Trans. of the Society of Instrument and Control Engineers
Vol.E-1, No.1, 145/153 (2001)

The Relationship of a Space Robot's Hand Trajectory and Its Attitude Variation

Katsuhiko YAMADA* and Kazuo TSUCHIYA**

The subject of this paper is a space robot that has a manipulator arm mounted on its body, which is an artificial satellite. Because the attitude of the space robot varies with the arm motion, it is necessary to understand the relationship of the arm trajectory and the attitude variation. In this paper, we evaluate the attitude variation and obtain a hand trajectory that minimizes it for the simple case where the arm mass is concentrated at the hand. We study here two coordinate systems of the hand: an inertial frame and a satellite-fixed frame. For the inertial frame, the attitude variation is proportional to the area surrounded by the hand trajectory and the origin. For the satellite-fixed frame, after the coordinate system is transformed, the attitude variation is also proportional to the area surrounded by the hand trajectory and the origin. In both cases, the hand trajectory that minimizes the attitude variation is approximated by a hyperbola. Numerical simulation is executed to examine these results.

Key Words: space robot, manipulator, hand trajectory, attitude variation, manipulatability

1. Introduction

The subject of this paper is an artificial satellite with manipulator arms, which is effectively a space robot. A space robot differs from a robot fixed on the ground because its satellite body is moved by the motion of its manipulator arm. In this paper we consider the approach of setting the hand trajectory to reduce the attitude disturbance of the satellite as a measure to decrease the satellite attitude variation caused by the arm motion.

Arm motion trajectory has already been studied with satellite attitude variation taken into consideration [1)~6)]. Vafa *et al.* proposed a method to obtain the arm motion trajectory from a disturbance map (DM), which shows the directions in which the satellite attitude disturbance is minimized or maximized in the arm joint space [1)]. This method was extended to Enhanced DM, a method that shows the disturbance 0 curve and the size of maximum disturbance. Enhanced DM was used to obtain the fuel's minimum trajectory [2)]. Yamada *et al.* obtained the hand trajectory from an optimum control problem by simplifying a space robot model [3)]. Nakamura *et al.* proposed a trajectory generating method, so called Bi-Directional approach, to obtain the satellite's final attitude and the arm's final joint angle simultaneously [4)]. In addition, Nenchev *et al.* defined a matrix that expresses a mapping to the hand velocity, without influencing the satellite attitude variation, as a Fixed-Attitude-Restricted Jacobian Matrix. They proposed obtaining the hand trajectory from a singular value decomposition of this matrix [5)]. In the meantime, although it is slightly different from the case to obtain the arm trajectory, Vafa *et al.* [1)] and Longman [6)] proposed controlling the satellite attitude by repeating small periodic motions of arm joint angles.

All of these methods emphasize the trajectory design method and/or the satellite attitude control method, and by designing the arm trajectory in such a way, they intend to reduce the satellite attitude variation or control the satellite attitude as desired. However, the trajectory obtained by such techniques cannot be approximated as a simple curve. Therefore, it might be difficult to grasp the physical relationship between the arm trajectory and the satellite attitude variation from the results. On the other hand, in order to design the arm trajectory for a space robot, it is important to intuitively understand the relationship between the arm trajectory and the satellite attitude variation.

In this paper, we conduct more basic study on the relationship between satellite attitude variation and arm motion trajectory by using a simplified space robot model. First, the differential relationship between the arm trajectory and the attitude variation derived from the momentum/angular momentum conservation law is shown in an integrated form. Then, the hand trajectory, which decreases the square integration of the satellite attitude variation along the trajectory, is given in a simple curve. These results describe a simple relationship between arm trajectory and satellite attitude variation, and they are useful in understanding the basic properties of the space robot. At the same time, when the arm's payload mass

* Mitsubishi Electric Corporation, Amagasaki
** Kyoto University, Kyoto

is greater than the mass of the arm, these results hold approximately. Therefore, they are useful as the first approximation for designing the optimum hand motion trajectory.

In this paper we assume that the motion of the space robot is restricted in a plane and that no external forces are exerted on the space robot during the arm motion; furthermore, we assume that its momentum and angular momentum are conserved at 0. Based on these assumptions, we derive in Section 2 the satellite attitude variation when the arm moves from one point to another in the inertial coordinates system (inertial frame). The next step is to simplify the model while assuming that only the hand mass is dominant over other parts of the arm and to derive the hand trajectory that reduces the satellite attitude variation analytically. In Section 3, a similar study is performed on the case where the hand moves from one point to another in the satellite-fixed coordinates system (satellite-fixed frame) instead of the inertial frame. When the arm is used to capture a floating object in space, it is necessary to move the hand in the inertial frame. Conversely, for insertion/removal of the module attached to the satellite, it is necessary to move the hand in the satellite-fixed frame. Therefore, for a space robot, there are two task frames that basically differ from each other. In this paper, we study the differences between these two frames from the point of the satellite attitude variation. In Section 4, a feasibility study of the obtained trajectories for a manipulator arm is described. Finally, in Section 5, numerical examples of these results are shown.

