## Unsteady Flow Rate Measurement of Air using Isothermal Chamber $^{\dagger}$

Kenji Kawashima\*, Toshinori Fujita\* and Toshiharu Kagawa\*

It is difficult to measure the flow rate of compressible fluids, because the volume of compressible fluids changes with temperature. Therefore, there is no easy and effective method on the unsteady flow rate measurement of compressible fluids. We have proposed a simple method to measure instantaneous flow rates of air using an isothermal chamber. In this paper, a simple method to measure the unsteady oscillating flow rate of air using the chamber is proposed. The isothermal chamber can almost realize isothermal condition due to larger heat transfer area by stuffing steel wool in it. Therefore, the flow rate of air discharged from the chamber is obtained only by measuring pressure in the chamber. The unsteady oscillating flow rate is generated by a servo valve installed on the chamber. At first, the measurement error of the method is examined. Then, the unsteady oscillating flow rate was measured by the proposed method against several frequencies. It became clear from the experiment that the proposed method can measure the unsteady oscillating flow rate up to  $40[\mathrm{Hz}]$  within 7% of errors.

Key Words: flow measurement, unsteady flow, compressible fluid, isothermal condition

#### 1. Introduction

It is difficult to measure the flow rate of compressible fluids such as air, because the density of them changes not only by pressure but also by temperature. Especially, the unsteady flow rate measurement of gases is very difficult. Effective method is yet to be established <sup>1) 2)</sup>. Calibration method of gaseous flow meters is defined in the JIS standard <sup>3)</sup>. However, the method is defined only on the steady flow. Therefore, the dynamic characteristics of flow meters are impossible to measure.

On the making process of the semi conductors, the gas flow control is needed. The dynamic characteristics of the flow meter become very important. Also there is a respirator which feedbacks the flow rate <sup>4)</sup>. A flow meter with a high response is demanded. Moreover, on the recent pneumatic servo systems, high response flow control valves are used. On designing the control system, it becomes very important to know the dynamic response of the flow control valves. In such cases, unsteady oscillatory flow measurement is needed. There exist many other cases needed to measure the dynamic characteristics of gaseous flow meters. From these circumstances, it is demanded to establish a simple method to measure the unsteady flow rate of compressible fluids.

We have already proposed a chamber which could almost realize an isothermal condition during air charge or discharge by stuffing steel wool into a normal chamber. We call this chamber an isothermal chamber. Moreover,

we have confirmed that the steady flow rate charge or discharge from the chamber could be measured within 1% of errors from measuring the pressure change in the chamber  $^{5)}$ .

In this paper, we propose a simple method to measure the unsteady oscillatory flow using the isothermal chamber. Firstly, the measurement error of the unsteady flow measurement of air using the isothermal is estimated. As the amplitude of the pressure becomes the measurement error, the selection of the tank volume is investigated. Secondly, since the temperature change in the chamber becomes the measurement error, characteristics of the isothermal chamber are calculated with simulation. After investigating the measurement errors, the effectiveness of the proposed method is confirmed experimentally. Finally, we show that the proposed method can measure the unsteady oscillatory flow rate of air up to 40Hz.

### Nomenclature

A: average opening of the servo valve  $[m^2]$ 

B: amplitude of the opening to the servo valve  $[m^2]$ 

 $C_v$  specific heat of air  $[J/(kg \cdot K)]$ 

f :input frequency to the servo valve [Hz]

G:mass flow rate [kg/s]

h:heat transfer ratio  $[W/(m^2 \cdot K)]$ 

 $k : \text{conversion factor } [\text{m}^3/\text{kg}]$ 

K: proportional constant

m:mass of the steel wool [kg]

 $R: gas constant [J/(kg \cdot K)]$ 

P: pressure in the isothermal chamber [Pa]

<sup>\*</sup> Faculty of engineering, Tokyo Institute of Technology

 $P_s$ :supply pressure [Pa]

Q: flow rate obtained from the chamber [m<sup>3</sup>/s]

 $Q_s$ : flow rate obtained from the valve displacement [m<sup>3</sup>/s]

S: heat transfer area [m<sup>2</sup>]

 $S_e$ : effective area of the servo valve [m<sup>2</sup>]

t:time [s]

T: thermal time constant [s]

v: displacement of the servo valve [V]

V: tank volume of the isothermal chamber [m<sup>3</sup>]

