Trans. of the Society of Instrument and Control Engineers
Vol. E-1, No. 1, 164/170 (2001)

Acoustic Bridge Volumeter†

TORIGOE Ippei* and ISHII Yasushi**

A new device is proposed for measuring the volume of a container (or the volume of an object placed within a known container) using gas compressibility laws. A container whose volume is to be measured, a reference container of known volume and a bypass tube are so arranged that their electric equivalents form a bridge circuit. A loudspeaker is mounted on the wall between the two containers so that one face thereof applies alternating compression to the volume undergoing measurement, while the opposite face applies compression of opposite phase to the reference volume. These compressions cause sound pressure within the two containers and the bypass tube. And there is the balance point within the bypass tube where the sound pressure vanishes; the bypass tube forms the ratio arms of the acoustic bridge circuit. The position of this balance point is a function of the ratio of the two volumes. Furthermore, it is affected neither by the static pressure, the gas composition, the temperature nor the compression amplitude. The volume to be measured can be obtained by simple calculation from the position of the balance point. An experimental apparatus was built; that included a container whose volume could be varied from $2 \times 10^{-3}\,m^3$ to $10 \times 10^{-3}\,m^3$ by filling with water and an electret microphone for picking up the sound pressure along the bypass tube. The success of the proposed method has been verified experimentally with this apparatus.

Key Words: volume measurement, gas laws, Boyle-Charles' law, sound pressure, acoustic impedance, acoustic bridge circuit.

1. Introduction

It is not easy to measure the volume of a container having a complex shape. Measurement of the internal dimensions of a container involves considerable time and intensive work. Methods in which liquid poured into a container is measured are not applicable to some types of container. For those containers to which the liquid method is not applicable, gas is employed as an alternative filling medium. Methods using gas are all based on the equation of state. A typical procedure of this method is as follows: a container is filled with a certain amount of gas, a known change in volume is made to this container, the original volume is calculated from the resulting change in pressure on the basis of Boyle's law. With this method, factors that might cause measurement errors include fluctuations of the gas temperature during the measurement, errors in pressure gauges, leaks in the measurement system or the container itself, a change in the volume due to the opening and closing of a valve, etc. In order to make high accuracy measurement, by eliminating all these errors, a large-scale system and a complicated measurement procedure are required. A simpler example of this method employs a piston or a mercurial column connected to a container which is moved to cause a change in the container volume [1), 2)].

The method based on Boyle's law requires a long time to complete the measurement, since the pressure must be measured after the temperature of the gas in the container reaches an equilibrium with the ambient temperature. Slight leaks in the container will result in a considerable measurement error. Another method has been developed to overcome these drawbacks; in this method an alternating change in volume is applied to a container, and the volume is calculated from a resulting dynamic change in pressure. Because a dynamic change in pressure is sound, this method can be called an acoustic volumetric measurement method. For example, there are reports on a porosity meter for acoustical materials[3)], a fuel gauge[4)] and a body plethysmograph that measures the volume of a human body[5)].

In the acoustic volumetric measurement, a change in the following quantities may cause measurement errors: the gas composition, the temperature, the static pressure, the sensitivity of a sound pressure sensor, the amplitude of the alternating volume change, etc. The effects of these quantities must be minimized or eliminated to increase the level of accuracy and stability of measurement. We previously proposed the following methods featuring the use of information contained in sound frequencies: (1) a method utilizing a Helmholtz resonator and a temperature compensation technique for this method[6)], (2) a method using the antiresonant frequency of an acoustical system and a temperature compensation technique [7), 8)]. We also proposed the following method featuring the use of sound pressure amplitude information. A reference container with a known volume and a measuring container whose volume is unknown are placed facing each other. A sound source is mounted between the two containers and driven sinusoidally to apply an alternating compression to each container. The resulting sound pressure in both containers is detected and the volume of the measuring container is calculated from the ratio between these sound pressure amplitudes[9)]. Since a ratiometric technique is being used, it becomes possible to measure the volume

---
† Presented 30th SICE Annual Conference (1997•7)
* Faculty of Engineering, Kumamoto University, Kumamoto-shi, Kumamoto
** Measurement Science Laboratory, Kawasaki-shi,Kanagawa

of a container without being affected by such factors as the gas composition, the temperature, the static pressure, and the amplitude of compression.

