficient for reachability in the classes contained in TC nets (DC nets, resp.). Moreover, we can replace the above condition (i) with the existence of a minimal solution in the classes contained in NDC nets (NIC nets, resp.).

While any normal Petri net is weakly persistent if the initial marking is sufficient, any TC net is weakly persistent for arbitrary initial marking. However, inverses of these nets are not necessarily weakly persistent.

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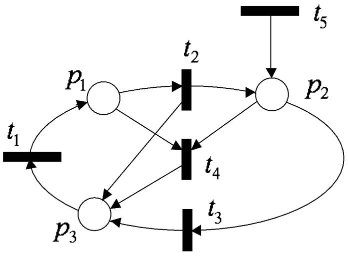

Fig. 1 An example of TC nets in which a minimal solution is not feasible.

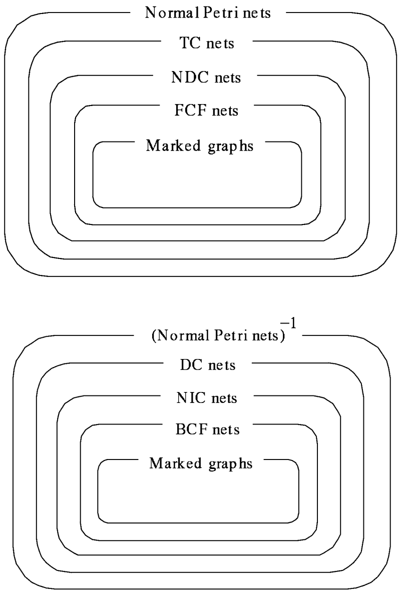

Fig. 2 Relations among classes of Petri nets.

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