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A Study on Unit Interpolation with Rational Analytic Bounded Functions†

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This paper considers the problem of unit interpolation with rational analytic bounded functions. Lower bounds of the degree and norm of the units which interpolates given data are derived, using the Nevanlinna-Pick matrix and the non-Euclidean distance. These results explain why stable controllers tend to have large degree/norm in the strong stabilization problem.

Key Words: unit interpolation, strong stabilization, Nevanlinna-Pick matrix, non-Euclidean distance

1. Introduction

The strong stabilization problem was formulated by Youla *et al.*[1)], where in addition to the closed loop stability the controller itself is required to be stable. They derived a necessary and sufficient condition for the strong stabilizability. Vidyasagar[2)] extended the result for multi-input multi-output systems. Also in 2), it was shown that there is a close relation between the strong stabilization problem and the simultaneous stabilization problem, *i.e.*, the problem of finding a single controller which stabilizes multiple plants.

Vidyasagar[2)] pointed out the following reasons why the strong stabilization problem is important. The transfer function of the closed feedback system has no further zero in the right half plane in addition to a zero of the plant if and only if it is stabilized by a stable controller. A right half plane zero in general causes the degradation of the sensitivity function, and therefore it should be avoided if possible. The simultaneous stabilization problem is important as the stabilization technique for the uncertain control system. Youla *et al.*[1)] also pointed out the former without giving a specific reason.

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When the strong stabilization is possible, the computation of a stable controller involves computing a unit in bounded analytic function which satisfies a certain interpolation condition. There are several algorithms for this computation. The algorithms found in the proof of the sufficient part in 1), 2) are, as is pointed out in 1), not for practical purposes. The algorithms by Cusumano *et al.*[3)] and Dorato *et al.*[4)] uses the interpolation theory by positive real functions, and as is shown in the papers they generally yields lower order controller. However, sometimes they produce high order controllers. Smith *et al.*[5)] showed by an example that the degree of strong stabilizing controller is not bounded by a function of the degree of the plant. Ghosh[6)] made the same observation for the simultaneous stabilization problem.

In this paper, we make a theoretical study on the degree of stable controllers in the strong stabilization problem, and show why the controller tends to have high order. For this purpose, we study the problem of unit interpolation with rational analytic bounded functions, and give bounds of the minimum degree of interpolants. We first show that when the number of interpolation point is more than one the degree of unit interpolants is not bounded by a function of the number of interpolation point. This is a similar result as in 5), 6). Then we use the Nevanlinna-Pick interpolation theory to derive lower bounds of the degree of unit interpolants. This implies that a careful approach is required for the strong stabilization. The strong stabilizability is sometimes not enough for practical purposes because a high order controller is inevitable in some situations. Another observation is that the tendency to yield a high order controller is an intrinsic property of the strong stabilization problem rather than a property of the

Algorithms [1)~4)].

2. Strong stabilization

Consider the feedback system shown in **Fig. 1**, where p is a plant and c is a controller. We assume that both p and c are single input single output systems. We say the feedback system is stable if any transfer functions from v_i to u_j $(i = 1, 2,\ j = 1, 2)$ are stable. In this case, we call c a stabilizing controller. The plant p is called strongly stabilizable if there is a stable stabilizing controller.

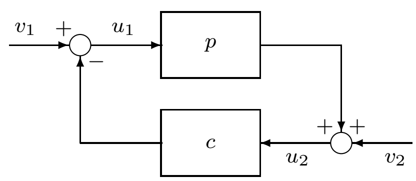

Fig. 1 Feedback system.

Proposition 1 (strong stabilizability [1), 2)]).
Consider the feedback system shown in Fig. 1. The plant p is strongly stabilizable if and only if the following condition (PIP condition) holds.
(PIP condition): The number of real poles of p between any two zeros on the nonnegative real axis including the infinity is even.

The condition is called the parity interlacing property, or the PIP condition.

Notice that designing a stable stabilizing controller is reduced to finding a unit satisfying interpolation constraints as the following procedure shows. First, let $p = n/d$ be coprime factorization over the ring of stable rational functions. Solve the Bezout identity $xy + yd = 1$ where x and y are stable rational functions. Then a stabilizing controller which are not necessarily stable is parameterized as $c = (x + dq)/(y - nq)$ where q is a stable rational function. Hence c is stable if and only if $y - nq$ is a unit. Suppose that the zeros of n (*i.e.,* the zeros of p) in the closed right half plane are simple. If we can find a unit u whose value at a zero z of n is $y(z)^{-1}$, then setting the free parameter $q = (y - u^{-1})/n$ gives a stable stabilizing controller.