In developing the method discussed in this paper, the above method featuring sound pressure amplitude was adopted as a basic concept. In addition, a bridge circuit was employed to the method with the prospect that the bridge configuration would work to cancel out the effects of influence quantities. In an acoustical system, sound pressure and volume velocity function in much the same way as the voltage and current function in an electrical circuit. A container is represented equivalently by a capacitor whose capacitance is proportional to the volume of the container. Measuring the volume of a container can be compared to measuring capacitance in an electrical circuit. This paper applies the bridge technique widely used in electrical measurements to an acoustical system in order to measure the volume.

2. Principle

2.1 Structure of the acoustic bridge volumeter

Fig.1 shows the configuration of the acoustic bridge volumeter. A measuring container whose volume is unknown (volume: V_2) and a reference container with a known volume (volume: V_1) are placed facing one another across a baffle plate. A sound source (loudspeaker) is mounted in this baffle. The two containers are connected via a bypass tube and a microphone is installed to detect the sound pressure inside the bypass tube. In this setup, when the speaker is driven sinusoidally, the volume of each container also fluctuates sinusoidally. The fluctuations in the two containers are equal in magnitude while opposite in phase. Since the two containers are connected via a bypass tube, the gas can move through this tube. The movement of gas in the bypass tube causes compression of the gas in each container. That is, a change in volume is expressed as the total of that caused by the displacement of the speaker diaphragm and that due to the movement of gas in the bypass tube. When the length of the bypass tube is sufficiently short compared with the wavelength at the speaker driving frequency, the whole of the gas in the bypass tube can be considered to vibrate in phase. Then, the volume change in each of the two containers caused by gas movement also has the same magnitude while opposite phase. Therefore, at a given moment, the volume change in each container is equal in magnitude but opposite in sign.

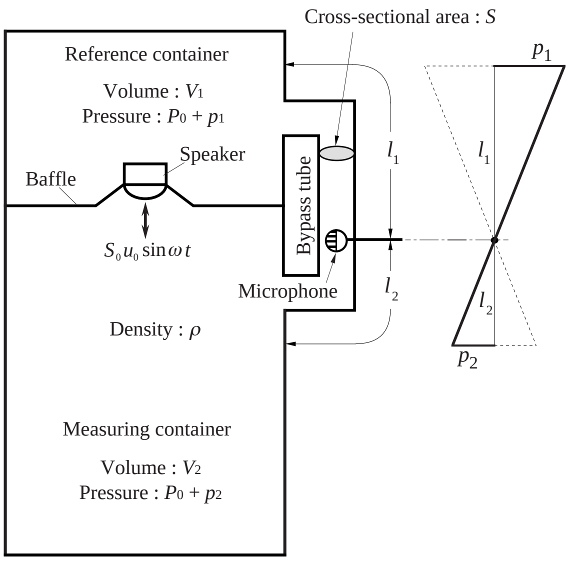


Fig.1 Configuration of the acoustic bridge volumeter.

When a sinusoidal change of the volume ΔV is applied to a container having a volume of V_i (i = 1,2), the resulting sound pressure p_i can be expressed as follows, on condition that the change in state of gas is adiabatic:

$$p_i = -\gamma \frac{P_0}{V_i} \Delta V \tag{1}$$

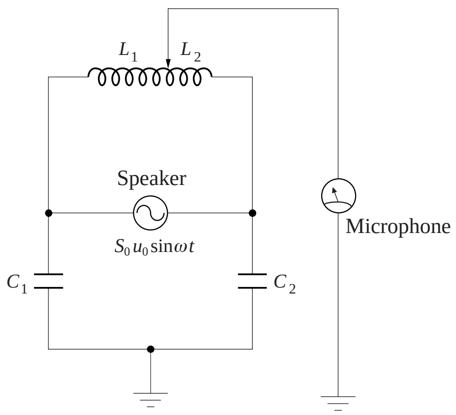
where, γ is the ratio of specific heat (1.40 for air at atmospheric pressure and room temperature) and P_0 is the static (average) pressure. The amplitude of sound pressure in each container is inversely proportional to the volume of each container, and of opposite sign.

Let us next consider the sound pressure distribution within the bypass tube. The sound pressure at the upper end of the bypass tube is equal to that inside the reference container, while the sound pressure at the lower end is equal to the sound pressure in the measuring container. Because the gas inside the bypass tube vibrates at an uniform acceleration, the sound pressure gradient along the bypass tube axis is uniform. This implies that there is a section across the bypass tube where the sound pressure vanishes — the cross section at "the balance point". Let the distance from the upper end to this section be l_1 and the distance from the lower end be l_2. It must be noted here that l_1 and l_2 are the acoustical distances that contain the "end correction". The graph on the right in Figure 1 is a sketch illustrating the sound pressure distribution within the bypass tube.

