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Maneuverability of Master-Slave Telemanipulation Systems

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Maneuverability of master-slave telemanipulation systems is difficult to evaluate exactly, since it seems intuitive sense for human operators. However, if we could not evaluate the system performance quantitatively, it would be impossible to decide what kind of master-slave system is desirable and compare the performance among the several control schemes. In this paper, we propose a way to evaluate the maneuverability of master-slave telemanipulation systems. We first analyze a one degree-of-freedom system considering the operator and object dynamics. Secondly, we define some ideal responses of master-slave systems and derive the conditions to achieve these responses. Lastly, a quantitative performance index is given in order to evaluate the maneuverability of the system. This index examines how close the actual response is to the ideal one. A numerical example is shown where three conventional control schemes are evaluated by the proposed performance index.

Key Words: master-slave manipulator, teleoperation, telemanipulation, maneuverability, bilateral control

1. Introduction

Research on master-slave teleoperation has a long history, back to the origin of the robotics research. Master-slave manipulators have been widely applied to many hazardous environments, such as nuclear power plants and bottoms of the sea, where remote operations are required. The application area of master-slave teleoperation is getting wider toward, for example, tasks in space. Total system including a master arm, a slave arm and their control schemes is called master-slave telemanipulation system. Hereafter, it will be called master-slave system for short.

Looking back the previous studies on master-slave systems, isomorphic configurations of master and slave arms had been taken over by different configurations since the performance of digital computers improved fast enough to execute coordinate transformation in real-time [1) 2)]. Control methodologies have been improved from unilateral control, which was used at the beginning of the development of servo manipulators, to bilateral control, which mimics the property of mechanical master-slave manipulators [3)]. Since then, however, we have been using somewhat classical control schemes, such as symmetric position servo type, force reflection type, and force reflecting servo type. Maneuverability of the present master-slave systems is still far from satisfactory, requiring the operators to have special skills and being difficult for them to continue the operation for long time [4)].

Certainly, the maneuverability of master-slave systems depends on the quality of mechanical design of the manipulators. However, it is also true that the maneuverability depends on what kind of control scheme is implemented. However, there has been little discussion about how to evaluate different control schemes quantitatively. This is because "maneuverability" is essentially an intuitive matter for the human operators and it is difficult to evaluate such an intuitive aspect quantitatively. Another problem is that theoretical analysis of master-slave systems is complex because both the operator dynamics and the remote environment dynamics should be taken into account. Hannaford [5)] also pointed out the importance of considering the whole system including not only the arm dynamics but also the remote object and operator dynamics for the system stability analysis.

In this paper, we propose a way to evaluate the maneuverability of master-slave systems quantitatively. For this purpose, we first suppose a simple master-slave system that has one degree-of-freedom (DOF). We also model the operator and remote environment dynamics [6) 7)]. Secondly, we define three ideal responses of master-slave systems by paying attention to position and force responses of the master and slave arms. We then derive the conditions to achieve these ideal responses [8)]. Lastly, we show performance indices that examine how close the actual responses are to the ideal ones. Using these indices, one can evaluate the maneuverability of the system quantitatively.

2. Modeling of One DOF System

2.1 Modeling of arms, remote environment, and operator

Most master-slave systems consist of arms with multiple DOF. In this paper, however, a one DOF system is considered in order to make the problem simple.

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Dynamics of the master arm and the slave arm is given by the following equations:

$$\tau_m + f_m = m_m \ddot{x}_m + b_m \dot{x}_m, \quad (1)$$

$$\tau_s - f_s = m_s \ddot{x}_s + b_s \dot{x}_s, \quad (2)$$

where x_m and x_s denote the displacements of the master and slave arms, respectively. In addition, m_m and b_m represent mass and viscous coefficient of the master arm, whereas m_s and b_s are those of the slave arm. Moreover, f_m denotes the force that the operator applies to the master arm, and f_s denotes the force that the slave arm applies to the remote object. Actuator driving forces of the master and slave arms are represented by τ_m and τ_s, respectively.

Dynamics of the remote environment with which the slave arm interacts is modeled by the following linear system:

$$f_s = m_w \ddot{x}_s + b_w \dot{x}_s + c_w x_s, \quad (3)$$

where m_w, b_w and c_w denote mass, viscous coefficient and stiffness of the environment, respectively. As can be seen that the displacement of the environment is represented by x_s in eq.(3), we assumed that the slave arm is rigidly contacting with the environment or firmly grasping the remote object so that it does not depart from the environment.