In what follows, we shall assume that there is no interpolation points on the imaginary axis including the infinity for the sake of simplicity.

Let $Z = \{z_1, \cdots, z_k\}$ be the set of zeros of n in the open right half plane, and let $\beta_j = y(z_j)^{-1}$, $j = 1, \cdots, k$. We call the pair $(Z, \{\beta_j : j = 1, \cdots, k\})$ as interpolation data. It is possible to restate the PIP condition of Proposition 1 in terms of interpolation data. Since a zero of p in the open right half plane is a zero of n and a pole of p in the open right half plane is a zero of d, the PIP condition holds if and only if the signs of d at the real zeros of n are constant. Then by the Bezout identity, it follows that the signs of y at the real zeros of n are constant. Hence the plant p satisfies the PIP condition if and only if the interpolation data $(Z, \{\beta_j : j = 1, \cdots, k\})$ has the property that the signs of β_j for the real z_j are constant. When this is the case, we say the interpolation data satisfies the PIP condition.

3. Degree of strongly stabilizing controller

Though the strong stabilizability condition of Proposition 1 completely decides whether there is a stable stabilizing controller, we must take some caution on the degree of the controller. As the following Theorem indicates, a stable controller of extremely high degree is unavoidable in some situations. The same observations are made by way of examples in 5), 6). This paper gives a new proof which offers an insight into the order of a unit.

Theorem 1. Consider the set $\mathcal{P}_k$ of plants of degree k which satisfy the PIP condition. For each plant $p \in \mathcal{P}_k$, let $\delta(p)$ be the minimum degree of strongly stabilizing controller. Then $\{\delta(p) : p \in \mathcal{P}_k\}$ is unbounded.

Proof. Let $p \in \mathcal{P}_k$, and c be a strongly stabilizing controller. Let $p = n_p/d_p$ and $c = n_c/d_c$ be coprime factorization over the ring of polynomials. We consider the case where n_p has at least two positive zeros $0 < \sigma_1 < \sigma_2 < \infty$, and is anti-Hurwitz, *i.e.,* the real part of the zeros of n_p are positive. Because the feedback loop is stable, the polynomial $\psi = d_p d_c + n_p n_c$ is Hurwitz. Note that d_c satisfies the interpolation constraints

$$d_c(\sigma_i) = \frac{\psi(\sigma_i)}{d_p(\sigma_i)}, \quad i = 1, 2. \tag{3.1}$$

Since c is stable, d_c is Hurwitz, and we may assume that the coefficients of d_c are positive. Thus it follows that

$$0 < d_c(\sigma_1) < d_c(\sigma_2). \tag{3.2}$$

If the minimum degree of strongly stabilizing controllers has an upper bound, then there is a positive integer N such that for any $p \in \mathcal{P}_k$ we can select a strongly stabilizing controller c for which $\deg(\psi) \leq N$. Then applying Lemma 1 below, we have

$$\log\left|\frac{\psi(\sigma_2)}{\psi(\sigma_1)}\right| \leq N\left(\frac{\sigma_2 - \sigma_1}{\sigma_1}\right). \tag{3.3}$$

Let $\mathcal{P}_k(n_p)$ denote the set of k-th order transfer functions which have n_p as a numerator and satisfy the PIP condition. Then it is easy to show that

$$\inf\left\{\left|\frac{d_p(\sigma_1)}{d_p(\sigma_2)}\right| : p = \frac{n_p}{d_p} \in \mathcal{P}_k(n_p)\right\} = 0. \tag{3.4}$$

Indeed, consider a sequence of transfer functions in $\mathcal{P}_k(n_p)$ which have a real pole approaching to σ_1 between σ_1 and σ_2. Hence from (3.3) and (3.4) there is a $p \in \mathcal{P}_k(n_p)$ such that

$$\frac{\psi(\sigma_1)}{d_p(\sigma_1)} > \frac{\psi(\sigma_2)}{d_p(\sigma_2)} > 0. \tag{3.5}$$

From (3.1) this implies $d_c(\sigma_1) > d_c(\sigma_2)$, which contradicts (3.2). □