W:mass of air [kg]

 $\bar{\theta}$  :average temperature in the chamber [K]

 $\theta_a$ : room temperature [K]

## 2. Unsteady flow Rate Measurement using Isothermal Chamber

#### 2.1 Principle

The principle of the proposed method is as follows: the state equation of compressible fluids in a chamber can be written as

$$PV = WR\bar{\theta} \tag{1}$$

Then, following equation is derived by differentiating Eq.(1).

$$V\frac{dP}{dt} = GR\bar{\theta} + WR\frac{d\bar{\theta}}{dt} \tag{2}$$

If the state of air in the chamber while charge or discharge remains isothermal, next equation is obtained from Eq.(2).

$$G = \frac{V}{R\theta_a} \frac{dP}{dt} \tag{3}$$

It is clear from Eq.(3) that if the volume of the isothermal chamber V and the room temperature  $\theta_a$  is known, we can obtain the mass flow rate G by measuring the pressure and differentiated the pressure. Then, the mass flow rate G is converted to the volumetric flow rate Q on the standard condition by the conversion factor k.

$$Q = kG \tag{4}$$

#### 2.2 Experimental Apparatus and Procedure

The experimental apparatus of the unsteady flow rate measurement is shown in Fig.1.In this research, we used the isothermal chamber which  $0.25[\mu m]$  diameter steel wool was stuffed  $300[kg/m^3]$ .

Firstly, compressed air at 500[kPa] is charged to the isothermal chamber and shut the hand valve. Secondly, opening the solenoid valve, the compressed air is discharge to atmosphere. At that time, unsteady oscillatory flow is generated by oscillates the servo valve using a function generator. The displacement of the spool of the servo

valve could be measured as a voltage from  $0\sim10[V]$ . Measuring the pressure during discharge and fed into a personal computer through an AD converter. The flow rate discharged from the chamber was obtained from Eq.(3). A semi conductive type pressure sensor was used on the measurement. The resolution of the sensor is 0.1[kPa]. On differentiating the pressure, 5 points of data was used. A low pass filter was used to smooth the pressure curve. The cutoff frequency of the filter was 2.5 times larger value than that of the input frequency. The sampling time was changed owing to the input frequency but at least 80 points data were measured in a cycle. The diameter of 6[mm] nylon tubes were used to connect each element.

Measurement method of the unsteady flow rate of compressible fluids is yet to be established. Therefore, we compared the proposed method with the flowing method which could be the standard value. The method is to measure the flow rate from the displacement of the spool of the servo valve and the pressure difference. The relation between the spool displacement and the effective area of the servo valve was measured in advance defined in the ISO standard <sup>6</sup>). A area type flow meter which accuracy is 2% was used in the measurement. The experimental result is shown in Fig.2. The black dots in the figure are the measured points. The solid line in the figure is a fitting curve with the third order function obtained from the least square method.

Measuring the pressure P during discharge and the spool displacement of the servo valve v, then using the solid line in Fig.2 the effective  $S_{\epsilon}(v)$  was obtained. Then on the choke condition, the flow rate is obtained from the following equation.

$$Q = KS_e(v)P\sqrt{\frac{273}{\theta_a}} \tag{5}$$

It is clear from Eq.(5) that the temperature term was included. Therefore the temperature of air which passed

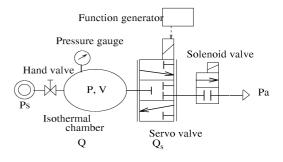


Fig. 1 Experimental Apparatus of the Unsteady Flow Rate Measurement

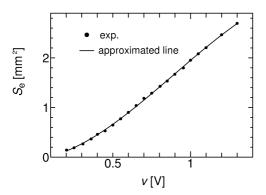


Fig. 2 Static characteristics of the servo valve

through the servo valve must be measured. However, on the isothermal chamber, the air is almost same as the room temperature. Even if there is decrease in temperature of 3[K], the measurement error remains less than 0.5%. Hence, the temperature of the air is assumed to be room temperature and obtained the flow rate. The total measurement errors of the method are considered to be less than 3%.

#### 3. Investigation of the Measurement Errors

#### 3.1 Factors of Measurement Errors

The factors of measurement errors of the proposed method is considered to be as follows:

- 1) Measurement error of the tank volume.
- 2) Measurement error of the room temperature.
- 3) Measurement error due to pressure sensor's accuracy and resolution.
- 4) Measurement error due to temperature change in the isothermal chamber.