Likewise, dynamics of the operator will be approximated by a simple spring-damper-mass system:

$$\tau_{op} - f_m = m_{op} \ddot{x}_m + b_{op} \dot{x}_m + c_{op} x_m, \quad (4)$$

where m_{op}, b_{op} and c_{op} denote mass, viscous coefficient, and stiffness of the operator, respectively, whereas τ_{op} means force generated by the operator's muscles. Similarly to eq.(3), the displacement of the operator is represented by x_m in eq.(4) because we assume that the operator is firmly grasping the master arm and he never release the master arm during the operation. It should be noted that the parameters of the operator dynamics may change during the operation. For example, Akazawa et al.[9] reported that b_{op} and c_{op} are proportional to the sum of the forces exerted by flexor and extensor muscles. Therefore, these parameters are actually not constant.

2.2 Generalized control schemes of master and slave arms

Let the following control schemes be considered as a general framework for determining the actuator inputs of the master and slave arms:

$$\tau_m = \begin{bmatrix} K_{mpm} + K'_{mpm}\frac{d}{dt} + K''_{mpm}\frac{d^2}{dt^2} & K_{mfm} \end{bmatrix} \begin{bmatrix} x_m \\ f_m \end{bmatrix}$$
$$- \begin{bmatrix} K_{mps} + K'_{mps}\frac{d}{dt} + K''_{mps}\frac{d^2}{dt^2} & K_{mfs} \end{bmatrix} \begin{bmatrix} x_s \\ f_s \end{bmatrix}, \quad (5)$$

$$\tau_s = \begin{bmatrix} K_{spm} + K'_{spm}\frac{d}{dt} + K''_{spm}\frac{d^2}{dt^2} & K_{sfm} \end{bmatrix} \begin{bmatrix} x_m \\ f_m \end{bmatrix}$$
$$- \begin{bmatrix} K_{sps} + K'_{sps}\frac{d}{dt} + K''_{sps}\frac{d^2}{dt^2} & K_{sfs} \end{bmatrix} \begin{bmatrix} x_s \\ f_s \end{bmatrix}, \quad (6)$$

where K_{mpm}, K'_{mpm}, K''_{mpm} and K_{mfm} are feedback gains of the master arm position, velocity, acceleration and force, whereas K_{mps}, K'_{mps}, K''_{mps} and K_{mfs} are gains of the slave arm position, velocity, acceleration and force, respectively. These eight gains specify the input τ_m. Similarly, K_{spm}, K'_{spm}, K''_{spm}, K_{sfm}, K_{sps}, K'_{sps}, K''_{sps}, and K_{sfs} specify the input τ_s. Equations (5) and (6) are extension of the formulation by Fukuda *et al.*[10] Based on their formulation, we added velocity and acceleration terms.

In eqs.(5) and (6), we assume an ideal situation where time delay due to the data transmission between the master and slave sites is negligible. Although it would be possible to set up a more general form by adding differential terms of force, we do not consider such terms for simplicity. Conventional control schemes such as symmetric position servo type, force reflection type and force reflecting servo type can be represented as a special case of eqs. (5) and (6) with appropriate gains.

2.3 Representation of master-slave system by two-terminal-pair network

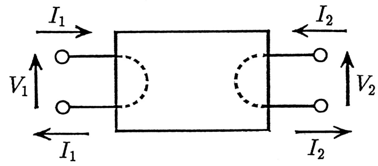

Fig. 1 Two-terminal-pair network.

Two-terminal-pair network is usually used for analyzing electrical circuits. Impedance matrix $\boldsymbol{Z}$ is defined from the relations between current and voltage of a two-terminal-pair network as follows:

$$V_1 = z_{11} I_1 + z_{12} I_2, \quad (7)$$

$$V_2 = z_{21} I_1 + z_{22} I_2, \quad (8)$$

$$\boldsymbol{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}, \quad (9)$$

where I_1 and I_2 denote current at each terminal pair, and V_1 and V_2 denote voltage at each terminal pair, as shown in **Fig.1**, respectively.

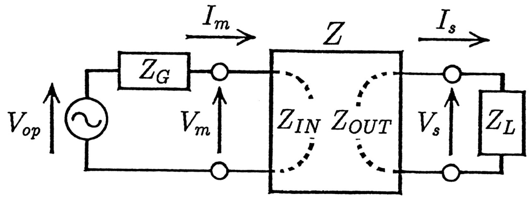

Fig. 2 Connection of power source and load to two-terminal-pair network.

