

Mass Measurement Using a Dynamic Vibration Absorber under Weightless Conditions

Takeshi MIZUNO* and Kenji ARAKI**

A new type of mass-measurement system under weightless conditions is proposed which uses a dynamic vibration absorber as a measuring device. In this system, an object to be measured is fixed to a rotating table (rotor) at a distance from the rotation axis. Since it makes the rotor unbalanced, a centrifugal force whose amplitude is proportional to the mass is generated during rotation. It works as a harmonic excitation and forces a structure supporting the table to vibrate. However, a dynamic vibration absorber attached to the structure is tuned or controlled to reduce the vibration to zero. When the structure does not move at all, the absorber mass vibrates in such a way that the product of the mass and the displacement amplitude is equal to the amount of unbalance, that is, the product of the mass to be measured and its distance from the rotation axis. Therefore, the mass of the object is determined by measuring the displacement amplitude of the absorber mass. In this study, the principles and features of the proposed mass-measurement system are clarified. Experiments are carried out with a developed apparatus having an active dynamic vibration absorber. This apparatus can perform measurement at various rotational speeds because the absorber's natural frequency can be tuned by a feedback parameter. Experimental results demonstrate the feasibility of the proposed mass-measurement system.

Keywords: mass measurement, weightless conditions, dynamic vibration absorber, centrifugal force, anti-resonance

1. Introduction

Various weightless or micro-gravity experiments have been carried out on the Space Shuttle. A number of new phenomena, which could not be observed on earth, have been disclosed in the experiments. The production of new materials and medicines in space is expected to start in the near future. Mass measurement under weightless conditions will be necessary to perform such production.

Conventional methods of mass measurement are classified into two types ¹⁾:

- (1) Compare the weight force acting on an object of unknown mass with those of reference weights,
- (2) Measure the change in length of a spring that is proportional to the weight force acting on an object.

Both of them cannot perform measurement in space because they are based on gravitational acceleration on earth.

There are several methods proposed for mass measurement under weightless conditions:

- (1) Measure the natural frequency of a spring-mass system.
- (2) Apply the concept of the dynamic measurement method ²⁾.

- (3) Measure the centrifugal force in rotating an object ³⁾.

The first method is sensitive to vibration transmitted from the surroundings; it takes longer time to measure in the presence of such vibration. The second method needs sophisticated mass-estimation algorithms including the compensation of sensor dynamics; it does not seem clear how to improve the accuracy of measurement. The third method has a critical problem that centrifugal forces generated during measurement may cause vibration in the surroundings; the distance of the center of gravity mass of an object from the rotation axis must be identified precisely. If the distance is unknown, two centrifugal forces must be measured whereby the object is placed at two different distances ³⁾.

In this paper, a new type of mass-measurement system is proposed in which a dynamic vibration absorber works as a device for both mass-measurement and vibration control. This method uses centrifugal force in principles so that the distance from the rotation axis must be identified. However, the vibratory force is cancelled by the absorber during measurement so that there is no vibration transmitted to the surroundings. It is, therefore, suitable for measuring in vibration-sensitive environments such as space structures.

The principles and features of the proposed mass-measurement system are clarified in this paper. An apparatus using an active dynamic vibration absorber with

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an electromagnetic servomechanism is developed. Several experiments carried out with the apparatus are reported.

2. Principles and Characteristics of Measurement

2.1 Using an Undamped Dynamic Vibration Absorber

A basic model of the proposed mass-measuring system that uses an undamped dynamic vibration absorber is shown in Fig.1. An object to be measured m_u is fixed to a turning table at a distance r from the axis of rotation. The frame supporting the table is mounted to the base through spring k_p . A dynamic vibration absorber consisting of an absorber mass m_a and spring k_a is attached to the frame. When the table is rotating at a velocity ω , the equations of motion are derived as follows.

$$m_p \ddot{x}_p(t) = -k_p x_p(t) - k_a(x_p(t) - x_a(t)) + m_u r \omega^2 \cos \omega t, \tag{1}$$

$$m_a \ddot{x}_a(t) = -k_a(x_a(t) - x_p(t)), \tag{2}$$

where m_p is the total mass suspended by the spring.