2. Inertial Task Coordinates

2.1 Modeling

We first consider the case where the hand task coordinates is an inertial frame. It is assumed that the space robot moves in a plane and that the momentum and the angular momentum during the arm motion are conserved at 0. Consider the case where the space robot has a manipulator with n degrees of freedom, the satellite is defined as body 0, and the arm links are specified as body 1 to body n in order from the satellite side. The origin O in the inertial frame is the CM (center of mass) of the entire space robot, and the following symbols are defined for each body:

- m_i : mass of body i
- i_i : moment of inertia around the CM of body i
- m_c : total mass of the space robot
- x_i, y_i : position of the CM of body i in the inertial frame
- θ_i : rotation angle of body i in the inertial frame
- m_m : total mass of the arm
- x_m, y_m : position of the CM of the arm in the inertial frame

Since the origin of the inertial frame is the CM of the space robot, the following relations hold by integrating the momentum conservation law.

$$\sum_{i=0}^{n} m_i x_i = 0, \quad \sum_{i=0}^{n} m_i y_i = 0 \tag{1}$$

In the mean time, since the angular momentum about the origin of the inertial frame is conserved at 0, the following relation holds

$$\sum_{i=0}^{n} \{ i_i \dot{\theta}_i + m_i (x_i \dot{y}_i - y_i \dot{x}_i) \} = 0 \tag{2}$$

where $\dot{}$ expresses the time differentiation. Eliminating x_0 and y_0 from (1) and (2), we obtain

$$i_0 \dot{\theta}_0 + \frac{m_m^2}{m_0}(x_m \dot{y}_m - y_m \dot{x}_m) + \sum_{i=1}^{n} \{ i_i \dot{\theta}_i + m_i (x_i \dot{y}_i - y_i \dot{x}_i) \} = 0 \tag{3}$$

2.2 Satellite Attitude Variation

From (3), $d\theta_0$ is expressed by the following differential form.

$$d\theta_0 = -\frac{m_m^2}{i_0 m_0}(x_m dy_m - y_m dx_m) - \sum_{i=1}^{n} \left\{ \frac{i_i}{i_0} d\theta_i + \frac{m_i}{i_0}(x_i dy_i - y_i dx_i) \right\} \tag{4}$$

It is a property of the system, whose angular momentum is conserved at 0, that time can be omitted by regarding it as an auxiliary variable. In such a system, the satellite attitude variation depends only on the arm's spatial trajectory and does not depend on the arm's time trajectory. Therefore, only the spatial trajectory of the hand is considered in the following study.

Consider that the hand moves from A to B, as shown in **Fig. 1**. If the satellite attitude variation is specified as $\Delta\theta_0$, $\Delta\theta_0$ can be expressed as the sum of the influence of the movement of the arm's CM and that of each link's CM from (4). Then, if we look at the first term of the right side of (4) for instance, this influence can be expressed as follows. As the hand moves from A to B, the CM of the arm also moves from C to D. Consider the case where the CM of the arm moves on the line from $D \to O$ (the origin) $\to C$. The first term of the right side does not contribute to the satellite attitude variation because $x_m dy_m - y_m dx_m$ becomes 0 on $D \to O$ and $O \to C$. Therefore, the attitude variation when the CM of the arm moves from $C \to D$ is equal to that when the center of the arm's mass moves

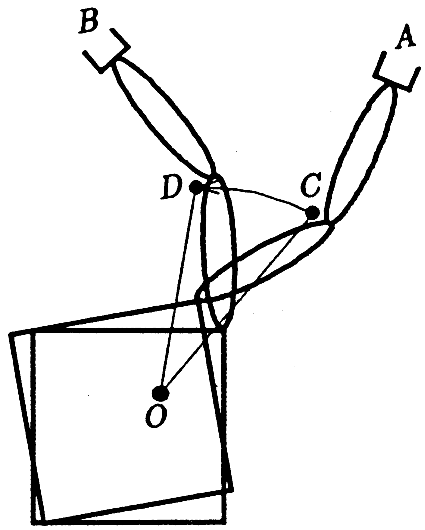

Fig. 1 Motion of a space robot

along the loop (i.e. $C \to D \to O \to C$). The attitude variation along the loop is expressed as follows by Stokes' theorem

$$-\frac{m_m^2}{i_0 m_0}\int_{\partial c_m}(x_m dy_m - y_m dx_m) = -\frac{2m_m^2}{i_0 m_0}\int_{c_m} dx_m \wedge dy_m = -\frac{2m_m^2}{i_0 m_0}S(c_m) \qquad (5)$$

where $\wedge$ is the exterior product, c_m is the domain surrounded by $C \to D \to O \to C$ (fan-shaped domain OCD), ∂c_m is its boundary, and $S(c_m)$ is its area. That is, the contribution of the first term of the right side to the satellite attitude variation is in proportion to the c_m area. This area has direction, and its positive direction is when the CM of the arm moves counterclockwise around the c_m.

We will apply this to each link when the hand moves from A to B. Defining c_i as the domain surrounded by the motion trajectory of the CM of link i, the line connecting the starting point to the origin, and the line connecting the end point to the origin, we can express the satellite attitude variation $\Delta\theta_0$ as follows

$$\Delta\theta_0 = -\frac{2m_m^2}{i_0 m_0}S(c_m) - \sum_{i=1}^{n}\left(\frac{i_i}{i_0}\Delta\theta_i + \frac{2m_i}{i_0}S(c_i)\right) \qquad (6)$$

where $\Delta\theta_i$ is the attitude variation of link i. That is, the satellite attitude variation can be expressed as the linear combination of the attitude variation of each link and the area of the domain surrounded by the CM of each link and that surrounded by the CM of the arm.