Let us consider a two-terminal-pair network where each terminal pair is connected to a power source and a load as shown in **Fig.2**. Regarding the power source as an operator, the load as a remote environment and the two-terminal-pair network itself as a master-slave system, the whole system can be represented by a electric circuit shown in **Fig.2**. The correspondences between the modeling in the previous section and the circuit representation in **Fig.2** are given as follows:

velocity of the master arm $\dot{x}_m$	$\longleftrightarrow$	current I_m
velocity of the slave arm $\dot{x}_s$	$\longleftrightarrow$	current I_s
operator's force τ_{op}	$\longleftrightarrow$	voltage V_{op}
force at the master side f_m	$\longleftrightarrow$	voltage V_m
force at the slave side f_s	$\longleftrightarrow$	voltage V_s

Representation of the master-slave system by a two-terminal-pair network is not a new idea [11]. However, the framework shown in **Fig.2**, where the operator and the remote environment are considered as a power source and a load connected to the network, was recently given by Raju [12] [13]. This circuit representation does not change the nature of the problem at all. However, this representation makes the formulations compact and easy to handle.

In addition to the above correspondences, the actuator driving forces τ_m and τ_s are rewrote to voltages T_m and T_s, respectively. Then, eqs.(1), (2), (5) and (6) are transformed from time domain into s-domain as follows:

$$T_m + V_m = (m_m s + b_m)I_m \triangleq Z_m I_m, \tag{10}$$

$$T_s - V_s = (m_s s + b_s)I_s \triangleq Z_s I_s, \tag{11}$$

$$\begin{aligned} T_m &= \begin{bmatrix} K''_{mpm}s + K'_{mpm} + K_{mpm}\frac{1}{s} & K_{mfm} \end{bmatrix} \begin{bmatrix} I_m \\ V_m \end{bmatrix} \\ &\quad - \begin{bmatrix} K''_{mps}s + K'_{mps} + K_{mps}\frac{1}{s} & K_{mfs} \end{bmatrix} \begin{bmatrix} I_s \\ V_s \end{bmatrix} \\ &\triangleq \begin{bmatrix} P_m & Q_m \end{bmatrix} \begin{bmatrix} I_m \\ V_m \end{bmatrix} - \begin{bmatrix} R_m & S_m \end{bmatrix} \begin{bmatrix} I_s \\ V_s \end{bmatrix}, \end{aligned} \tag{12}$$

$$\begin{aligned} T_s &= \begin{bmatrix} K''_{spm}s + K'_{spm} + K_{spm}\frac{1}{s} & K_{sfm} \end{bmatrix} \begin{bmatrix} I_m \\ V_m \end{bmatrix} \\ &\quad - \begin{bmatrix} K''_{sps}s + K'_{sps} + K_{sps}\frac{1}{s} & K_{sfs} \end{bmatrix} \begin{bmatrix} I_s \\ V_s \end{bmatrix} \\ &\triangleq \begin{bmatrix} P_s & Q_s \end{bmatrix} \begin{bmatrix} I_m \\ V_m \end{bmatrix} - \begin{bmatrix} R_s & S_s \end{bmatrix} \begin{bmatrix} I_s \\ V_s \end{bmatrix}. \end{aligned} \tag{13}$$

Eliminating T_m and T_s from eqs.(10), (11), (12) and (13), we obtain the following equation:

$$\begin{bmatrix} Z_m - P_m & -R_m \\ -P_s & -(Z_s + R_s) \end{bmatrix} \begin{bmatrix} I_m \\ -I_s \end{bmatrix} = \begin{bmatrix} 1 + Q_m & -S_m \\ Q_s & -(1 + S_s) \end{bmatrix} \begin{bmatrix} V_m \\ V_s \end{bmatrix}. \tag{14}$$