2.2 Equivalent circuit

The acoustical system in Fig.1 can be represented equivalently as **Fig.2** in an appropriate frequency range. Providing that the vibration velocity of the diaphragm is $u_0 \sin \omega t$ and the effective area is S_0, a speaker can be represented by a current source having an intensity of $S_0 u_0 \sin \omega t$. The electrical analogue for the acoustical compliance of the container is a capacitor whose capacitance is $V_i / \gamma P_0$ (i = 1, 2). The acoustical inertance of the bypass tube of length l_i and cross-sectional area S is represented by a coil having inductance $\rho l_i / S$ (ρ is the density of gas). The microphone set to detect the sound pressure in the bypass tube works as a galvanometer (null detector). The balance equation for this bridge circuit is

$$\frac{j\omega\rho l_1}{S} \cdot \frac{\gamma P_0}{j\omega V_2} = \frac{j\omega\rho l_2}{S} \cdot \frac{\gamma P_0}{j\omega V_1} \tag{2}$$



$L_1 = \rho l_1 / S$, $L_2 = \rho l_2 / S$,
$C_1 = V_1 / \gamma P_0$, $C_2 = V_2 / \gamma P_0$

Fig.2 Equivalent circuit.

Therefore, the relationship between the position of the balance point in the bypass tube and the volume of the containers can be expressed as follows:

$$\frac{l_1}{l_2} = \frac{V_2}{V_1} \tag{3}$$

As equation (3) shows, if the position of the balance point is measured, the ratio of the two volumes can be found, and therefore the volume of the measuring container can be calculated.

2.3 Approximation validity

The equivalent circuit shown in Figure 2 contains some approximations. The conditions under which the equivalent representation applies are as follows:

(1) Infinitesimal vibration

The fluid system shown in Fig.1 can be regarded as an acoustical system only if the velocity of fluid is extremely small and linear acoustical theory can thus apply. When the flow velocity increases inside the bypass tube, the kinetic energy of the fluid will increase and the dynamic pressure — a term proportional to the square of the particle velocity — will not be negligible. Although the dynamic pressure can be eliminated with the help of signal processing techniques such as phase sensitive detection, to maintain a higher signal-to-noise ratio, it is necessary to control the intensity of the dynamic pressure to within certain limits. In the experiments mentioned in the next section, the intensity of the dynamic pressure was 1% or less of the sound pressure inside the reference container.

(2) Lumped parameter system

An acoustical system which is by nature a distributed constant system can be approximated by a lumped parameter system, as shown in Figure 2, if the operating frequency is sufficiently low. In other words, containers can be represented as capacitors and a tube can be represented as an inductance, provided the characteristic size L of the container or the tube is much smaller than the wavelength λ. The error resulting from the lumped element approximation is of the order of $(kL)^2/2$ (see the appendix). Where, k $(=\omega/c)$ is the wave number, c is the sonic speed, and $kL = 2\pi L/\lambda$. The value of $(kL)^2/2$ in the experiments in the next section was about 0.02.

(3) Losses

In Figure 2, the bypass tube is represented by a lossless inductance. In a real tube, however, there exists resistance due to the viscosity of air. Also, in a real container, there is some conductance due to heat conduction. Let us estimate the approximate level of losses, assuming that the cross section of the bypass tube is circular.

<Viscous resistance>

The acoustic impedance of a tube of length l can be calculated as follows [10]:

$$\frac{j\omega\rho l}{S}\left\{1+\frac{\delta_v}{a}(1-j)\right\} \qquad \text{where} \quad \delta_v \equiv \sqrt{\frac{2\mu}{\rho\omega}} \tag{4}$$

a is the tube inner radius, μ is the viscosity coefficient of the gas, ρ is the density of the gas, and ω is the angular frequency of sound. From Eq. (4) we can see the following: owing to viscosity, a real component (resistance) arises in the impedance of the bypass tube, and the imaginary component is also affected and increases. The effect of the viscosity on both components is of the order of δ_v / a — the ratio of the boundary layer thickness to the tube inner radius. The value of δ_v / a was about 0.04 in the experiments described in the next section.