To obtain the stationary solutions, we represent the variables in such a form:

$$x_p(t) = X_p \cos \omega t, \tag{3}$$

$$x_a(t) = X_a \cos \omega t. \tag{4}$$

From (1) to (4), we get

$$X_p = \frac{1 - \frac{\omega^2}{\omega_a^2}}{(1 + \frac{k_a}{k_p} - \frac{\omega^2}{\omega_p^2})(1 - \frac{\omega^2}{\omega_a^2}) - \frac{k_a}{k_p}} \cdot \frac{m_u r \omega^2}{k_p}, \tag{5}$$

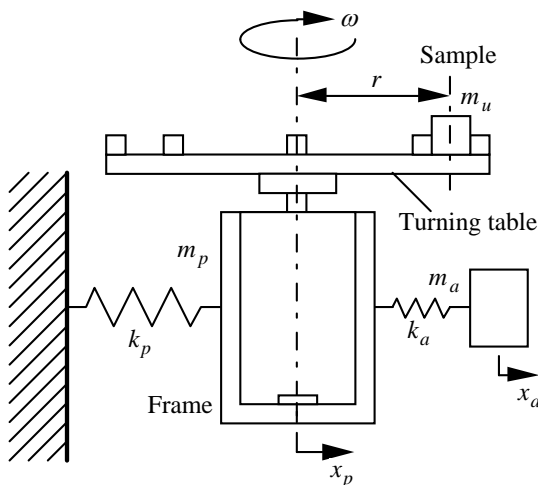


Fig.1 Mass measurement system with an undamped dynamic vibration absorber

$$X_a = \frac{1}{(1 + \frac{k_a}{k_p} - \frac{\omega^2}{\omega_p^2})(1 - \frac{\omega^2}{\omega_a^2}) - \frac{k_a}{k_p}} \cdot \frac{m_u r \omega^2}{k_p}, \tag{6}$$

where

$$\omega_p = \sqrt{\frac{k_p}{m_p}}, \tag{7}$$

$$\omega_a = \sqrt{\frac{k_a}{m_a}}. \tag{8}$$

When the rotational frequency ω is equal to the absorber's natural frequency ω_a , it is derived from (5) that

$$X_p = 0, \tag{9}$$

so that the steady-state vibration of the frame becomes zero.

From (6), the absorber mass vibrates to satisfy

$$X_a = -\frac{m_u r}{m_a}. \tag{10}$$

From (10), we get

$$m_u = \left| \frac{m_a X_a}{r} \right|. \tag{11}$$

Therefore, the mass of the object is given by dividing the product of the absorber mass m_a and its displacement amplitude $|X_a|$ by the distance of the object from the rotation axis r .

The principle of measurement is summarized as follows.

- (1) Fix an object on the turning table at a predetermined position.
- (2) Coincide the rotating speed of the table with the absorber's natural frequency so as to stop the vibration of the frame.
- (3) Measure the displacement amplitude of the absorber mass.

2.2 Using an Active Dynamic Vibration Absorber

As explained in the previous section, the absorber's natural frequency must equal the rotational frequency when an undamped dynamic vibration absorber is used. One method of achieving it is to use an active dynamic vibration absorber and adjust its natural frequency by changing the feedback gain. When an active dynamic vibration absorber is used, however, the principle of measurement can be more generalized. It will be explained in the following.

Figure 2 shows a basic model when an active dynamic vibration absorber is used. The spring k_p in the passive absorber is replaced by an actuator. Electromagnet, linear motor and piezoelectric device are available as an actuator.

To make general formulation, the actuator is modeled as a device generating force $F(t)$. Then the equations of motion are:

$$m_p \ddot{x}_p(t) = -k_p x_p(t) - F(t) + m_u r \omega^2 \cos \omega t, \quad (12)$$

$$m_a \ddot{x}_a(t) = F(t). \quad (13)$$

Assuming that the motion of the frame is regulated by active control as

$$x_p(t) = 0, \quad (14)$$

we get

$$m_a \ddot{x}_a = m_u r \omega^2 \cos \omega t, \quad (15)$$

independently of the applied control method. Substituting (4) into (15) gives

$$X_a = -\frac{m_u r}{m_a} \quad \therefore m_u = \left| \frac{m_a X_a}{r} \right|. \quad (16)$$

Therefore, m_u is estimated according to the same equation as an undamped dynamic vibration absorber is used.