Noting that I_1, I_2, V_1, and V_2 in **Fig.1** correspond to I_m, $-I_s$, V_m, and V_s in **Fig.2**, respectively, elements of the impedance matrix of the master-slave system are obtained from eq.(14) as follows:

$$z_{11} = \frac{(1 + S_s)(Z_m - P_m) + S_m P_s}{(1 + S_s)(1 + Q_m) - S_m Q_s} \triangleq \frac{N_{11}}{D_Z}, \tag{15}$$

$$z_{12} = \frac{-(1 + S_s)R_m + S_m(Z_s + R_s)}{(1 + S_s)(1 + Q_m) - S_m Q_s} \triangleq \frac{N_{12}}{D_Z}, \tag{16}$$

$$z_{21} = \frac{(1 + Q_m)P_s + Q_s(Z_m - P_m)}{(1 + S_s)(1 + Q_m) - S_m Q_s} \triangleq \frac{N_{21}}{D_Z}, \tag{17}$$

$$z_{22} = \frac{(1 + Q_m)(Z_s + R_s) - Q_s R_m}{(1 + S_s)(1 + Q_m) - S_m Q_s} \triangleq \frac{N_{22}}{D_Z}. \tag{18}$$

The determinant $|\boldsymbol{Z}|$ is given by

$$|\boldsymbol{Z}| = \frac{(Z_m - P_m)(Z_s + R_s) + P_s R_m}{(1 + S_s)(1 + Q_m) - S_m Q_s} \triangleq \frac{D_Y}{D_Z}. \tag{19}$$

The admittance matrix is obtained by inverting $\boldsymbol{Z}$ as follows:

$$\boldsymbol{Y} = \boldsymbol{Z}^{-1} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{N_{22}}{D_Y} & \dfrac{-N_{12}}{D_Y} \\ \dfrac{-N_{21}}{D_Y} & \dfrac{N_{11}}{D_Y} \end{bmatrix}. \tag{20}$$

Dynamics of the operator and the remote environment can also be represented as a form of impedance:

$$Z_L = m_w s + b_w + c_w \frac{1}{s}, \tag{21}$$

$$Z_G = m_{op} s + b_{op} + c_{op} \frac{1}{s}. \tag{22}$$

Equations (21) and (22) are obtained from the simple modeling of the operator and the remote environment in section 2. 1. Of course, one can suppose more appropriate impedance models for Z_L and Z_G if necessary. When we need to evaluate Z_L and Z_G numerically, however, we use eqs.(21) and (22).

3. Ideal Responses of Master-Slave Systems

3.1 Definition of ideal responses

In this section, before evaluating the performance of master-slave systems, we discuss what the ideal response of master-slave systems is. If the definition of the ideal response is valid, it would be possible to evaluate the performance of the system by examining how close the actual system response is to the ideal one.

DEFINITION : *The following three responses are defined as the ideal responses of master-slave systems.*

Ideal response I : *If the position responses, x_m and x_s, to an operator's input τ_{op} are identical, whatever dynamics the remote environment has, it is said that the system realizes ideal response I.*

Ideal response II : *If the force responses, f_m and f_s, to an operator's input τ_{op} are identical, whatever dynamics the remote environment has, it is said that the system realizes ideal response II.*

Ideal response III : *If both the position responses, x_m and x_s, and the force responses, f_m and f_s, to an operator's input τ_{op} are identical at the same time, whatever dynamics the remote environment has, it is said that the system realizes ideal response III.*

Obviously, when the position responses and force responses are identical between the master and the slave, the resultant position and force responses coincide with the responses when the operator directly interacts with the remote environment. Therefore, if the ideal response III is realized, the operator can maneuver the system as if he were interacting with the remote environment himself. In this sense, the ideal response III can be regarded as a final goal of master-slave systems. This ideal situation could be described as *ideal kinesthetic coupling*, *object teleperception*, or *transparent system.*

3.2 Conditions for ideal responses

The concept of the two-terminal-pair network is widely used to design electric filters. The master-slave system can also be regarded as a sort of mechanical filter between the operator and the remote environment. Here, we define some transmission coefficients in order to derive the conditions of the ideal responses.

First, we define a velocity transmission coefficient, T_i, which specifies how the velocity is transmitted from the master side (I_m) to the slave side (I_s), as follows:

$$T_i \triangleq \frac{I_m}{I_s}. \tag{23}$$

From eqs.(15) through (18) and the relationship of $V_s = Z_L I_s$, the velocity transmission coefficient is given by

$$T_i = \frac{z_{22} + Z_L}{z_{21}} = \frac{N_{22} + D_Z Z_L}{N_{21}}. \tag{24}$$

Since $T_i \equiv 1$ for any Z_L is necessary for realizing the ideal response I, the following conditions can be obtained.