< Heat conduction>

When an alternating compression is applied to a container, the temperature of gas in the container fluctuates around the quiescent value in accordance with the pressure fluctuation. For actual containers, heat conduction occurs between the gas and the wall, and thus the change of gas state in containers is not adiabatic in the strict sense. Assuming that the thermal conductivity and the heat capacity of the container wall is infinite and that the temperature of the wall is isothermal, we find the impedance of a container having a volume V to be given by

$$\frac{\gamma P_0}{j\omega V}\left\{1-(\gamma-1)(1-j)\frac{S_R\delta_t}{2V}\right\} \tag{5}$$

where δ_t is the thickness of a thermal boundary layer, κ is the thermal conductivity of the gas, c_p is the specific heat at constant pressure, and S_R is the inner surface area of the container (see the appendix). This equation indicates the following: owing to heat conduction at the wall, a real component arises in the impedance and the capacitance increases. The effect of heat conduction on the impedance of a container is of the order of $(\gamma - 1) S_R \delta_t / 2V_0$; that is, it is proportional to the ratio between the volume of the thermal boundary layer and the volume of the container. For the containers in the experiments described later, the conductance was about 2% of the susceptance.

(4) End correction

<Additional mass>

In accordance with the vibration of the gas within the bypass tube, the gas in the containers also vibrates in the vicinity of both ends of the bypass tube. The effective length of the bypass tube is longer than the geometrical one ("end correction"). The distances l_1 and l_2 which indicate the position of the balance point include the end correction. If a tube is connected to finite-volume containers having different shapes, as is the case with the present study, it is impossible to clarify the amount of end correction analytically. Studies have been conducted to investigate the end correction in the configuration in which a tube is connected to a finite-volume container [10), 11), 12), 13)]. From the results of these studies, it seems reasonable to suppose that, for the containers in our experiments, the correction is about $8a/3\pi$ which is the value when the tube is mounted in an infinite baffle, and that the shape of the containers has only a minimal effect and thus the corrections at both ends of the bypass tube are nearly equal.

<Additional volume>

When the gas in the container is compressed and its pressure rises, the gas within the bypass tube is also compressed. Therefore, an increase in the volume due to the bypass tube must be added to the effective volume of the container. The additional volume is approximately $Sl_i/2$ since the pressure gradient in the bypass tube is uniform. For the containers in our experiments , the additional volume was less than 0.05% of V_i, which is extremely small and therefore considered to be negligible.

2.4 Nullifying the influence quantities

The acoustic bridge volumeter can measure the volume without being affected by influence quantities such as the gas composition, the temperature, the static pressure, the sensitivity of the microphone, the amplitude of speaker vibration, etc. This is because the acoustic bridge volumeter employs a null method and performs comparative measurement. There are advantages due to this feature; in the design stage, the performance requirements for the sound source and a sound pressure sensor can be relaxed. In the experimental apparatus described in the next section, a low-priced speaker and an electret microphone were used.

In addition, even if there are departures in a real system from the conditions for appropriate approximation, the departures will not directly affect the measurement results. For example, the impedance of the bypass tube can be expressed as the sum of the end correction and the impedance present over the entire length of the tube. Since the amount of end correction is of the order of the tube diameter, if the length of the bypass tube is made much larger than its diameter, it can be presumed that the impedance of the bypass tube is nearly proportional to its length. If this be the case, the balance equation for the bridge circuit is still given by Eq.(3). Even if the viscous resistance is large and not negligible, the effect on the volume measurement is minimal. Particularly, the effect of all influence quantities, including the effect of approximation, can be cancelled by making the bridge highly symmetrical and thereby bringing the balance point to the midpoint of the bypass tube.

3. Experiment

Several experiments were performed using a prototype to verify the principle of the acoustic bridge volumeter. In **Fig. 3**, the experimental setup is shown; the containers consist of an acrylic pipe having a diameter of 192 mm and a thickness of 12 mm, aluminum top and baffle plates of 10 mm in thickness as well as an aluminum base plate of 22 mm in thickness each mounted on the acrylic pipe. The reference container is set in an upper position and is about 100 mm in height and 2.9×10^{-3} m^3 in volume. The measuring container is set at a lower position and is about 340 mm in height. A valve is installed on the measuring container so that its volume can be varied in the range from 2.0×10^{-3} to 10.1×10^{-3} m^3 by pouring in water. A full-range speaker of 70 mm diameter is mounted in the baffle plate to apply a sinusoidal compression to each container at a frequency of 40 Hz; the sound pressure level within the reference container was 118 dB.