2.3 When Arm Mass is Concentrated at the Hand

Equations (4) and (6) show the relationship between the arm motion and the satellite attitude variation for the space robot during a planar motion. However, it is not easy to determine the hand trajectory from these equations. We studied in detail the hand trajectory and the satellite attitude variation while considering that the arm mass is concentrated at the hand. Although the arm mass is not actually centered on the hand, when the arm takes hold of a large-mass payload, it can be regarded as an approximation of the case. When the hand mass is designated as m_1 and its position as (x_1, y_1), and if the other arm mass is ignored, (4) becomes as follows.

$$d\theta_0 = -\mu(x_1 dy_1 - y_1 dx_1), \quad \mu = \frac{m_1 m_c}{i_0 m_0} \qquad (7)$$

Then, the plane where the hand moves is converted to the following plane (x, y):

$$x = \sqrt{\mu}x_1, \quad y = \sqrt{\mu}y_1 \qquad (8)$$

Substitution of (8) into (7) yields

$$d\theta_0 = -(xdy - ydx) \qquad (9)$$

If the same discussion is tried on (9) as in the previous clause, the satellite attitude variation in this case equals -2 times the area of the domain (with direction) surrounded by the hand motion trajectory on the (x, y)-plane and the origin. Therefore, if this domain is defined as c, it holds

$$\Delta\theta_0 = -2S(c) \qquad (10)$$

Furthermore, if the hand moves from A to B in the (x, y)-plane as shown in **Fig. 2**, the area of the domain surrounded by the hand motion trajectory and line AB should be closer to the area of $\triangle OAB$ in order to decrease the final attitude variation of the satellite. If the length of the hand trajectory is restricted, the desirable trajectory would be the one that surrounds a domain with the maximum possible area, and the problem to obtain the trajectory would be reduced to an isoperimetric problem. Therefore, under the condition of the constant trajectory length, the arc of a circle, which goes through the starting point and the end point, is considered a candidate trajectory to decrease the final attitude variation of the satellite.

If we consider the case where the satellite attitude is controlled during the arm motion, it is desirable to have a trajectory that can decrease the satellite attitude variation as much as possible during the arm motion. This trajectory can be obtained by $A \to O \to B$ in Fig. 2, although it is not a realistic trajectory. Thus, we consider minimizing the following integration, which takes

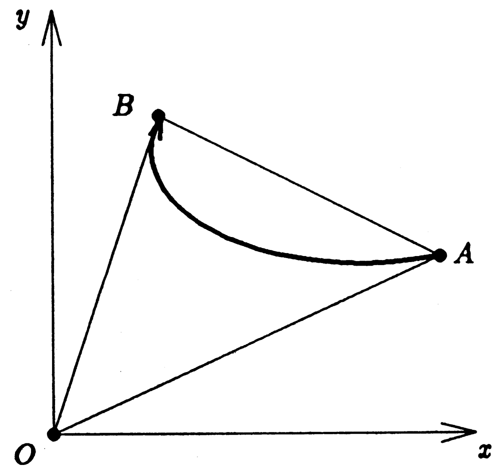

Fig. 2 Hand trajectory in (x, y)-plane

the length of trajectory into consideration

$$\int_A^B ds, \quad ds^2 = dx^2 + dy^2 + a^2 d\theta_0^2 \tag{11}$$

where a is an arbitrary parameter, and $d\theta_0$ is given by (9). In the case of $a = 0$, the straight line from A to B obviously becomes a solution. Since ds^2 in (11) is positive, it becomes a Riemannian metric. Consequently, the trajectory from A to B can be obtained as a geodesic on the manifold with this metric.

If the independent variable is specified as x^i and the Riemannian metric is specified as

$$ds^2 = \sum_{i,j} g_{ij} dx^i dx^j \tag{12}$$

the geodesic equation is generally obtained as follows [7)]

$$\frac{d^2x^i}{ds^2} + \sum_{k,l,m} \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right) \frac{dx^k}{ds} \frac{dx^l}{ds} = 0 \tag{13}$$

where g^{im} is a tensor that satisfies $\sum_m g^{im} g_{mj} = \delta_j^i$ (unit tensor). If (9) and (11) are substituted into (13) and the polar coordinates (r, ϕ) are applied as the independent variables instead of (x, y), the geodesic equation becomes as follows.

$$\frac{d^2r}{ds^2} - r(1 + 2a^2r^2) \left(\frac{d\phi}{ds} \right)^2 = 0 \tag{14}$$

$$\frac{d^2\phi}{ds^2} + \frac{2(1 + 2a^2r^2)}{r(1 + a^2r^2)} \frac{d\phi}{ds} \frac{dr}{ds} = 0 \tag{15}$$

Meanwhile, the metric expression in the polar coordinates becomes as follows.

$$ds^2 = dr^2 + r^2(1 + a^2r^2) d\phi^2 \tag{16}$$

The following relation is obtained from (15).

$$r^2(1 + a^2r^2) \frac{d\phi}{ds} = \gamma = \text{constant} \tag{17}$$

Therefore, combining the above and the expression (16) produces an equation as follows.

$$\left| \frac{dr}{ds} \right| = \sqrt{1 - \frac{\gamma^2}{r^2(1 + a^2r^2)}} \tag{18}$$