[Conditions for ideal response I]

(A) $D_Z = 0$ (25)

(B) $N_{21} = N_{22} \neq 0$ (26)

Next, we define a force transmission coefficient, T_v, which specifies how the force is transmitted from the master side (V_s) to the slave side (V_s), as follows:

$$T_v \triangleq \frac{V_m}{V_s}. \tag{27}$$

Similarly to eq.(24), T_v is obtained from eq.(20) and the relationship of $V_s = Z_L I_s$ as follows:

$$T_v = \frac{y_{22} + \dfrac{1}{Z_L}}{-y_{21}} = \frac{N_{11} Z_L + D_Y}{N_{21} Z_L}. \tag{28}$$

Since $T_v \equiv 1$ for any Z_L is necessary for realizing the ideal response II, the following conditions are obtained.

[Conditions for ideal response II]

(C) $D_Y = 0$ (29)

(D) $N_{21} = N_{11} \neq 0$ (30)

Note that T_v cannot be defined when $Z_L = 0$. It will be shown later that the conditions (C) and (D) are valid in such a special case (see the footnote in section 4.1).

When all of the conditions for the ideal responses I and II are satisfied, the system realizes the ideal response III. Letting $x_m = x_s \triangleq x$ and $f_m = f_s \triangleq f$ in eqs.(3) and (4), it is obvious that x and f become the responses when the operator directly interacts with the remote environment. In fact, the input impedance from the operator side is given by

$$Z_{IN} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_L} = \frac{D_Y + N_{11} Z_L}{N_{22} + D_Z Z_L}. \tag{31}$$

Substituting the conditions (A), (B), (C) and (D) into eq.(31), we get

$$Z_{IN} \equiv Z_L, \tag{32}$$

showing that the operator can feel the remote environment impedance through the system, i.e., the system is transparent and realizes ideal kinesthetic coupling.

[Conditions for ideal response III]

All of conditions (A), (B), (C) and (D).

Due to the conditions (A) and (C), impedance matrix and admittance matrix cannot be defined when the system realizes the ideal response III. In this case, T_i and T_v cannot be defined by using z_{ij} and y_{ij} as in eqs.(24) and (28).

Therefore, we should check the consistency of the derived conditions.

Let us define another matrix called chain matrix. In **Fig.1**, let the following relations be considered:

$$V_1 = k_{11}V_2 + k_{12}(-I_2), \tag{33}$$

$$I_1 = k_{21}V_2 + k_{22}(-I_2). \tag{34}$$

Chain matrix is defined by

$$\boldsymbol{K} \triangleq \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}. \tag{35}$$

Chain matrix, which also specifies the property of two-terminal-pair networks, is used when the output of a two-terminal-pair network is connected to the input of another two-terminal-pair network. In the case of master-slave systems, the chain matrix can be represented by

$$\boldsymbol{K} = \frac{1}{z_{21}}\begin{bmatrix} z_{11} & |\boldsymbol{Z}| \\ 1 & z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{N_{11}}{N_{21}} & \dfrac{D_Y}{N_{21}} \\ \dfrac{D_Z}{N_{21}} & \dfrac{N_{22}}{N_{21}} \end{bmatrix}. \tag{36}$$

Note that elements of $\boldsymbol{K}$ correspond to the conditions (A), (B), (C) and (D), respectively. Substituting conditions (A) and (B), we get $I_1 = -I_2$, namely $T_i \equiv 1$. Likewise, substituting conditions (C) and (D), we obtain $V_1 = V_2$ or $T_v \equiv 1$. When all conditions (A), (B), (C), and (D) are satisfied, we get

$$\boldsymbol{K} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{37}$$

Therefore, the chain matrix can be defined even when the system realizes the ideal responses. It was also shown that the derived condition is consistent with this ideal case.

3.3 Consideration of position and force scaling factors

We have been implicitly assuming that position and force scaling factors are identical between the master and the slave. Of course, we may have a situation where position and force scaling factors are different, i.e., manipulating micro objects or handling heavy ones. We can deal with such situations by setting $T_i = \alpha$ and $T_v = \beta$ instead of setting them one, where α and β denote position (velocity) transmission ratio and force transmission ratio, respectively. Introducing these transmission ratios, we can derive new conditions of the ideal responses that include α and β. For example, a new transparent situation corresponding to eq.(32) is given by

$$Z_{IN} \equiv \frac{\beta}{\alpha} Z_L. \tag{38}$$

In this way, one can formulate more generally by introducing α and β. In this paper, however, we will consider the case when $\alpha = 1$ and $\beta = 1$ for simplicity.