Equation (12) indicates that the vibration of the supporting structure must be eliminated during measurement. There are several control methods applicable for this purpose:

- (1) Use the absorber as an undamped dynamic vibration absorber.
- (2) Apply the theory of output regulation with internal stability⁴⁾⁻⁶⁾.
- (3) Apply the repetitive or periodically learning control method^{7), 8)}.

The first method has been already explained in the

previous section. In the second and third methods, the control input is constructed to cancel the centrifugal force. The second methods are subdivided into two methods:

- (2-1) Build a disturbance observer and cancel the effects of the disturbance asymptotically using the estimated signals^{4), 5)}.
- (2-2) Insert a model of disturbance into the feedback loop⁶⁾.

According to the third method, the control input canceling the periodical excitation is made iteratively by modifying the input based on the results during the last period⁷⁾⁻⁹⁾.

When an active dynamic vibration absorber is used, therefore, various active control methods are applicable to eliminate the vibration of the frame.

2.3 Measurement Using a Counter Weight

Equations (10) and (16) indicate that the motion of the absorber mass becomes larger in proportion to the mass of an object to be measured. When the vibration amplitude of the absorber mass becomes so large to exceed the operation range of the spring or the actuator, measurement cannot be performed.

When the stroke of the absorber mass is limited by a value of X_{amax} , the measurement range is given by

$$m_u \leq \left| \frac{m_a X_{amax}}{r} \right|. \quad (17)$$

This range can be widened by decreasing r . However, it causes measurement inaccuracy because measurement accuracy becomes more sensitive to the value of r .

A method of overcoming this problem is to use a counter weight for reducing the centrifugal force during measurement. This method is effective when the approximated mass of an object can be estimated before measurement. Figure 2 shows the basic model of measurement system using a counter weight. A weight of known mass m_c is attached at a distance r' on the turning table opposite to the object m_u . The equation of motion corresponding to (12) becomes

$$m_p \ddot{x}_p(t) = -k_p x_p(t) - F(t) + (m_u r - m_c r') \omega^2 \cos \omega t. \quad (18)$$

When the vibration of the frame is eliminated, the absorber mass vibrates to satisfy