Lemma 1. Let ψ be an N-th degree Hurwitz polynomial, and $0 < \sigma_1 < \sigma_2$. Then we have

$$\log \left| \frac{\psi(\sigma_2)}{\psi(\sigma_1)} \right| \le N \left(\frac{\sigma_2 - \sigma_1}{\sigma_1} \right). \tag{3.6}$$

Proof. Without loss of generality, we can assume that ψ is monic. Let $\psi(s) = \prod_{j=1}^{N} (s - z_j)$, $\operatorname{Re} z_j < 0$. Then we have

$$\begin{aligned} \log \left| \frac{\psi(\sigma_2)}{\psi(\sigma_1)} \right| &= \sum_{j=1}^{N} \log \left| \frac{\sigma_2 - z_j}{\sigma_1 - z_j} \right| \qquad (3.7) \\ &\le \sum_{j=1}^{N} \log \frac{|\sigma_2 - \sigma_1| + |\sigma_1 - z_j|}{|\sigma_1 - z_j|} \\ &\le \sum_{j=1}^{N} \frac{\sigma_2 - \sigma_1}{|\sigma_1 - z_j|} \\ &\le N \left(\frac{\sigma_2 - \sigma_1}{\sigma_1} \right). \end{aligned}$$

□

4. Degree and norm of unit interpolant

In Section 3, we have shown that the degree of strongly stabilizing controller sometimes becomes quite large depending on interpolation data. In another word, the PIP condition does not guarantee if there is a strongly stabilizing controller of moderate degree. In this section, we scrutinize the situation, and decide which interpolation data yields such undesirable strongly stabilizing controllers. To do this, we will derive two lower bounds of unit interpolants given interpolation data; namely a lower bound of (i) the minimum degree, and (ii) the minimum norm.

4.1 Positive real interpolation problem

The following Proposition shows a necessary and sufficient condition for the existence of a positive real function [7].

Proposition 2.[7] Let $Z = \{z_1, \cdots, z_n\}$ be a subset in the open right half plane. Consider the interpolation data $(Z, \{\beta_1, \cdots, \beta_n\})$. Then there is a holomorphic function f in the open right half plane such that $\operatorname{Re} f(s) > 0$, $\operatorname{Re} s > 0$ and $f(z_j) = \beta_j$ if and only if the following matrix

$$\left[\frac{\beta_i + \overline{\beta}_j}{z_i + \overline{z}_j} \right] \tag{4.1}$$

is positive definite.

Proposition 3. Let Z and β_j be defined as Proposition 2. Suppose that $\operatorname{Re} \beta_j > 0$. If there is a holomorphic function f in the open right half plane such that $f(z_j) = \beta_j$, then $\rho(z_i, z_j) > \rho(\beta_i, \beta_j)$ holds, where $\rho(z_i, z_j)$ is the non-Euclidean distance in the right half plane defined by

$$\rho(z_i, z_j) = \log \frac{1+w}{1-w}, \quad w = \frac{|z_j - z_i|}{|z_j + \overline{z}_i|}. \tag{4.2}$$

Proof. It is a consequence of the fact that the holomorphic function which maps the open right half plane into itself is non-expansive if the non-Euclidean distance is introduced. However, we shall prove the proposition using Proposition 2 to show that this is a special case of the Navenlinna-Pick condition. Since 2×2 principle minors of the Navenlinna-Pick matrix (4.1) are positive, we have

$$\frac{|z_i + \overline{z}_j|^2}{\operatorname{Re} z_i \operatorname{Re} z_j} > \frac{\left|\beta_i + \overline{\beta}_j\right|^2}{\operatorname{Re} \beta_i \operatorname{Re} \beta_j}. \tag{4.3}$$

Notice that the right hand side of (4.2) is monotone increasing as a function of $0 \le w < 1$, and that

$$\begin{aligned} 2\frac{1+w^2}{1-w^2} &= \frac{(\operatorname{Re} z_i + \operatorname{Re} z_j)^2 + (\operatorname{Im} z_i - \operatorname{Im} z_j)^2}{(\operatorname{Re} z_i)(\operatorname{Re} z_j)} - 2 \qquad (4.4) \\ &= \frac{|z_i + \overline{z}_j|^2}{\operatorname{Re} z_i \operatorname{Re} z_j} \end{aligned}$$

holds. Hence it follows that the inequality (4.3) is equivalent to

$$\rho(z_i, z_j) > \rho(\beta_i, \beta_j). \tag{4.5}$$

□

In what follows, we shall utilize these propositions on the positive real interpolation condition, and study the degree and norm of unit interpolants.