3.4 Guideline for designing control schemes to realize ideal responses

Now, let us discuss the possibility of designing a new control scheme that can realize the ideal responses. In eqs.(5) and (6), we introduced acceleration terms to generalize the framework of control schemes. Compared to position and velocity, however, acceleration is difficult to measure and we would like to exclude the acceleration terms from the control scheme if possible.

However, it is impossible to satisfy the condition (C) when the acceleration signals are not used in eqs.(12) and (13), i.e., when $K''_{mpm} = K''_{mps} = K''_{spm} = K''_{sps} = 0$ in P_m, R_m, P_s, and R_s. Consequently, the following fact is obtained.

PROPOSITION : *In the framework of (5) and (6), any control scheme without acceleration terms cannot realize the ideal response II nor III.*

4. Evaluation of Maneuverability and Stability

4.1 Evaluation of maneuverability

In this section, we propose a performance index of maneuverability based on the concept of the ideal responses introduced in the previous section.

Let us consider four transfer functions from the operator's force τ_{op} (V_{op}) to the master side displacement x_m (I_m/s), to the slave side displacement x_s (I_s/s), to the master side force f_m (V_m), and to the slave side force f_s (V_s), respectively. Denoting these four function by $G_{mp}(s)$, $G_{sp}(s)$, $G_{mf}(s)$, and $G_{sf}(s)$, they are given by

$$G_{mp}(s) = \frac{s[N_{22} + D_Z Z_L]}{s^2[D_Y + N_{11}Z_L + N_{22}Z_G + D_Z Z_L Z_G]}, \tag{39}$$

$$G_{sp}(s) = \frac{s[N_{21}]}{s^2[D_Y + N_{11}Z_L + N_{22}Z_G + D_Z Z_L Z_G]}, \tag{40}$$

$$G_{mf}(s) = \frac{s^2[D_Y + N_{11}Z_L]}{s^2[D_Y + N_{11}Z_L + N_{22}Z_G + D_Z Z_L Z_G]}, \tag{41}$$

$$G_{sf}(s) = \frac{s^2[N_{21}Z_L]}{s^2[D_Y + N_{11}Z_L + N_{22}Z_G + D_Z Z_L Z_G]}. \tag{42}$$

By using these transfer functions, one can evaluate how well the actual system realizes the ideal responses as follows.

[Performance index of maneuverability]

The following two indices are defined:

$$J_p = \int_0^{\omega_{max}} F(G_{mp}(j\omega), G_{sp}(j\omega))W(\omega)d\omega, \quad (43)$$

$$J_f = \int_0^{\omega_{max}} F(G_{mf}(j\omega), G_{sf}(j\omega))W(\omega)d\omega, \quad (44)$$

where $F()$ denotes an appropriate function representing the difference of two transfer functions and $W()$ means a weighting function with respect to frequency. ω_{max} means the maximum frequency of the manipulation bandwidth of human operators. One can evaluate the maneuverability of master-slave systems by checking how small these indices are.

When the system realizes the ideal response I, index J_p is zero. When the system realizes the ideal response II, index J_f is zero. Consequently, if both J_p and J_f are close to zero, the response of that system is close to the ideal response III.

Examples of J_p and J_f are

$$J_p = \int_0^{\omega_{max}} |G_{mp}(j\omega) - G_{sp}(j\omega)| \left| \frac{1}{1+j\omega T} \right| d\omega, \quad (45)$$

$$J_f = \int_0^{\omega_{max}} |G_{mf}(j\omega) - G_{sf}(j\omega)| \left| \frac{1}{1+j\omega T} \right| d\omega, \quad (46)$$

where $F()$ is simple absolute difference and $W()$ is the gain of first-order-lag to put larger weight on the low frequency area than on the high frequency area. T ($T\omega_{max} > 1$) denotes time constant of the first-order-lag.

One problem when evaluating the maneuverability of the system by eqs.(43) and (44) is that indices J_p and J_f contain Z_L and Z_G. Therefore, even if we fix the control scheme, J_p and J_f may vary according to the change of operator and remote environment dynamics.

Therefore, it might be better to consider another indices which contain neither Z_L nor Z_G. On the other hand, it would be reasonable to include the dynamics of the operator, who maneuvers the system, in the indices, because the operator dynamics could be a criterion for evaluating the system.