An acrylic pipe having a wall thickness of 3 mm is connected to elbows to make a]-shaped bypass tube of diameter 16 mm and length 120 mm; this tube is attached to the side wall of the containers. A microphone probe tube made of brass (2 mm inside diameter and 3 mm outside diameter) is inserted coaxially into the bypass tube. A small aperture is drilled in the side face of this probe tube to feed the sound pressure in the bypass tube to an electret microphone mounted on the top of the probe tube. A step motor was employed to slide the

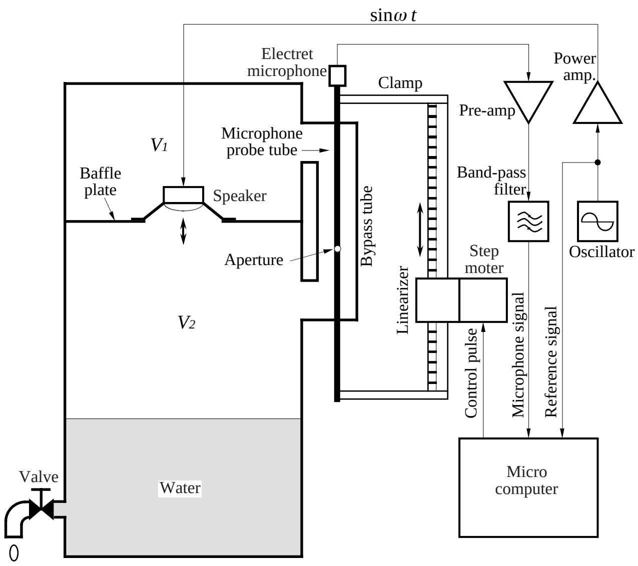


Fig.3 Experimenatal setup.

microphone probe tube in the axial direction in increments of 0.02 mm.

First, the amplitude of the sound pressure inside the bypass tube was measured. **Fig.4** shows the results when V_2 was about 6.9×10^{-3} m^3. The negative amplitude means that the phase of the sound was inverted. The sound pressure was linearly distributed along the axis of the bypass tube, as was predicted in section 2.1.

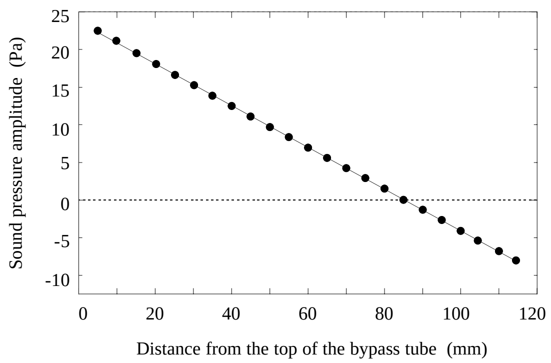


Fig.4 Sound pressure distribution along the bypass tube.

Next, the measuring container was filled with water to upper limit, and the position of the balance point in the bypass tube was measured as the water was released in increments 0.5×10^{-3} m^3 (500cc). For the]-shaped bypass tube in this setup, it is impossible to calculate geometrically the acoustic equivalent length even after the end correction is left out of consideration. Thus, for the first place, l_1' and l_2', distances from the corner of the]-shaped pipe to the balance point, were measured. Let the distance from the corner of the]-shaped pipe to the acoustic end be Δl, then the acoustic distances between the balance point and both ends of the tube are given by $l_1 = l_1' + \Delta l$, $l_2 = l_2' + \Delta l$ (see **Fig. 5**). Moreover, the correct value of the volume of the reference container V_1 is not yet known. A regression line was fitted to the values measured at 17 points using Eq. (3). The results are plotted in **Fig.6**; in which the estimated value of V_1 and Δl are used. **Fig.7** is an enlarged plot of the residuals from the regression line. The percentage of residuals was less than 0.25% in this experiment. Figures 6 and 7 indicate that the acoustic bridge volumeter is capable of measuring the volume of a container in terms of the balance point position.

Finally, the following experiments were performed to observe the effects of influence quantities that may cause measurement errors. Measurements were made at certain volume and at a room temperature of 302K. Then, the room temperature was lowered to 292K and the same volume was measured again; both results were the same within the measurement uncertainty. To investigate the effect of the shape of containers, the volume of the measuring container was read, at several levels of water, for two cases, one in which the experimental apparatus was set upright and one in which it was tilted by 45 degrees.