4.2 Degree of unit interpolants

In this section, we study upper and lower bounds of the minimum degree of unit interpolants. The upper bound is identical to that given in 3). The implication of the lower bound is that the degree of unit interpolants is not bounded by a function of the number of interpolation points. This contrasts the fact that an H^∞ interpolation problem always have a solution whose degree is equal to the number of interpolation points if a solution exists.

We say that interpolation data $(Z, \{\beta_1, \cdots, \beta_n\})$ satisfy the conjugate condition if $\overline{z}_j = z_i$ implies $\overline{\beta}_j = \beta_i$.

Theorem 2. Let $Z = \{z_1, \cdots, z_n\}$ be a subset of the open right half plane. Let $(Z, \{\beta_1, \cdots, \beta_n\})$ be interpolation data which satisfy the conjugate condition and the PIP condition. Suppose that if there is a real $z_i \in Z$ then $\beta_i > 0$. Let m be a positive integer, and consider the

Nevanlinna-Pick matrix

$$\left[\frac{\beta_i^{1/m}+\overline{\beta}_j^{1/m}}{z_i+\overline{z}_j}\right], \tag{4.6}$$

where the m-th root is constructed in such a way that if β_i and β_j are conjugate, so are $\beta_i^{1/m}$ and $\beta_j^{1/m}$. Consider all possible branches of the m-th root (there are m^q combination if there are $2q$ non-real complex numbers in Z). Let m_0 be the minimum integer such that the matrix (4.6) is positive definite for a possible choice of the branches. Then any unit interpolants have at least m_0 degree. Furthermore, there is a (nm_0) degree unit interpolant.

Proof. The last assertion is due to 3). To show the claim on the lower bound, suppose that there is an m-th order unit interpolant. Since f is a unit, we can define holomorphic logarithm of f which we denote $\log f$. Note that the phase of f is bounded from above and below by $m\pi/2$ and $-m\pi/2$, respectively. Hence $f^{1/m} := \exp((1/m)\log f)$ is positive real, and satisfy $f^{1/m}(z_i)=\beta_i^{1/m}$. Hence from Proposition 2, the matrix

$$\left[\frac{f^{1/m}(z_i)+\overline{f^{1/m}(z_j)}}{z_i+\overline{z}_j}\right] \tag{4.7}$$

is positive real. Thus there is at least one positive definite matrix in (4.6). □

Applying Proposition 3, we obtain cruder but simpler lower bounds.

Theorem 3. Let $(Z=\{z_1,\cdots,z_n\},\{\beta_1,\cdots,\beta_n\})$ be interpolation data which satisfy the conjugate condition and the PIP condition. Suppose that if there is a real $z_i\in Z$ then $\beta_i>0$. Then the following integers m are lower bounds of the minimum degree of unit interpolants.

(i) Let $z_i, z_j\in Z$ be two distinct real numbers. Define m as the smallest integer greater than

$$\frac{\log(\max\{\beta_i,\beta_j\})-\log(\min\{\beta_i,\beta_j\})}{\log(\max\{z_i,z_j\})-\log(\min\{z_i,z_j\})}. \tag{4.8}$$

(ii) Let $z_i\in Z$ be a non-real number. Define m as the smallest integer greater than

$$\frac{|\arg(\beta_i)|}{|\arg(z_i)|}, \tag{4.9}$$

where $\arg(z)$ is the imaginary part of the principle value $\log z$ $(-\pi<\arg(z)\le\pi)$.

Proof. (i): The non-Euclidean distance between the two positive numbers z_i and z_j is

$$\rho(z_i,z_j)=\log\left(\frac{\max\{z_i,z_j\}}{\min\{z_i,z_j\}}\right). \tag{4.10}$$

Proposition 3 implies that if there is a unit interpolants for the interpolation data $\left(Z,\left\{\beta_1^{1/m},\cdots,\beta_n^{1/m}\right\}\right)$ then

$$\rho(z_i,z_j)>\rho(\beta_i^{1/m},\beta_j^{1/m}). \tag{4.11}$$

Using (4.10), we see that the smallest integer m for which (4.11) holds is given by (4.8).