Now, let us consider two special cases when $Z_L = 0$ and $Z_L = \infty$. The former case(1) corresponds to the situation when the slave arm is free. The latter case corresponds to the situation when the slave arm is constrained by a rigid environment. In these special cases, the differences between two transfer functions in eqs.(43) and (44) are given as follows:

[$Z_L = 0$]

$$G_{mp}(s) - G_{sp}(s) = \frac{s[N_{22} - N_{21}]}{s^2[D_Y + N_{22}Z_G]}, \quad (47)$$

$$G_{mf}(s) - G_{sf}(s) = \frac{s[D_Y]}{s^2[D_Y + N_{22}Z_G]}, \quad (48)$$

[$Z_L = \infty$]

$$G_{mp}(s) - G_{sp}(s) = \frac{D_Z}{s[N_{11} + D_Z Z_G]}, \quad (49)$$

$$G_{mf}(s) - G_{sf}(s) = \frac{s[N_{11} - N_{21}]}{s[N_{11} + D_Z Z_G]}. \quad (50)$$

Making eqs.(47), (48), (49) and (50) zero corresponds to the conditions (B), (C), (A) and (D), respectively. Substituting eqs.(47) through (50) into eqs.(45) and (46), we can get the performance indices that do not contain Z_L.

4.2 Numerical examples of evaluation

Let us evaluate the maneuverability of the conventional control schemes such as symmetric position servo type, force reflection type, and force reflecting servo type by the proposed indices. Parameters of the master and slave arms are given by

$$m_m = m_s = 2.0[\text{kg}],\ b_m = b_s = 0.2[\text{Ns/m}].$$

The following three kinds of remote environments are considered.

[case 1]: $m_w = 1.0$[kg], $b_w = 2.0$[Ns/m], $c_w = 10.0$[N/m]
[case 2]: $m_w = 10$[kg], $b_w = 50$[Ns/m], $c_w = 1000$[N/m]
[case 3]: $m_w = 1.0 \times 10^4$[kg], $b_w = 2.0 \times 10^4$[Ns/m], $c_w = 4.0 \times 10^4$[N/m]

In case 1 we supposed a relatively soft environment, whereas a relatively hard one in case 2, and in case 3 we supposed a nearly rigid one. To simplify the problem, we set the parameters of the operator constant as follows:

$$m_{op} = 1.0[\text{kg}],\ b_{op} = 2.0[\text{Ns/m}],\ c_{op} = 10.0[\text{N/m}].$$

Control gains of each scheme were given as follows:

[Symmetric Position Servo Type]

$K_{mpm} = K_{mps} = -500$[N/m], $K'_{mpm} = -50$[Ns/m],
$K_{spm} = K_{sps} = 500$[N/m], $K'_{sps} = 50$[Ns/m]

[Force Reflection Type]

$K_{mfs} = 1.0$, $K_{spm} = K_{sps} = 500$[N/m],
$K'_{sps} = 50$[Ns/m]

[Force Reflecting Servo Type]

$K_{mfm} = 2.5$, $K_{mfs} = 3.5$,
$K_{sps} = K_{spm} = 500$[N/m], $K'_{sps} = 50$[Ns/m]

(1) In the case when $Z_L = 0$, we get $G_{sf} = 0$ from eq.(42). Substituting the conditions (C) and (D) into eq.(41), we get $G_{mf}(s) = 0$, which means $f_m = f_s = 0$, i.e., the ideal response II. Therefore, conditions (C) and (D) are valid even in this case.

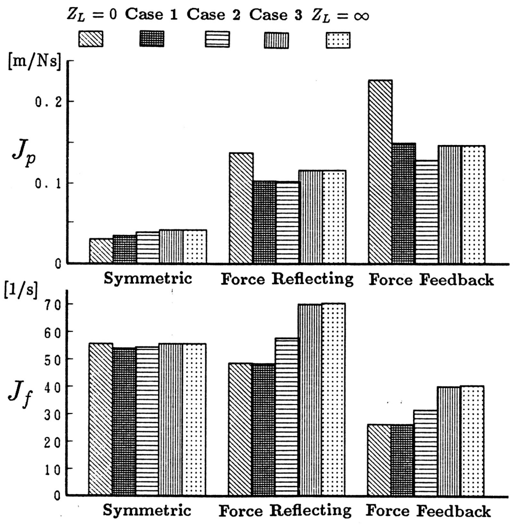

Fig. 3 Numerical example of maneuverability index

Other gains that are not specified explicitly as above are zero.