Or, by eliminating ds from those relations, the following relation is obtained

$$\left| \frac{dr}{d\phi} \right| = \frac{r}{\gamma} \sqrt{(1 + a^2r^2)\{r^2(1 + a^2r^2) - \gamma^2\}} \tag{19}$$

$$\frac{d^2r}{d\phi^2} = \frac{r(1 + 2a^2r^2)(-\gamma^2 + 2r^2 + 2a^2r^4)}{\gamma^2} \tag{20}$$

Although the hand trajectory cannot be derived as an elementary function from these equations, it is possible to understand the trajectory property to some extent. For instance, since $d\phi/ds$ is a fixed sign, either of the hand motion ϕ, or the satellite attitude variation θ_0 shows a monotonous increase while the other shows a monotonous decrease. Moreover, if the curvature is specified as κ, it becomes as follows.

$$\kappa = -\frac{2a^2(1 + a^2r^2)|\gamma|}{\{(1 + a^2r^2)^2 - a^2\gamma^2\}^{3/2}} < 0 \tag{21}$$

Therefore, the trajectory resides inside $\triangle OAB$. From this result and the meaning of a, it is understood that as a increases, the trajectory becomes asymptotically $A \to O \to B$.

Now, we try to approximate the trajectory by using a simple curve with such a property. As the 6th power term (the highest power term) and the 4th power term of r in the square root of (19) are both positive, it is assumed that there is no large difference in solution behavior even though the 6th power term is omitted. In this case, this differential equation can be solved as follows

$$r^2 = \frac{2\gamma^2}{1 - a^2\gamma^2 + \sqrt{1 + 6a^2\gamma^2 + a^4\gamma^4} \cos\{2(\phi - \phi_0)\}} \tag{22}$$

where ϕ_0 is constant and γ and ϕ_0 are determined by the starting point and end point of the hand trajectory. Equation (22) is a hyperbola equation. When a is 0, it becomes a straight line as a special case, and as a increases, r at $\phi = \phi_0$ approaches 0. This means that the trajectory changes from $A \to B$ to $A \to O \to B$. Therefore, the approximation of the hyperbola in (22) can express qualitatively the trajectory minimizing integration (11). This is confirmed by the numerical example given later.

3. Satellite-Fixed Task Coordinates

3.1 Modeling

In this section, we study the case where the hand moves in the satellite-fixed frame, assuming that, for instance, the manipulator hand attaches a payload to the satellite. The origin of the satellite-fixed frame is the CM of the satellite, and the following symbols are defined in addition to those used in Section 2:

x_{bi}, y_{bi} : CM position of body i in the satellite-fixed frame

θ_{bi} : rotation angle of body i in the satellite-fixed frame

x_{bc}, y_{bc} : CM position of the space robot in satellite-fixed frame

u_i, v_i : CM velocity of body i (in the inertial frame) expressed in the satellite-fixed frame

As it is based on the assumption that the momentum and the angular momentum of the space robot are conserved at 0, the angular momentum about the CM of the satellite also becomes 0. Therefore, the following equation holds

$$i_0\dot{\theta}_0 + \sum_{i=1}^{n}\{i_i(\dot{\theta}_0 + \dot{\theta}_{bi}) + m_i(x_{bi}v_i - y_{bi}u_i)\} = 0 \quad (23)$$

where θ_0 is the rotation angle of the satellite in the inertial frame. The following equations also hold based on the momentum conservation.

$$\sum_{i=0}^{n} m_i u_i = 0, \quad \sum_{i=0}^{n} m_i v_i = 0 \quad (24)$$

On the other hand, u_i and v_i ($i \geq 1$) are expressed as follows.

$$\begin{aligned} u_i &= u_0 + \dot{x}_{bi} - \dot{\theta}_0 y_{bi} \\ v_i &= v_0 + \dot{y}_{bi} + \dot{\theta}_0 x_{bi} \end{aligned} \quad (25)$$

Using (24) and (25), u_0 and v_0 become as follows.

$$\begin{aligned} u_0 &= -\dot{x}_{bc} + y_{bc}\dot{\theta}_0 \\ v_0 &= -\dot{y}_{bc} - x_{bc}\dot{\theta}_0 \end{aligned} \quad (26)$$

Substituting (25) and (26) into (23) and eliminating u_i and v_i, we obtain

$$i_c\dot{\theta}_0 - m_c(x_{bc}\dot{y}_{bc} - y_{bc}\dot{x}_{bc}) + \sum_{i=1}^{n}\{i_i\dot{\theta}_{bi} + m_i(x_{bi}\dot{y}_{bi} - y_{bi}\dot{x}_{bi})\} = 0 \quad (27)$$

$$i_c = i_0 + \sum_{i=1}^{n}\{i_i + m_i(x_{bi}^2 + y_{bi}^2)\} - m_c(x_{bc}^2 + y_{bc}^2)$$

The symbol i_c is the moment of inertia of the space robot about its CM. Equation (27) is the basic equation in this case.

Equation (27) is associated with (3) in Section 2 except that the sign of the 2nd term in the right side is reversed. That is, in (3), the movement of the CM of the arm rotates the satellite in the same direction as that of each link of the arm. On the other hand, in (27), the movement of the CM of the space robot in the satellite-fixed frame rotates the satellite in the reverse direction to that of each link of the arm. Therefore, when the hand moves in the satellite-fixed frame, it is expected that the satellite attitude variation becomes relatively smaller. In the next step, we consider this in detail concerning the case where the arm mass is concentrated at the hand.