The factor 1) and 2) are independent from the frequency. The errors due to 1) and 2) are considered to be less than 0.3% and 0.1% respectively <sup>5)</sup>. On the unsteady flow rate measurement, the factor 3) effects larger compared with the steady flow measurement. The reason is that when the same amplitude voltage was input to the servo valve, as the frequency becomes faster the amplitude of the pressure during discharge becomes smaller. Therefore, the measured flow is affected by the resolution of the pressure sensor. On the proposed method, the tank volume must become smaller as the frequency of the oscillatory flow becomes faster. Then, next the factor 3) is discussed and an index to select the tank volume is investigated.

#### 3.2 Error due to pressure measurement

The noise level of the pressure sensor used in this experiment is smaller than the resolution and the linear-

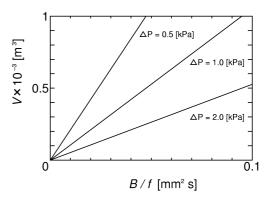


Fig. 3 Indexes to select the volume of the isothermal chamber

ity is very high. Therefore, the measurement error due to pressure measurement is mainly the effect of the resolution. The effect is discussed in the following. The pressure during discharge can be written as follows from Eq.(3),(4) and (5) assuming that the chamber is perfectly isothermal condition and the valve opening is given by  $S_e = A - B \sin(2\pi f t)$ .

$$P = P_s e^{\frac{KR\theta_a}{kV}} \sqrt{\frac{273}{\theta_a}} \left( At + \frac{B}{2\pi f} (\cos(2\pi ft) - 1) \right)$$
 (6)

Hence, the amplitude of the pressure  $\triangle P$  is given by the next equation.

$$\triangle P = P_s \left( 1 - e^{\frac{KR\theta_a}{kV}} \sqrt{\frac{273}{\theta_a}} \frac{B}{2\pi f} \right) \tag{7}$$

From the above equation, it is clear that the pressure amplitude is given by the amplitude of the valve opening B, frequency of the unsteady oscillatory flow f and the volume of the isothermal chamber V.

In this experiment, we used the pressure sensor which resolution is 0.1[kPa] so that to keep the error lower than 0.5%,1% and 2%, the pressure amplitude of at least 2,1 and 0.5[kPa] is needed respectively. Consequently, indexes of selecting the tank volume of the chamber are obtained from Eq.(7). The results are shown in Fig.3. The higher the frequency f becomes and smaller the input amplitude B becomes, the smaller volume is needed.

#### 3.3 Error due to temperature change

To investigate the measurement error due to the temperature change, characteristics of the isothermal chamber was calculated numerically. On the numerical analysis, we assumed that the heat conductivity rules the heat transfer characteristics. Then we calculated with the numerical model proposed in the previous paper <sup>5)</sup>.

Fig.4 shows the comparison of the experimental and the calculated results. Experiment was done using the apparatus shown in Fig.1. The volume of the chamber used in the experiment was  $0.2 \times 10^{-3} [\mathrm{m}^3]$ . The input voltage

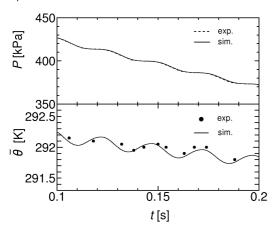


Fig. 4 Comparison of pressure and temperature curves between simulation and experiment

to the servo valve was  $0.75 - 0.5sin(2\pi ft)$  and using the stop method 5) the average temperature in the chamber was measured. The room temperature was 293[K] at the experiment. Fig.4 shows the results at the frequency of 40Hz. The lateral axis shows the time from the beginning of discharge. The upper figure indicates the pressure curves and the lower figure shows the average temperature in the chamber. In the lower figure, the black dots show the experimental results and the solid line shows the calculated result. It is clear from Fig.4 that as the pressure decrease becomes larger the temperature drop becomes larger. On the other hand, when the pressure decrease is small the temperature rises. The temperature decrease is less than 1[K] on the whole, which suggests that the chamber remains almost isothermal condition. The experimental results and the calculations show good agreements that indicate the effectiveness of the simulation.