(ii): The non-Euclidean distance between z and $\overline{z}$ for a non-real number z is

$$\rho(z,\overline{z})=\log\frac{1+\sin(|\arg(z)|)}{1-\sin(|\arg(z)|)}. \tag{4.12}$$

Suppose that $z_j=\overline{z}_i$ is a non-real number. If there is a unit interpolants for the interpolation data $\left(Z,\left\{\beta_1^{1/m},\cdots,\beta_n^{1/m}\right\}\right)$, then from Proposition 3 it follows that

$$\rho(z_i,\overline{z}_i)>\rho(\beta_i^{1/m},\overline{\beta}_i^{1/m}). \tag{4.13}$$

Notice that the functions $\log\{(1+w)/(1-w)\}$, $0\le w<1$ and $\sin\theta$, $0\le\theta<\pi/2$ are monotone increasing. Thus using (4.12) we see that (4.13) is equivalent to

$$|\arg(z_i)|>\left|\arg(\beta_i^{1/m})\right|. \tag{4.14}$$

The right hand side of (4.14) is minimized when we take the principle value of $\beta_i^{1/m}$. In this case $\left|\arg(\beta_i^{1/m})\right| = |\arg(\beta_i)|/m$. Hence the minimum integer m for which (4.14) holds is the minimum integer greater than (4.9).

□

4.3 Norm of unit interpolants

In this section, we study the minimum norm of unit interpolants, and derive lower bounds of the norm as in Section 4.2. We employ the H^∞ norm which is defined as

$$\|f\|_\infty=\sup\{|f(s)|:\mathrm{Re}\,s>0\}.$$

The following theorem is due to 8), but is included for reference.

Theorem 4.[8] Let $Z=\{z_1,\cdots,z_n\}$ be a subset of the open right half plane, and $(Z,\{\beta_1,\cdots,\beta_n\})$ be interpolation data which satisfies both the PIP and conjugate conditions. Suppose that if there is a real $z_i\in Z$ then $\beta_i>0$. Let M be a positive number and consider the following Nevanlinna-Pick matrix

$$\left[\frac{\log(M/\beta_i)+\overline{\log(M/\beta_j)}}{z_i+\overline{z}_j}\right], \tag{4.15}$$

where the m-th root is constructed in such a way that if β_i and β_j are conjugate, so are $\beta_i^{1/m}$ and $\beta_j^{1/m}$. Consider all possible branches of the m-th root (there are m^q combination if there are $2q$ non-real complex numbers in Z). Let M_0 be the infimum of M for which the matrix (4.6) is positive definite for a possible choice of the branches. Then the norm of any unit interpolants is greater than or equal to M_0.

The proof of Theorem 4 uses the fact that if there is a unit interpolant whose norm is less than M then there is a positive definite function satisfying the interpolation data $(Z,\{\log(M/\beta_1),\cdots,\log(M/\beta_n)\})$.

Applying Proposition 3, we obtain cruder but simpler lower bounds just as Theorem 3.

Theorem 5. Let $(Z = \{z_1, \cdots, z_n\}, \{\beta_1, \cdots, \beta_n\})$ be interpolation data which satisfy the conjugate condition and the PIP condition. Suppose that if there is a real $z_i \in Z$ then $\beta_i > 0$. Then the following positive numbers M are lower bounds of the norm of unit interpolants.

(1) Let $z_i, z_j \in Z$ be two distinct real numbers. Let $z_+ = \max\{z_i, z_j\}$, $z_- = \min\{z_i, z_j\}$, $\beta_+ = \max\{\beta_i, \beta_j\}$, and $\beta_- = \min\{\beta_i, \beta_j\}$. Define M as

$$M = \exp\left(\frac{z_+ \log(\beta_+) - z_- \log(\beta_-)}{z_+ - z_-}\right). \tag{4.16}$$

(2) Let $z_i \in Z$ be a non-real number. Define M as

$$M = |\beta_i| \exp\left(\frac{\mathrm{Re}\, z_i \, |\arg(\beta_i)|}{|\mathrm{Im}\, z_i|}\right), \tag{4.17}$$

where $\arg(z)$ is the imaginary part of the principle value $\log z$ $(-\pi < \arg(z) \leq \pi)$.