3.2 When Arm Mass is Concentrated at the Hand

Because i_c, the coefficient of $\dot{\theta}_0$, is not constant in (27), it is difficult to handle this case. We will assume here that the arm mass is concentrated at the hand as in 2.3. The arm mass is specified as m_1 and the arm position is specified as (x_{b1}, y_{b1}), while ignoring the arm mass of any other part except the hand. Then, (27) becomes

$$\left\{i_0 + \frac{m_0 m_1}{m_c}(x_{b1}^2 + y_{b1}^2)\right\}\dot{\theta}_0 + \frac{m_0 m_1}{m_c}(x_{b1}\dot{y}_{b1} - y_{b1}\dot{x}_{b1}) = 0 \quad (28)$$

From this, $d\theta_0$ is obtained as follows.

$$d\theta_0 = -\frac{\mu_b(x_{b1}dy_{b1} - y_{b1}dx_{b1})}{1 + \mu_b(x_{b1}^2 + y_{b1}^2)}, \quad \mu_b = \frac{m_0 m_1}{i_0 m_c} \quad (29)$$

The next step is to convert the hand motion plane to (x_d, y_d) through (x_b, y_b) as follows.

$$x_b = \sqrt{\mu_b}x_{b1}, \quad y_b = \sqrt{\mu_b}y_{b1} \quad (30)$$

$$x_d = \frac{x_b}{\sqrt{1 + x_b^2 + y_b^2}}, \quad y_d = \frac{y_b}{\sqrt{1 + x_b^2 + y_b^2}} \quad (31)$$

If x_d and y_d in (31) are used, (29) becomes as follows.

$$d\theta_0 = -(x_d dy_d - y_d dx_d) \quad (32)$$

Therefore, the attitude variation in this case equals -2 times the area of the domain c_d surrounded by the hand trajectory and the origin in the (x_d, y_d)-plane as is the case in 2.3.

$$\Delta\theta_0 = -2S(c_d) \quad (33)$$

If this area is smaller, the final satellite attitude variation can be reduced. Changing (x_b, y_b) and (x_d, y_d) into polar coordinates (r_b, ϕ_b) and (r_d, ϕ_d), respectively, we obtain the following relation from (31)

$$r_d = \frac{r_b}{\sqrt{1 + r_b^2}}, \quad \phi_d = \phi_b \quad (34)$$

Namely, when the hand trajectory in the inertial frame is the same as in the satellite-fixed frame, the attitude variation is smaller in the satellite-fixed frame. Also, the hand motion in the (x_d, y_d)-plane is limited to the inside of a circle with a radius of 1. Therefore, if the hand moves at an angle of ψ in the (x_b, y_b)-plane monotonously, the absolute value $|S(c_d)|$ representing the area of the domain c_d in the (x_d, y_d)-plane is smaller than $\psi/2$ and thus $|\Delta\theta_0| < \psi$. That is, there exists a maximum value of the attitude variation in this case. When $|\Delta\theta_0|$ gets closer to ψ, the hand has an extremely large moment of inertia about the CM of the satellite. When trying to rotate the hand against the satellite, the satellite rotates against the inertial frame, with the hand itself not rotating in the inertial frame but only at a certain angle ψ against the satellite.

As in Section 2, here we investigated the trajectory that reduces the satellite attitude variation during hand motion. Assuming that the hand moves from A to B in the (x_b, y_b)-plane, we will obtain the trajectory that minimizes the following integration in consideration of the length of the trajectory

$$\int_A^B ds, \quad ds^2 = dx_b^2 + dy_b^2 + a^2 d\theta_0^2 \tag{35}$$

where a is an arbitrary parameter. When polar coordinates (r_b, ϕ_b) are applied, the Riemannian metric of (35) becomes as follows

$$ds^2 = dr_b^2 + r_b^2 \left\{ 1 + \frac{a^2 r_b^2}{(1+r_b^2)^2} \right\} d\phi_b^2 \tag{36}$$

As in Section 2, the geodesic equation is derived corresponding to (17) as

$$\frac{r_b^2\{(1+r_b^2)^2 + a^2 r_b^2\}}{(1+r_b^2)^2} \frac{d\phi_b}{ds} = \gamma_b = \text{constant} \tag{37}$$

Using this relation, the hand trajectory in this case is given as follows.

$$\left| \frac{dr_d}{d\phi_d} \right| = \frac{r_d}{\gamma_b} \sqrt{(1-r_d^2)\alpha\{r_d^2\alpha - \gamma_b^2(1-r_d^2)\}} \tag{38}$$

$$\alpha = 1 + a^2 r_d^2 (1 - r_d^2)$$

where (r_b, ϕ_b) are converted into (r_d, ϕ_d) using (34). To obtain an approximate solution of (38), we omit the 6th power or the higher terms of r_d in the square root of (38) by using $r_d < 1$. Then, (38) can be integrated and the following solution is obtained for r_b

$$r_b^2 = \frac{2\gamma_b^2}{1 - a^2\gamma_b^2 + \sqrt{1 + 6a^2\gamma_b^2 + 8a^2\gamma_b^4 + a^4\gamma_b^4}} * \\ * \frac{}{\cos\{2(\phi_b - \phi_{b0})\}} \tag{39}$$

where ϕ_{b0} is a constant. This is almost the same as (22), and a hyperbola also becomes an approximate solution.