$$X_a = -\frac{m_u r - m_c r'}{m_a}. \quad (19)$$

Hence, if the counter weight is selected to satisfy

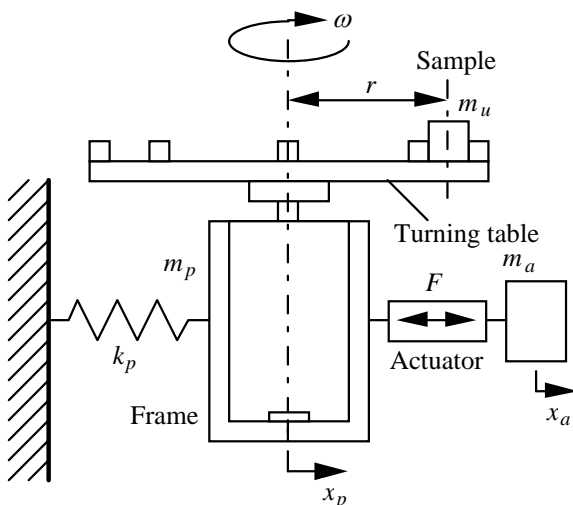


Fig.2 Mass measurement system with an active dynamic vibration absorber

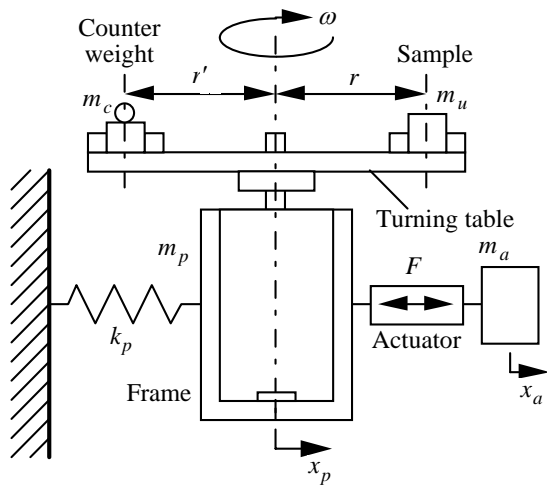


Fig.3 Mass measurement system using a counter weight

$$\frac{m_u r - m_a X_{a \max}}{r'} \leq m_c \leq \frac{m_u r + m_a X_{a \max}}{r'} \quad (20)$$

the absorber mass remains in the operating range of the spring or the actuator. From (19), the mass of the object is obtained as

$$m_u = m_c \left(\frac{r'}{r} \right) - \frac{m_a X_a}{r} \quad (21)$$

To calculate the correct value of m_u according to (21), not only the absolute value but also the sign of X_a are necessary. It can be determined by checking the phase between the motions of the absorber mass and the counter weight. In the case of same phase, the mass to be measured is calculated by

$$m_u = m_c \frac{r_c}{r} + m_a \frac{|X_a|}{r} \quad (\text{see Fig.4(a)}). \quad (22)$$

Otherwise, it is calculated by

$$m_u = m_c \frac{r_c}{r} - m_a \frac{|X_a|}{r} \quad (\text{see Fig.4 (b)}). \quad (23)$$

2.4 Basic Characteristics

In the proposed measuring system, the centrifugal force acting on the frame is canceled by the reaction force produced by the absorber so that there is no stationary vibratory force transmitted to the foundation through the spring k_p . Therefore, it suits measurement in flexible structures such as space stations. Other important characteristics are:

- (1) Measurement sensitivity can be adjusted by changing the distance r . For example, smaller mass can be measured by increasing the distance.

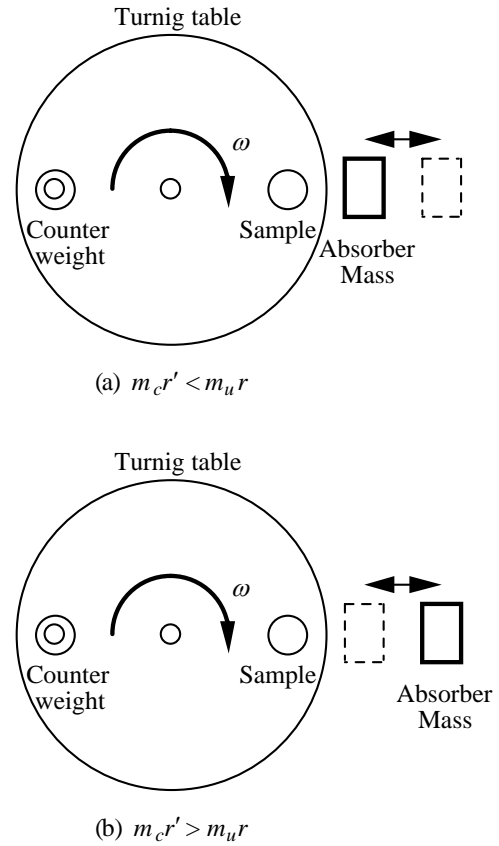


Fig.4 Relation between the angular position of a counter weight and the vibration of the absorber mass (Top view)

- (2) Sensitivity is also adjusted by changing the absorber mass. Since the vibration amplitude of the absorber mass is inversely proportional to its mass, the displacement to be detected can be made larger by using a smaller absorber mass.
- (3) The range of measurement can be widened by using known weights as a counter weight. This enables the proposed system to measure relatively large mass.
- (4) Advanced control theories can be applied to remove vibration when an active dynamic vibration absorber is used.
- (5) When the distance r is unknown or changes during rotation, measurement must be carried out two or more times at different distances and at different rotational speeds.

3. Experimental Methods

3.1 Experimental Setup

The schematic drawing of the experimental setup is shown in Fig.5. This setup has been used for study on the balancing machine with a dynamic vibration absorber¹⁰⁾. In this research, it is operated as a mass measurement system.