We confirmed that the simulation is effective so that the error due to the temperature change is estimated using the simulation's results. The flow rate obtained from Eq.(2)  $Q_0$  which considered the temperature change using the results shown in Fig.4 and the flow rate Q which obtained using Eq.(3) are compared. The ratio of Q and  $Q_0$  is given by the following equation.

$$\frac{Q}{Q_0} = \frac{1}{\frac{\theta_a}{\theta} \left(1 - \frac{P}{\theta} \frac{d\bar{\theta}}{dt} / \frac{dP}{dt}\right)} \tag{8}$$

Fig.5 shows the compared results between  $Q_0$  and Q. The upper figure shows the flow rates and the lower figure shows the error due to the temperature change against the average flow rate  $Q_m$ . As the flow rate  $Q_0$  obtained from Eq.(2) and  $Q_s$  shows good agreement, in the figure  $Q_s$  is shown.

It is clear from Fig.5 that as the flow rate becomes larger

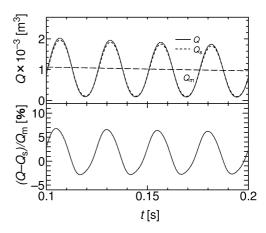


Fig. 5 Estimation of error according to temperature change

the error becomes larger. It is obvious from Eq.(8) that to neglect the temperature drop is to estimate the flow rate larger than the real value. It is also seen at the bottom of the oscillatory flow, since the temperature rises, the opposite phenomena could be seen. Even the maximum error against the average flow rate is 7%, the error against the maximum flow rate is less than 5%. From the above investigation, it is known that the total error is governed by the error due to the temperature change.

It is clear from Eq.(8) that the error due to the temperature change is determined by the ratio of the pressure change and the temperature change. Therefore, it became clear that the error is almost the same even the tank volume is changed. Although, as the tank volume becomes smaller decrease in the flow rate becomes faster, the larger tank volume is recommended. The size of the tank volume is determined by Fig.3. It became clear from the simulations that the maximum error due to the temperature change is determined by (A + B)/A and f.

Fig.6 shows the maximum error due to the temperature change determined by (A+B)/A and f. The error is obtained using the calculated results. Since the error of the

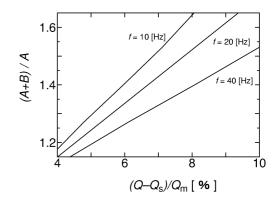


Fig. 6 Relation between frequencies and errors according to temperature change

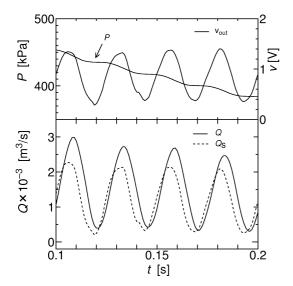


Fig. 7 Unsteady flow rate measurement with the normal chamber

proposed method is governed by the temperature change, the total error of the proposed method is considered to be evaluated by the temperature change. The maximum error becomes larger as the amplitude of the valve opening becomes larger which means the larger flow rate is measured. Also the error becomes larger as the frequency becomes higher. Even though the method is effective on practical use as the error is less than 5%

From the above investigation, when the input frequency was less than 10[Hz] the input to the servo valve was given as  $0.75-0.25sin(2\pi ft)$  and a  $1\times10^{-3}[\text{m}^3]$  isothermal chamber is used. When the input frequency was higher than 10[Hz] the input to the servo valve is given as  $0.75-0.5sin(2\pi ft)$  and a  $0.2\times10^{-3}[\text{m}^3]$  isothermal chamber was used.

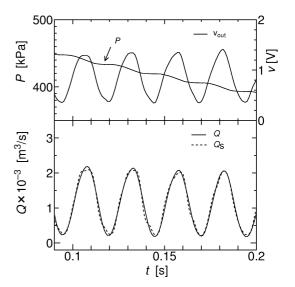
# 4. Experimental Results and Discussions of Unsteady Flow Rate Measurement

## 4.1 In the case of normal chamber

To confirm the effectiveness of the isothermal chamber when the unsteady flow rate is measured from the pressure change using the normal chamber is investigated. If the discharge from a normal chamber is assumed to be polytropic, the phase would not show any delay even in a normal chamber. However, it is known that the discharge of air from the chamber is not always a polytropic process <sup>7) 8)</sup>. The energy equation is given as follows in a normal chamber.

$$C_v W \frac{d\bar{\theta}}{dt} = GR\bar{\theta} + hS(\theta_a - \bar{\theta})$$
(9)