4. Realization of Trajectory by the Arm

In the previous sections, we derived the hand trajectories to reduce the satellite attitude variation under the approximation where the arm mass is concentrated at the hand. In this section, we briefly study the possibility of realizing these trajectories by the actual arm joint movement. We considered how to realize the hand trajectory by an arm of two degrees of freedom as shown in **Fig. 3**. The joint angles of the arm are specified as q_1 and q_2, the link lengths are specified as l_1 and l_2, and the position of the arm's fixed point in the satellite-fixed frame is specified as (a_1, a_2). If the hand task coordinates are the inertial frame with the origin at the CM of the space robot, the hand coordinates (x_1, y_1) are shown as below. For simplicity, the hand mass is specified as m_1 and the arm mass except for the hand can be ignored, as in the previous sections, and the axes of the inertial frame and the satellite-fixed frame are the same when the satellite attitude angle θ_0 is 0.

$$\begin{aligned} x_1 &= \nu\{a_1\cos\theta_0 - a_2\sin\theta_0 + l_1\cos(\theta_0+q_1) \\ &\quad + l_2\cos(\theta_0+q_1+q_2)\} \\ y_1 &= \nu\{a_1\sin\theta_0 + a_2\cos\theta_0 + l_1\sin(\theta_0+q_1) \\ &\quad + l_2\sin(\theta_0+q_1+q_2)\} \end{aligned} \tag{40}$$

where $\nu = m_0/(m_0+m_1)$. Taking the time derivative of the above equation and eliminating $\dot{\theta}_0$ by the angular momentum conservation, we obtain

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \tag{41}$$

where J is the generalized Jacobian matrix [5),8)]. In order to realize the trajectory from this relation, $\det J \neq 0$ is necessary. In this case, $\det J$ is obtained as follows.

$$\det J = \frac{\nu^2 i_0 l_1 l_2 \sin q_2}{i_c} \tag{42}$$

where i_c is the moment of inertia of the space robot about its CM. In case of $\sin q_2 \neq 0$, the trajectory can be realized if the joint angle limitation is ignored.

On the other hand, if the hand task coordinates are the satellite-fixed frame, the hand coordinates (x_{b1}, y_{b1}) are expressed as follows.

$$\begin{aligned} x_{b1} &= a_1 + l_1\cos q_1 + l_2\cos(q_1+q_2) \\ y_{b1} &= a_2 + l_1\sin q_1 + l_2\sin(q_1+q_2) \end{aligned} \tag{43}$$

The following relation is obtained corresponding to (41)

$$\begin{bmatrix} \dot{x}_{b1} \\ \dot{y}_{b1} \end{bmatrix} = J_b \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \tag{44}$$

where J_b is the normal Jacobian matrix used for a ground

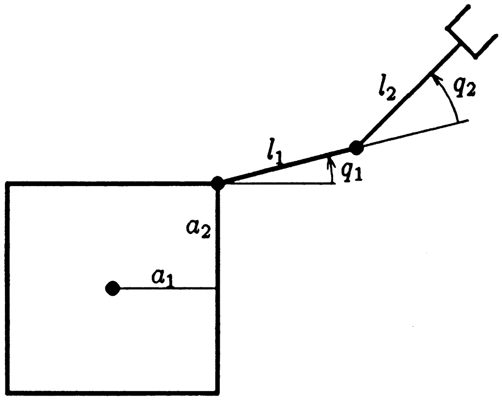

Fig. 3 A space robot with a manipulator of two degrees of freedom

manipulator. In this case, $\det J_b$ becomes as follows

$$\det J_b = l_1 l_2 \sin q_2 \tag{45}$$

Therefore, as long as $\sin q_2 \neq 0$, the trajectory can also be realized.

Equations (42) and (45) show the manipulatability of each point on the trajectory, and the degree to which the trajectory can be realized can be evaluated quantitatively by these values [5),8),9)]. From (42), if the hand task coordinates are placed in the inertial frame, as the hand mass increases, the manipulability decreases, while from (45), if the hand task coordinates are placed in the satellite-fixed frame, the manipulability is not influenced by the mass property. This is because the hand trajectory is designed against the satellite in the latter case.

It is difficult to obtain the trajectory while taking manipulability into consideration. However, it is easy to evaluate the obtained trajectory using the manipulability measures. From this point of view, we consider the feasibility of the trajectory quantitatively in Section 5.

5. Numerical Results

In this section, the trajectories obtained in Sections 2 and 3 are calculated using a space robot model. Define symbols as follows: satellite mass is m_0, moment of inertia of the satellite about its CM is i_0 and hand mass is m_1. Specifications of the space robot model are: $m_0 = 1000$ [kg], $m_1 = 200$ [kg] and $i_0 = 1000$ [kgm^2]. The arm mass is ignored in comparison with the hand mass. Also, in Fig. 3, it is specified as: $l_1 = l_2 = 2$ [m] and $a_1 = a_2 = 1$ [m]. We consider here the case to move the hand from $A' =(4, 0)$[m] to $B' =(2, 3)$[m] in the satellite-fixed frame. Correspondingly, if task coordinates are the inertial frame, the hand is moved from $A =(10/3, 0)$[m] to $B =(5/3, 2.5)$[m]. When the directions of the satellite-