Fig.3 shows the indices J_p and J_f defined by eqs.(45) and (46) in the above three cases as well as the cases when $Z_L = 0$ and ∞, which are given by eqs.(47) through (50). We set $\omega_{max} = 100$[Hz] and $1/T = 50$[Hz].

From **Fig.3**, one can see that symmetric position servo type shows small J_p but J_f is large. Force reflecting servo type gives larger J_p than that of symmetric position servo type, i.e., worse position response performance, but J_f is smaller, showing that force response was improved. Force reflection type gives larger J_p and J_f than those of symmetric position servo type when the remote environment impedance is large.

Since **Fig.3** is just one example with the particular gains, we cannot get any general conclusions from this example. However, the evaluation result by the proposed indices meets our intuition and seems reasonable.

As shown in **Fig.3**, indices J_p and J_f give different values according to the remote environment parameters. If we could estimate the parameter range of the target environment in advance, we can calculate the indices J_p and J_f by using a representative value from that parameter range. On the contrary, if we cannot estimate the object parameters beforehand, we can evaluate the system using indices J_p and J_f when $Z_L = 0$ and ∞. Of course, it is very important to use appropriate parameters of the human operator to get valid evaluation results. Since the parameters of the human operator, such as b_{op} and c_{op}, may fluctuate during the task, we need to select a representative value appropriately.

In eqs.(45) and (46), we evaluated absolute difference of the two transfer functions. It would be possible to evaluate the difference relatively with respect to the ideal situation. Namely, we can use the following indices:

$$J_p = \int_0^{\omega_{max}} \left| \frac{G_{mp}(j\omega) - G_{sp}(j\omega)}{G_{ip}(j\omega)} \right| \left| \frac{1}{1 + j\omega T} \right| d\omega, \tag{51}$$

$$J_f = \int_0^{\omega_{max}} \left| \frac{G_{mf}(j\omega) - G_{sf}(j\omega)}{G_{if}(j\omega)} \right| \left| \frac{1}{1 + j\omega T} \right| d\omega, \tag{52}$$

where $G_{ip}(s)$ and $G_{if}(s)$ denote the transfer functions of ideal response III, which are given by

$$G_{ip}(s) = \frac{1}{s[Z_L + Z_G]}, \tag{53}$$

$$G_{if}(s) = \frac{s[Z_L]}{s[Z_L + Z_G]}. \tag{54}$$

It should be noted, however, that $G_{if}(s) = 0$ when $Z_L = 0$ and $G_{ip}(s) = 0$ when $Z_L = \infty$ and one cannot evaluate the system by using eqs.(53) and (54).

4.3 Evaluation of stability

For precise analysis of stability, it is necessary to consider the whole system including the operator and object dynamics. Characteristic polynomial of four transfer functions in (39) through (42) is given by

$$H(s) = s^2[D_Y + N_{11}Z_L + N_{22}Z_G + D_Z Z_L Z_G]. \tag{55}$$

Of course, if all roots of the characteristic equation $H(s) = 0$ are in the left half side of complex plane, the system is stable. However, $H(s)$ is a fourth-order polynomial with respect to s containing many parameters, such as the control gains and the dynamics parameters of the human operator and the remote environment. Therefore, it would be difficult to obtain a general stability condition that is valid even when Z_L and Z_G fluctuate.

5. Conclusion

The main results of this paper can be summarized as follows:

- A simple one DOF master-slave system was modeled, where both the operator dynamics and the remote environment dynamics were taken into account. A general framework of control schemes was formulated including acceleration terms of the master and slave arms, as well as position, velocity and force terms.
- We discussed the ideal situation of master-slave systems mathematically and three ideal responses were defined. Conditions to achieve these ideal responses were derived. It was shown that acceleration measurement is necessary to make the force responses coincide between the master and slave.

• Performance indices, which evaluate the system maneuverability quantitatively, were proposed. The proposed indices examine how close the actual responses are to the ideal ones.

Using the results of this paper, one can evaluate the maneuverability of master-slave systems with various control schemes quantitatively. The result of the quantitative evaluations could be a guideline for designing a new control scheme that gives better maneuverability than the conventional schemes. For the future works, we would like to design a new control scheme that realizes the ideal responses and extend the discussion into a multiple DOF case.

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