Since the heat is transferred with heat convection and con-



**Fig. 8** Unsteady flow rate measurement with the isothermal chamber(f=40[Hz])

duction in the normal chamber, energy equation is given using the heat transfer ratio h. Here, as the heat transfer ratio depends on the shape of the chamber and discharge speed, the value is not constant. The transfer function from the pressure change to the flow rate is given as follows from Eq.(2) and Eq.(9).

$$G = \frac{V}{R\theta} \frac{Ts+1}{\kappa Ts+1} \frac{dP}{dt} \tag{10}$$

Here  $\kappa=1.4$  and  $T=C_vW/(h\cdot S)$ . T is called the thermal time constant <sup>9)</sup>. It is clear from Eq.(10) that the transfer function is a phase delay system. Therefore, if the unsteady flow rate is measured using Eq.(3) in a normal chamber, it is considered not only the amplitude but also the phase would become different from the real value.

The unsteady oscillatory flow rate is measured using Eq.(3) with a normal chamber which volume is  $0.2 \times 10^{-3}$  [m<sup>3</sup>]. The result at the frequency of 40[Hz] is shown in Fig.7. The lateral axis shows the time from the beginning of discharge. The upper figure shows the pressure curve and the displacement of the spool of the valve and the lower figure shows the flow rates. The dotted line shows the flow rate obtained using Eq.(5). At this time the temperature is assumed to be the room temperature. In practical, the temperature decreases about 25[K] owing to the sudden expansion of air during discharge. Therefore the dotted line includes the error due to the temperature change. Even though, there is no delay in phase with the real value because  $Q_s$  is obtained from the spool movement.

When we focus on the lower figure of Fig.7, the solid line and the dotted line shows the phase difference. That is the solid line is the phase delay system as Eq.(10). Also

the flow gain becomes larger. This is because the flow rate given as solid line neglected the temperature change. It is clear from section 3.3 that to neglect the temperature decrease is to estimate the flow rate larger than the real value. It became known that when the flow rate is measured from the pressure change with a normal chamber, not only the gain but also phase shows big error.

#### 4.2 In the case of isothermal chamber

The measured result of the unsteady oscillatory flow using Eq.(3) with the isothermal chamber is shown in Fig.8. The figure is at the frequency of 40[Hz].

When we focus on the lower figure of Fig.8, there is a little difference in flow rate at the top and the bottom. The difference between Q and  $Q_s$  is about 7% at the maximum. The difference is almost the same as the measurement error due to the temperature change as shown in Fig.5. Therefore the difference is considered to be of the temperature change. Although, when we compare the result with the result of the normal chamber shown in Fig.7, there is no phase delay in Fig.8. The effectiveness of the isothermal chamber on the unsteady oscillatory flow measurement is evident.

From the result shown in Fig8, it became clear that the unsteady oscillatory flow rate up to 40[Hz] could be measured using the isothermal chamber.

## 5. Conclusions

The following have been accomplished as a result of this study:

- (1) Measurement method of the unsteady oscillatory flow rate of air using the isothermal chamber is proposed and the measurement errors are investigated.
- (2) The problem of the unsteady flow rate measurement from the pressure change with a normal chamber is pointed out and by comparing the result using the isothermal chamber, the effectiveness of the isothermal chamber became clear.
- (3)The proposed method has been shown to be effective on the unsteady oscillating flow rate up to  $40[\mathrm{Hz}]$  by experiments.

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## Kenji Kawasніма (Member)



He received doctor degree of engineering from Tokyo Institute of Technology in 1997. He is now an associate professor in Tokyo Institute of Technology. His research interests are flow measurement, fluid control, process control and robotics. He is a member of SICE, JSME and JHPS.

#### Toshinori Fujita (Member)



He received master degree of engineering from Kanazawa University in 1988. He is now a researcher in Tokyo Institute of Technology. His research interests are Pneumatic equipments and Pneumatic servo control systems. He is a member of SICE, JSME and JHPS. He is a doctor of engineering.

## Toshiharu Kagawa (Member)



He received master bachelor degree of engineering from Tokyo Institute of Technology in 1974. He is now a professor in Tokyo Institute of Technology. His research interests are fluid control systems, flow measurement and process control. He is a member of SICE, JSME and JHPS. He is a doctor of engineering.

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