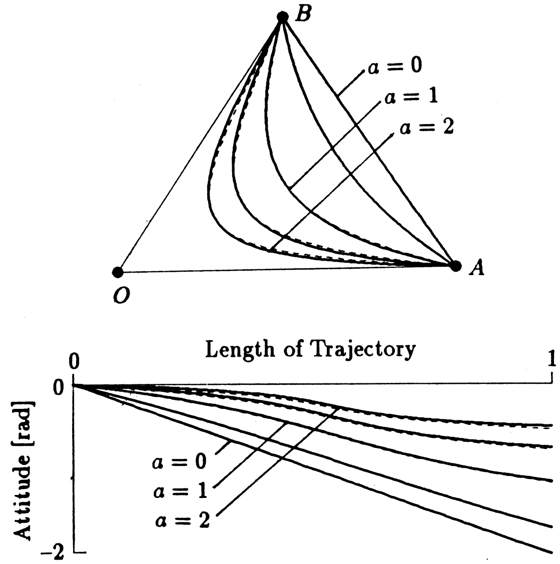

Fig. 4 Hand trajectories in the inertial frame (upper) and attitude variations of the satellite (lower) when the trajectories are given in the inertial frame (solid line ··· solutions to minimize (11), dotted line ··· hyperbola approximations)

fixed frame and the inertial frame are the same, positions A and B coincide with those of A' and B', respectively.

Figure 4 shows the exact trajectory that minimizes (11) in case of the inertial frame (solid line) and its approximation by a hyperbola (dotted line). The upper part of Fig. 4 shows the trajectory in the inertial frame, and the lower part shows the relation between the length of the trajectory and the satellite attitude variation when the trajectory length is normalized to 1. Exact solutions show the cases of $a = 0, 0.5, 1.0, 1.5, 2.0$ in (11). In the hyperbola approximation, the values of a in (22) are chosen to match the minimum of r with the exact solutions.

Also, **Fig. 5** shows the exact trajectory that minimizes (35) in the case of a satellite-fixed frame (solid line) and its approximation by a hyperbola (dotted line). The upper part of the figure shows the trajectory in the satellite-fixed frame, and the lower part shows the relationship between the length of the trajectory and the satellite attitude variation when the trajectory length is normalized to 1. Exact solutions show the cases of $a = 0, 1.0, 2.0, 3.0, 4.0$ in (35). In the hyperbola approximation, the values of a in (39) are chosen to match the minimum of r with the exact solutions. **Table 1** shows the correspondence of a between the exact solutions and the approximate ones in Fig. 4 and Fig. 5. As shown in Table 1, if the minimum values of r are matched, there is a considerable gap of

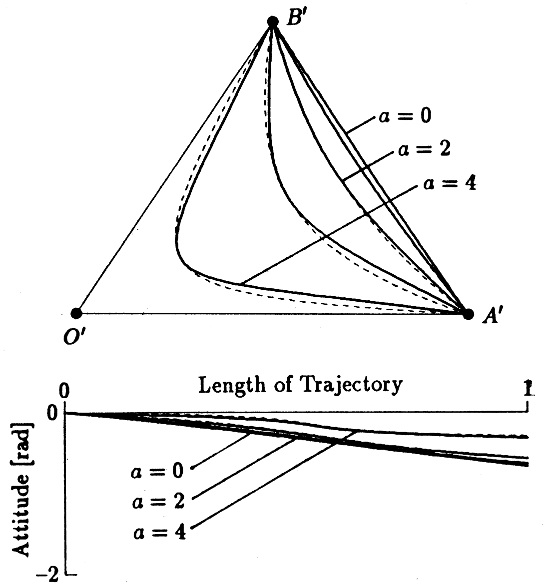

Fig. 5 Hand trajectories in the satellite-fixed frame (upper) and attitude variations of the satellite (lower) when the trajectories are given in the satellite-fixed frame (solid line ⋯ solutions to minimize (35), dotted line ⋯ hyperbola approximations)

Table 1 Correspondence of the values of a between exact solutions and approximate solutions

		a				
Inertial frame	Exact	0	0.5	1	1.5	2
	Approx.	0	0.612	1.36	2.15	2.97
Satellite-fixed frame	Exact	0	1	2	3	4
	Approx.	0	0.138	0.320	0.827	3.15

a between the exact solutions and the approximate ones. Therefore, the hyperbolas are not accurate approximations quantitatively, but are good approximations qualitatively as shown in Fig. 4 and Fig. 5. In both cases, as a increases, the trajectories curve toward the origin and the attitude variations are decreased. When Fig. 4 and Fig. 5 are compared, the satellite attitude variation becomes considerably small in the case of the satellite-fixed frame.

Figure 6 shows the result of realizing each trajectory (exact solution) in Fig. 4 by the arm joints with no attitude control of the satellite. The upper part shows the manipulability of (42) at each point on the trajectory when the length of the trajectory is normalized to 1 and the lower part shows the satellite movement on the trajectory of $a = 1$. The manipulability becomes 0 in the vicinity of the end point of the trajectory in the case of $a = 0$ and $a = 0.5$. This means that the hand cannot reach the end point because the satellite attitude variation is too large. Also, as a increases, the trajectory gets too close to the satellite, which decreases the manipulability. Consequently, the order of $a = 1$ is appropriate, and the satellite movement in this case is shown in the lower part of Fig. 6.