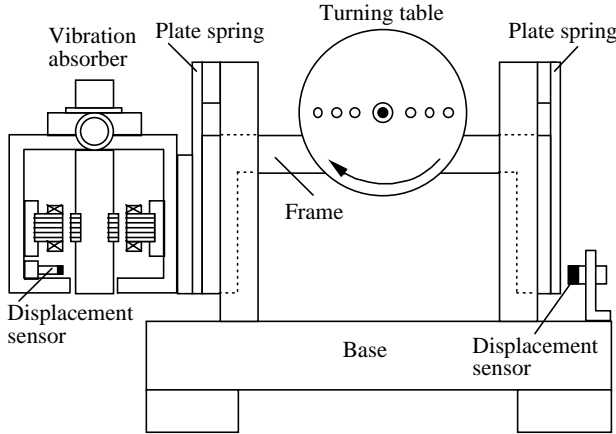


Fig.5 Schematic diagram of experimental setup

A motor fixed to the frame drives a disc-type rotor that works as a turning table. An active dynamic vibration absorber is attached to the frame that is mounted to the base by a pair of plate springs. The absorber has a lever hanging from a fixed point O with a ball-bearing pivot as an absorber mass. The attractive forces of a pair of electromagnets are used to control the motion of the lever.

3.2 Dynamics of the Setup

An object with a mass of m_u is fixed to the rotor at a distance r from the rotation axis. When the rotational speed is ω , the equations of motion are ⁵⁾:

$$(m_1 + m_2)\ddot{x}_1(t) + c_1\dot{x}_1(t) + k_1x_1(t) + m_2\ddot{x}_2(t) = m_u r \omega^2 \cos \omega t, \tag{24}$$

$$m_2\ddot{x}_1(t) + (m_2 + \frac{I}{l_g^2})\ddot{x}_2 - k_2x_2(t) = -k_\mu i(t), \tag{25}$$

where x_1 : displacement of the frame in the horizontal direction, x_2 : displacement of the center of gravity mass of the lever relative to the frame, m_1 : total mass suspended by the plate springs, m_2 , I : mass and moment of inertia of the lever, k_1 , c_1 : stiffness and damping constant between the frame and the base, l_g : distance of the mass center of the lever from O, k_2 , k_μ : coefficients of the electromagnets, and i : control components of the exciting current. The values of the coefficients are listed in Table 1.

To obtain the stationary solution, we represent

$$x_2(t) = X_2 \cos \omega t. \tag{26}$$

When the vibration of the frame is eliminated, substituting (26) into (24) gives

$$m_u = -\frac{m_2 X_2}{r}. \tag{27}$$

Table 1 Coefficients of the experimental setup

Parameter	Value
m_1	16.7 kg
m_2	1.13 kg
k_1	5.32×10^5 N/m
c_1	1.72×10^2 Ns/m
I	2.82×10^{-3} kgm ²
l_g	3.37×10^{-2} m
k_2	2.07×10^5 N/m
k_μ	1.08×10^2 N/A