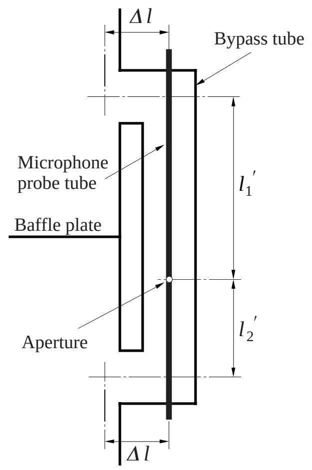


Fig.5 Acoustical equivalent length of the bypass tube. ($l_1 = l_1' + \Delta l$, $l_2 = l_2' + \Delta l$).

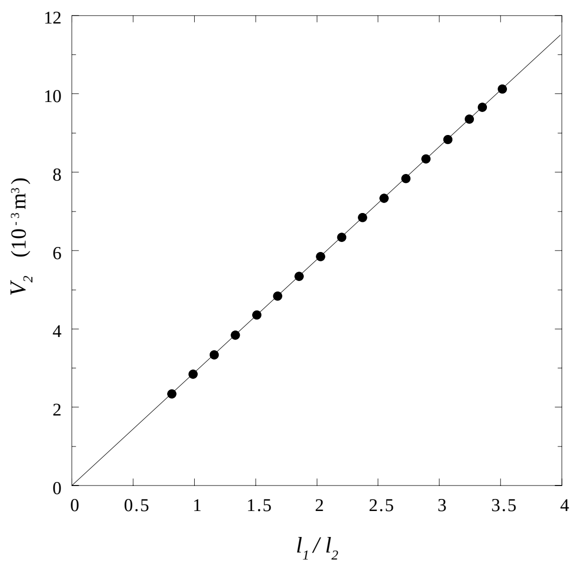


Fig.6 A plot of the container volume vs. (l_1 / l_2).

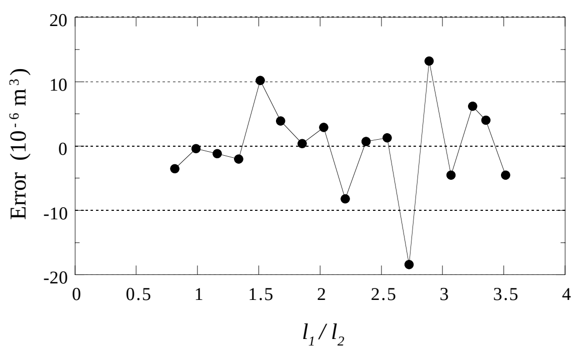


Fig.7 Residual errors.

No difference was observed between the two cases. The influence of a change in the gas composition was investigated as follows: firstly, measurement was made with an empty container; a piece of dry ice was then put in the measuring container to sublimate and fill the two containers with carbon dioxide (γ = 1.30); finally, the second measurement was made. No difference due to the change of the gas composition was observed.

4. Conclusion

A new method, named acoustic bridge volumeter, is proposed for measuring the volume of containers. We considered theoretically the principle and the influence quantities of the method. We also built an experimental apparatus and performed several experiments with this apparatus to verify the principle. The results indicate that the acoustic bridge volumeter is simple and superior in that a high level of accuracy and stability can be achieved.

Besides the prototype presented in this paper, there are many other possible embodiments of the principle. Elements other than acoustic inductances can be used on the ratio arms of the acoustic bridge — acoustic resistive elements, for example. If extreme measurement accuracy is required, the position of the microphone should be fixed at the midpoint of the ratio arms, and the reference volume should be adjusted to match the volume to be measured.

Because of the precursory nature of this study, we have concentrated on the fundamental aspects of the proposed method, and the experiments presented above are far from complete. Future work in this study should include: (i) the design of constructions for practical applications; (ii) tests in an extended range of measurement conditions; (iii) the assessment of the overall uncertainty in measurement.

We would like to express our sincere appreciation to Mr. HIRAOKA Jin (Toray Industries) and Mr. KIMURA Naozumi (Kyushu Matsushita Electric Industrial Co.) for their assistance in performing the experiments.