Proof. (1): If there is a positive real function satisfying the interpolation data $(Z, \{\log(M/\beta_1), \cdots, \log(M/\beta_n)\})$, then Proposition 3 implies that

$$\rho(z_i, z_j) > \rho(\log(M/\beta_i), \log(M/\beta_j)). \tag{4.18}$$

From (4.10),

$$\rho(z_i, z_j) = \log(z_+/z_-), \tag{4.19}$$

$$\rho(\log(M/\beta_i), \log(M/\beta_j)) = \log\left(\frac{\log(M/\beta_-)}{\log(M/\beta_+)}\right). \tag{4.20}$$

Since

$$\frac{\log(M/\beta_-)}{\log(M/\beta_+)} < \frac{z_+}{z_-}, \tag{4.21}$$

we have (4.16).

(2): Suppose that $z_j = \overline{z}_i$ is a non-real number. If there is a positive real function satisfying the interpolation data $(Z, \{\log(M/\beta_1), \cdots, \log(M/\beta_n)\})$, then from Proposition 3 it follows that

$$\rho(z_i, \overline{z}_i) > \rho(\log(M/\beta_i), \log(M/\overline{\beta}_i)). \tag{4.22}$$

Notice that the functions $\log\{(1+w)/(1-w)\}$, $0 \leq w < 1$, $\sin\theta$, $0 \leq \theta < \pi/2$, and $\tan\theta$, $0 \leq \theta < \pi/2$ are monotone increasing. Thus using (4.12) we see that (4.13) is equivalent to

$$\tan(|\arg(z_i)|) > \tan(|\arg(\log(M/\beta_i))|). \tag{4.23}$$

The right hand side of (4.23) is minimized when we take the principle value of $\log(M/\beta_i)$. In this case (4.23) is equivalent to

$$\left|\frac{\mathrm{Im}\, z_i}{z_i}\right| > \frac{|\arg(\beta_i)|}{\log M - \log|\beta_i|}, \tag{4.24}$$

and hence (4.17) is a lower bound. □

Note. Theorems 2–5 give lower and upper bounds of the minimum degree and the norm of unit interpolants. Let us summarize the implication of these results to the strong stabilization problem. As was pointed out in Section 2, designing a strongly stabilizing controller is reduced to finding a unit satisfying interpolation constraints. If we find a unit $u(s)$ which interpolates $d(z)$ at the right half plane zero z of $p = n/d$, then $c(s) = (u(s) - d(s))/n(s)$ is a strongly stabilizing controller. Note that the interpolation data depends on the co-prime factorization of p, and the degree and the norm of the unit interpolant are different from those of the controller. However, a simple calculation shows that

$$\deg(c) \geq \deg(u) - \deg(d) - \deg(n),$$
$$\|c\|_\infty \geq \frac{\|u\|_\infty - \|d\|_\infty}{\|n\|_\infty}.$$

If we use the normalized coprime factorization [9] for example, the above inequalities give lower bounds of the minimum degree and the norm of strongly stabilizing controller.

5. Examples

Example 1. Let $Z = \{1, 2\}$, $\beta_1 = 1$, and $\beta_2 = 1 \times 10^4$. From Theorem 3, we have $m_0 > (\log\beta_2)/(\log 2) \approx 13.3$, and hence we conclude that any unit interpolant has degree larger than or equal to 14. From Theorem 5, we have $M_0 \geq \exp(2\log(\beta_2)) = 1 \times 10^8$. Using the algorithm in 2), a unit interpolant is

$$u(s) = \left(1 + 0.96582\frac{s-1}{s+1}\right)^{33},$$

whose degree is 33, and $\|u\|_\infty = 4.8634 \times 10^9$.

Example 2. Let $Z = \{1 \pm 0.1j\}$, and $\beta_1 = \overline{\beta}_2 = 1 + j$. From Theorem 3, we have $m_0 > |\arg(1+j)| / |\arg(1+0.1j)| \approx 7.88$, and hence we conclude that any unit interpolant has degree larger than or equal to 8. From Theorem 5, we have $M_0 \geq 3.643 \times 10^3$. Using the algorithm in 2), a unit interpolant is

$$u(s) = \left(1 + 0.04571\frac{s-0.96196}{s+0.96196}\right)^{345},$$

whose degree is 345, and $\|u\|_\infty = 4.9761 \times 10^6$.

6. Conclusions

In this paper, it was shown that a stable controller which solves the strong stabilization problem sometimes has large degree depending on the interpolation data. In order to specify which data tends to yield high order controllers, non-Euclidean distance was exploited to compute lower bounds of the degree and norm of unit interpolants.

The results indicate that for the interpolation data which yields large lower bounds, strong stabilizing controllers are sometimes impractical.

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