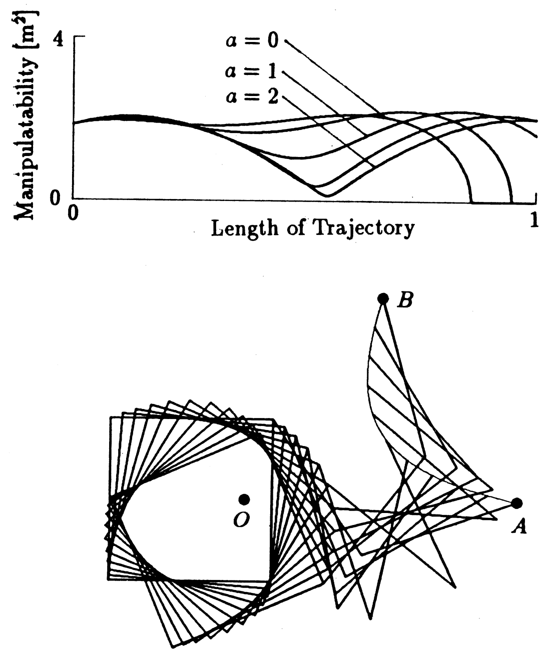

Fig. 6 Manipulability measures of the hand trajectories (upper) and the motion of the satellite in case of $a = 1$ (lower) when the trajectories are given in the inertial frame

Furthermore, **Fig. 7** shows the result of realizing each trajectory (exact solution) in Fig. 5 by the arm joints in the case of the satellite-fixed frame. The upper part shows the manipulability of (45) and the lower shows the satellite movement on the trajectory of $a = 2$ in the inertial frame. The hand movement in the satellite-fixed frame is shown in Fig. 5, while in the inertial frame, the hand movement decreases considerably due to the satellite rotation as shown in the lower part of Fig. 7. Since the satellite rotation angle is in proportion to the area of the fan-shaped domain surrounded by the hand trajectory in the inertial frame and the CM of the space robot mass O in both Fig. 6 and Fig. 7, the satellite attitude variation is smaller in the case of the satellite-fixed frame.

6. Conclusions

We performed a basic study on the relationship between hand motion trajectory and satellite attitude variation with a planar space robot model whose momentum and angular momentum are conserved at 0. The followings gives a summary of our conclusions.

(1) When the hand task coordinates are the inertial frame, the relationship (6) holds between the arm

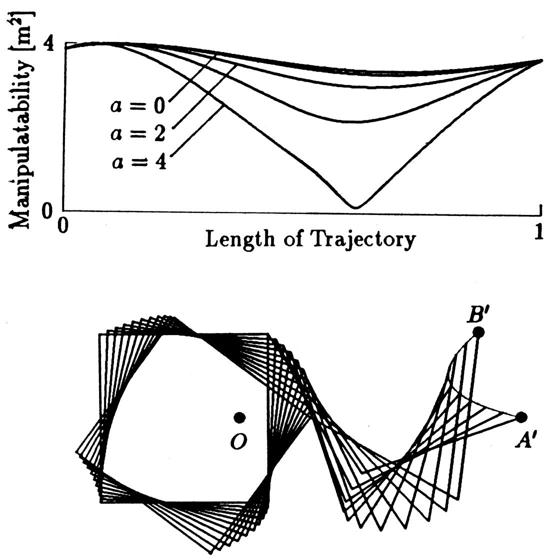

Fig. 7 Manipulability measures of the hand trajectories (upper) and the motion of the satellite in the case of $a = 2$ (lower) when the trajectories are given in the satellite-fixed frame

motion trajectory and the satellite attitude variation. Especially, if the arm mass is concentrated at the hand, attitude variation is in proportion to the area of the domain surrounded by the hand trajectory and the origin (CM of the space robot). A hyperbolic trajectory is suitable for decreasing the attitude variation during hand motion.

(2) When the hand task coordinates are the satellite-fixed frame and the arm mass is concentrated at the hand, the attitude variation is in proportion to the area of the domain surrounded by the hand trajectory and the origin (CM of the satellite) in the (x_d, y_d)-plane of (31). Therefore, if the hand trajectories are the same in the satellite-fixed frame and in the inertial frame, the attitude variation in the satellite-fixed frame becomes smaller than that of the inertial frame. A hyperbolic trajectory is also suitable for decreasing attitude variation during hand motion.

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Katsuhiko Yamada (Member)

Received the B.S., M.S. and Ph.D. degrees in mechanical engineering from University of Tokyo, Tokyo, in 1978, 1980 and 1990, respectively. Since 1980, he has worked in the Advanced Technology R & D Center of Mitsubishi Electric Corporation. His current research interests include dynamics analysis and control of spacecraft.

Kazuo Tsuchiya (Member)

Received the B.S., M.S. and Ph.D. degrees in aeronautical engineering from Kyoto University, Kyoto, in 1966, 1968 and 1974, respectively. From 1968 to 1990, he worked in the Central Research Laboratory of Mitsubishi Electric Corporation. From 1990 to 1996, he was a Professor in the Department of Computer-Controlled Mechanical Systems at Osaka University, Osaka. Since 1996, he has been a Professor in Department of Aeronautics and Astronautics at Kyoto University, Kyoto. His current research interests include dynamics analysis and control of mechanical systems and neural networks.

Reprinted from Trans. of the SICE
Vol. 28 No. 3 374/382 1992