References

1) Isobe and Sekiguchi: Portable volumeter, Oyo Buturi(A monthly publication of the Japan Society of Applied Physics), **27**-11, 683/685 (1958)
2) Miwa and Hidaka (Society of Powder Technology, Japan, Ed.): Particle density, Handbook of powder technology , 51/57, Nikkan Kogyo Shinbun (1986)
3) R.W.Leonard: Simplified Porosity Measurements, J. of Acoust. Soc. America, **20**, 39/41 (1948)
4) I.H.Cohn and W.E.Dunn: Precision Techniques for Measuring Liquid Quantity, Control Engineering, **15**-1, 51/55 (1968)
5) A.B.Debois, et al.: A rapid plethysmographic method for measuring thoracic gas volume, A comparison with a nitrogen washout method for measuring functional residual capacity in normal subjects, J. Clin. Invest., **35**, 322/326 (1956)
6) Ishii: Volume measurement using an acoustic resonator, Proc. 27th SICE Annual Conference, 131/132 (1988)
7) Yanagida,Ishii and Torigoe: Volume measurement utilizing the antiresonant frequency of an acoustic system, Proc. Acoust. Soc. Japan. Annual Conference, 501/502 (1989.10)
8) Yanagida,Ishii and Torigoe: Volume measurement utilizing the antiresonant frequency of an acoustic system (2nd. report), Proc. Acoust. Soc. Japan. Annual Conference, 437/438 (1990.9)
9) Torigoe and Ishii: Volume measurement using Boyle-Charles' law, Proc. Acoust. Soc. Japan. Annual Conference, 253/254 (1982.10)
10) L.E.Kinsler and A.R.Frey: Fundamentals of Acoustics, 241, John Wiley & Sons (1950)
11) U.Ingard: On the radiation of sound into a circular tube, with an application to resonators, J. of Acoust. Soc. America, **20**, 665/682 (1948)
12) Izumi and Narikiyo: End correction of an aperture neck of acoustic resonator, J. of the Acoust. Soc. Japan., **46**-7, 559/566 (1990)
13) Ichimiya,Endoh and Ishikawa: Volume measurement utilizing sound, Trans. Jpn. Soc. Mechanical Engineering (C), **57**-541, 2883/2893 (1991)

Appendix

1. Errors due to lumped element approximation

<Impedance of a container>

The acoustic impedance Z_l of a cylindrical container of depth L and cross-sectional area S_C is given by

$$Z_l = \frac{1}{j\omega}\frac{\gamma P_0}{S_c L} \tag{A.1}$$

This container can be regarded also as an acoustic pipe of sectional area S_C and length L with one end closed. The acoustic impedance Z_d, as seen from the other end, can be calculated as follows:

$$Z_d = \frac{1}{j}\frac{\rho c}{S_c}\cot kL \tag{A.2}$$

If kL is extremely small, the ratio of Z_l / Z_d is expressed approximately as

$$\frac{Z_l}{Z_d} \approx 1 + \frac{(kL)^2}{3} \tag{A.3}$$

< Particle velocity at both ends of the bypass tube>

Let us estimate the particle velocity at both ends of the bypass tube shown in Figure 1, viewing the bypass tube as an acoustic pipe of cross sectional area S and overall length L (= $l_1 + l_2$) and considering that each end is terminated by each container. The ratio of the particle velocity at the upper end u_1 to that at the lower end u_2 , u_1 / u_2 , is expressed irrespective of the intensity of the sound source as follows:

$$\begin{aligned}\frac{u_2}{u_1} &\approx 1 + \frac{V_2 - V_1}{V_2 + V_1 + SL}\cdot\frac{(kL)^2}{2} \\ &\approx 1 + \frac{V_2 - V_1}{V_2 + V_1}\cdot\frac{(kL)^2}{2}\end{aligned} \tag{A.4}$$

The difference in the particle velocity is of the order of $(kL)^2/2$.

2. Impedance of a container having an isothermal wall

The acoustic impedance of a container having a volume V and an isothermal wall (temperature T_0) is calculated.

Let us start with the distribution of excess temperature above an isothermal wall. Assume that there is an ideal gas above a plane, infinitely large, isothermal wall (y = 0) and that the pressure of the gas fluctuates in phase. The equation of energy conservation can be expressed in terms of enthalpy as

$$\rho \frac{Dh}{Dt} = \frac{DP}{Dt} - \operatorname{div} \mathbf{q} \qquad \text{(A.5)}$$

where ρ (= $\rho_0 + d\rho$) is the gas density, h is the enthalpy per unit mass, P (= $P_0 + p$) is the pressure, and $\boldsymbol{q}$ is the heat flux vector. Assuming that a change in pressure p is sinusoidal and extremely small, the second- or higher-order terms can be neglected. Then Eq. (A.5) can be modified as follows using Fourier's heat conduction law and the relationship between enthalpy and temperature ($dh = c_p dT \equiv c_p \theta$) :