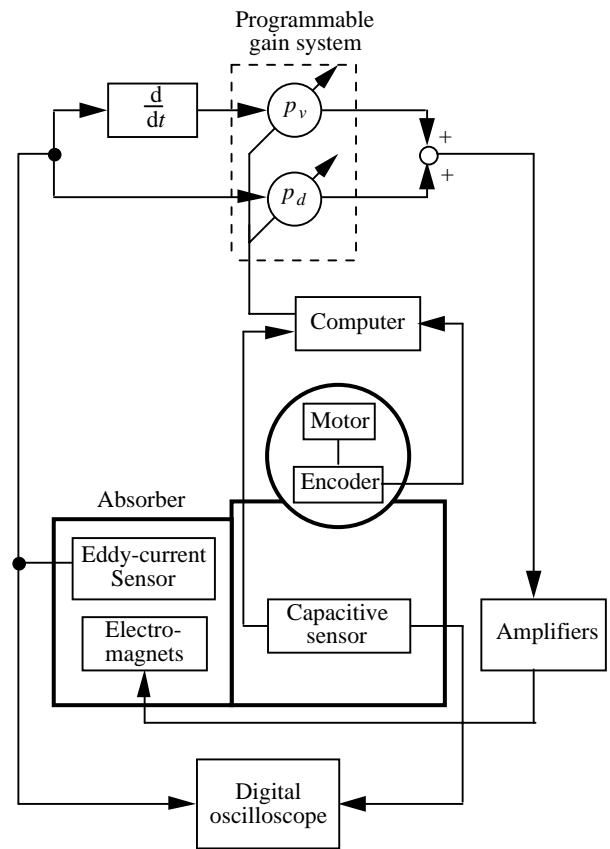


Fig.6 Outline of the control and measurement system

Therefore, the mass of an object is given by dividing the product of mass m_2 and displacement amplitude of the lever $|X_2|$ by the distance r .

3.3 Control and Measurement System

To eliminate the vibration of the frame, we use the absorber as an undamped dynamic vibration absorber and coincide its natural angular frequency with the rotational speed. The control input is, therefore, given by

$$i(t) = p_d x_2(t). \tag{28}$$

The natural angular frequency ω_2 of the absorber becomes

$$\omega_2 = \sqrt{\frac{k_\mu p_d - k_2}{m_2 + \frac{I}{l_g^2}}} \tag{29}$$

It can be adjusted by the feedback gain p_d .

In practice, the absorber system falls into instability when only the displacement feedback is used because of the phase lag in the power amplifier. It is necessary to add velocity feedback as

$$i(t) = p_d x_2(t) + p_v \dot{x}_2(t) \tag{30}$$

In the experiment, the gain p_v is minimized so far as the system is stable. As a result, the vibration of the frame is kept within 0.3 μ m in measuring.

The outline of the experimental system is shown in Fig.4. The relative displacement of the lever is detected by an eddy-current sensor with a resolution 1 μ m (catalogue value). The detected signal is inputted to a programmable gain system with multiplying D/A converters. It calculates the control input according to eq.(30) and outputs it to a pair of power amplifiers. Each power amplifier controls the current through the coil in proportion to its input.

The displacement and velocity feedback gains, p_d and p_v are adjusted by a supervisory computer. Since the output signals of the encoder attached to the motor are inputted to the computer, the gains can be adaptively changed in response to changes in the rotational speed.

The displacement of the frame is detected by a capacitive sensor with a resolution of 0.1 μ m (catalogue value). This signal is used to check whether the vibration of the frame is eliminated enough.

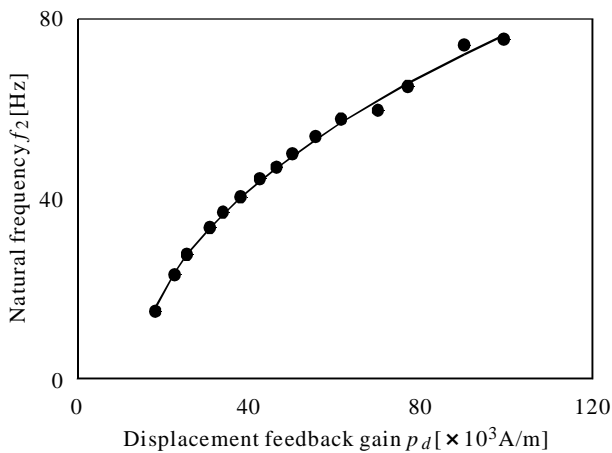


Fig.7 Natural frequency of the absorber as a function of displacement feedback gain

4. Experimental Results

4.1 Characteristics of the Absorber

The natural frequency of the absorber, $f_2 (= \omega_2 / 2\pi)$, is measured for each p_d as shown in Fig.7. This result shows the absorber can adjust its resonant frequency from 15Hz to 75Hz.