$$j \rho_0 \omega c_p \theta = j\omega p + \kappa \operatorname{div} \operatorname{grad} \theta \qquad \text{(A.6)}$$

where κ is the heat conductivity of the gas. Since the temperature distribution is developed in the vertical direction, the second term on the right hand side of (A.6) reduces to $\kappa \partial^2\theta / \partial y^2$. With the boundary conditions ($y = 0$: $\theta = 0$, $y \to \infty$: $\theta \to \theta(t)$) taken into consideration, the solution of Eq (A.6) is obtained:

$$\theta = \frac{p}{\rho_0 c_p} \left\{ 1 - \mathrm{e}^{-(1+j)y/\delta_t} \right\} \quad \text{where} \quad \delta_t \equiv \sqrt{\frac{2\kappa}{\rho\omega c_p}} \qquad \text{(A.7)}$$

δ_t is the thickness of the thermal boundary layer. The influence of the isothermal wall on the distribution of the gas temperature extends to a distance of the order of δ_t from the wall. At $y = 2\pi\delta_t$, for instance, the second term on the right hand side of (A.7) that represents the effect of the wall decreases to 0.2% of the first term that represents the value at a point far from the wall.

Let us next consider the acoustic impedance of a container having a volume V . Neglecting the higher orders of variation, we have the following equation for the sinusoidal time variation of the state of an ideal gas:

$$-\frac{j\omega\rho}{\rho_0} = -\frac{j\omega p}{P_0} + \frac{j\omega\theta}{T_0} \qquad \text{(A.8)}$$

Integrating the left hand term of (A.8) over the container volume and using the continuity equation

$$-\frac{j\omega\rho}{\rho_0} = \operatorname{div} \mathbf{u} \qquad \text{(A.9)}$$

we find

$$\iiint_V \operatorname{div} \mathbf{u} \, \mathrm{d}v = \iint_{S_R} u_n \, \mathrm{d}s = -U \qquad \text{(A.10)}$$

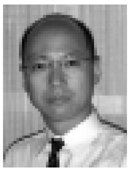
where $\boldsymbol{u}$ is the particle velocity, S_R is the container inner surface, u_n is the particle velocity component normal to S_R, and U is the volume velocity applied to the container. Let us next turn to the right hand side of (A.8). If the characteristic dimension of the container is large enough in comparison with δ_t, the excess temperature distribution is given by equation (A.7) almost everywhere in the boundary layer. Substituting (A.7) into θ and assuming that the pressure varies in phase within the container, we can integrate the right hand side of (A.8) over the container volume;

$$-\frac{j\omega V}{\gamma P_0} \left\{ 1 + (\gamma - 1)(1 - j) \frac{S_R \delta_t}{2V} \right\} p \qquad \text{(A.11)}$$

here S_R is the inner surface area of the container. Equating (A.10) to (A.11) and using the fact that the term due to heat conduction is small ($\varepsilon << 1$), we find the acoustic impedance of the container to be given by

$$\frac{\gamma P_0}{j\omega V} \left\{ 1 - \varepsilon(1 - j) \right\} \quad \text{where} \quad \varepsilon \equiv (\gamma - 1) \frac{S_R \delta_t}{2V} \ . \qquad \text{(A.12)}$$

TORIGOE Ippei (Member)



He received the B.E.degree in 1977, the M.E.degree in 1979, and the Dr.Eng.degree in 1989, all from The University of Tokyo. Since 1990 he has been an Asociate Professor at Kumamoto University. His current research interests are in the area of measurement utilizing sound, micro fluidics, and non-destructive test using sound.

Dr. Torigoe is a Member of the Acoustical Society of Japan, and the Society of Instrument and Control Engineers.

ISHII Yasushi (Member)

He graduated from The University of Tokyo with the Dr.Eng.degree in 1959. From 1959 to 1992, he was with The University of Tokyo as an Assistant Professor, as an Asociate Professor and as a Professor. He has been the President of Measurement Science Laboratory, Kanagawa, Japan, since he founded it in 1992. His research interests include signal processing and measurement utilizing sound.

Dr. Ishii is a Member of the Acoustical Society of Japan, and the Society of Instrument and Control Engineers.

Reprinted from Trans. of the SICE, Vol.30, No.11, 1303/1309 (1994)