4.2 Mass Measurement

The mass of a sample used in the experiment is

$$m_u = 1.256 \text{ [g]} .$$

It is attached to the disk at

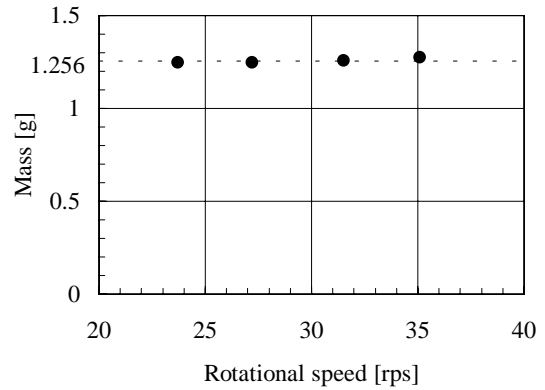
$$r = 40 \text{ [mm]} .$$

Since some of the parameters of the system are difficult to be measured directly, the scale parameter between $|X_2|$ and m_u was calibrated based on a preparatory measurement. In this measurement, the rotor was driven at a speed of 27.2[rps] in the counter-clockwise direction.

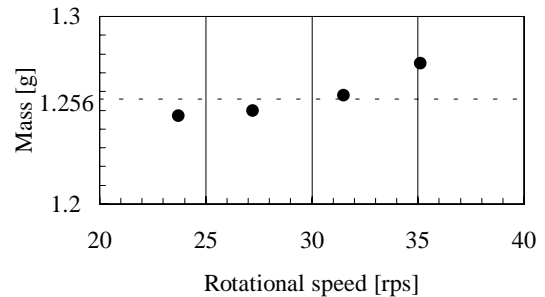
After the calibration, the direction of rotation was changed to be clockwise. The rotational speed is selected as

- (a) 23.7 [rps], (b) 27.2 [rps],
- (c) 31.5 [rps], (d) 35.1 [rps].

The amplitude $|X_2|$ is measured at each rotational speed when the vibration of the frame is minimized. The mass of



(a) Mass measured at various speeds



(b) Detail of (a)

Fig.8 Measurement results

the sample is estimated from the measured $|X_2|$ as

- (a) 1.247 [g], (b) 1.250 [g],
(c) 1.258 [g], (d) 1.275 [g].

The results of measurement are shown in **Fig.8**. The differences between the actual and measured values are within ± 20 [mg].

The results shown by Fig.8(b) indicate that measurement error tends to be larger when the measurement rotational speed differs more from the calibration speed. Since the resonant frequency of the frame-spring system ω_1 is given by

$$\begin{aligned}\omega_1 &= \sqrt{\frac{k_1}{m_1}} \\ &= 2\pi \times 28.4 \text{ [rad/s]},\end{aligned}$$

the difference between ω_1 and ω_2 is rather large in the cases of (a) and (d). In such cases, the motion of the absorber includes some low-frequency fluctuations when the velocity gain p_v is set to be small so that the precise measurement of $|X_2|$ is difficult. Except these cases, the measurement errors are within 0.5%. It indicates that the operation conditions should be selected for ideal antiresonance to occur when an undamped dynamic vibration absorber is used.

The acceleration acting on the sample during measurement is calculated as

- (a) 0.89×10^3 [m/s²], (b) 1.17×10^3 [m/s²],
(c) 1.57×10^3 [m/s²], (d) 1.95×10^3 [m/s²].

They are about 90 to 200 times the gravitational acceleration on earth. To avoid such high acceleration, the rotational speed should be low. However, the vibration of the frame is so small at rotational speeds far below ω_1 that it is difficult to check whether antiresonance occurs. Therefore, apparatus whose ω_1 is low enough should be used to perform measurements at low frequencies.

5. Conclusions

It was proposed to use a dynamic vibration absorber for mass measurement under weightless conditions. The principles and characteristics of the measurement were clarified. The effectiveness of the method using an undamped dynamic vibration absorber was confirmed experimentally with an apparatus having an active dynamic vibration absorber with an electromagnetic servomechanism.

A future work is to develop more accurate measurement systems by applying advanced control methods such as output regulation and periodically learning control to eliminate the vibration of the frame. It is also planned to

carry out the measurement of objects whose position of the center of mass is uncertain.

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Translated from Trans. of the SICE, Vol.32, No.8, 1145/1151 (1996)