Trans. of the Society of Instrument and Control Engineers
Vol.E-1, No.1, 1/7 (2001)

Flow Rate Measurement of Compressible Fluid using Pressure Change in the Chamber†

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It is not so easy to measure a flow rate of compressible fluids directly, because not only the pressure but also the temperature must be measured. In this paper, a chamber called an "Isothermal Chamber" is proposed. Then, a simple method to measure compressible fluids using this chamber is proposed. The isothermal chamber is a chamber that can almost realize isothermal conditions due to the larger heat transfer area and heat transfer coefficient by stuffing the steel wool into it. As the state during charge or discharge is almost isothermal, an instantaneous flow rate could be obtained using only a pressure in the chamber. At first, the characteristic of the isothermal chamber was examined by experiments and simulations. Then, the steady flow rate of air was measured by the proposed method. To confirm the effectiveness of the method, the measured results were compared with the results obtained from the sonic venturi nozzle. It became clear that the proposed method can measure the steady flow rate of air within 1 percent of errors.

Key Words: flow measurement, instantaneous flow, compressible fluid, isothermal change

1. Introduction

In industry, the flow rate is one of the most important quantities whose measurement requires high accuracy and high dynamic response. The principle of flow rate measurements can be divided into direct and indirect measurements. A lot of methods have been proposed [1) 2)], but most of them were indirect measurement. Therefore, the measured value obtained by those methods must have been calibrated by comparing with the direct measurements. As a result, the accuracy of measurements depended on the direct measurements.

There are two methods in the direct measurement of gases. One is weigh procedures(ISO 5024,1981) and the other is volumetric procedures(ISO 8959/2, 1986). Both methods require large equipments. Therefore, the measurement are not so easy.

In this paper, a chamber called an "Isothermal Chamber" is developed. Then, we propose a simple method to measure instantaneous flow rates of compressible fluids using this chamber. The isothermal chamber can almost realize isothermal condition due to larger heat transfer area and heat transfer coefficient by stuffing steel wool in it. As the process while charge or discharge remains almost isothermal, instantaneous flow rate could be obtained measuring only pressure in the chamber. The idea to realize isothermal condition by increasing the heat transfer area have proposed by Otis [4)], and it was applied to reduce energy consumption in accumulators. However, the idea has not been applied to the flow rate measurement. The method to measure the flow rate from the pressure can be seen on the test of the compressors [2)]. Even though, the pressure response depends on temperature change and could not measure the instantaneous flow. The proposed method can measure the instantaneous flow rate simply only by measuring the pressure in the chamber.

At first, we explain the principle of the proposed method. Secondly, the characteristic of the isothermal chamber is examined by experiments and simulations. Then, the estimation of the errors is done. Finally, we confirm the effectiveness and the simplicity of the proposed method from the experiments comparing with the measured value obtained by a sonic nozzle [5)].

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nomenclature

a: thermal conductivity [m^2/s]
A_r: aspect ratio
C: specific heat of the steel wool [J/(kg· K)]
C_v: specific heat of air [J/(kg· K)]
D: diameter of a nozzle [m]
G : mass flow rate [kg/s]
k : conversion factor [m^3/kg]
l: distance between material[m]
l_s: diameter of the material [m]
m: mass of the stuffed element [kg]
R : gas constant [J/(kg· K)]
P : pressure in the chamber [Pa]

P_s : supply pressure [Pa]
S_e: effective are of the nozzle [m^2]
S_r: heat transfer area [m^2]
Q : flow rate obtained from the proposed method[m^3/s]
Q_0 : flow rate considering the temperature change in the chamber[m^3/s]
Q_r : flow rate obtained from the sonic nozzle [m^3/s]
t: time [s]
V_0 : tank volume [m^3]
V : volume of the isothermal chamber [m^3]
W : mass in the chamber [kg]
$\bar{\theta}$: average temperature in the chamber[K]
θ_a : room temperature [K]
θ_s : temperature of the material [K]
λ_a:heat conductivity of air [W/(m · K)]
ρ_s:density of the material [Kg/m^3]

2. Flow Rate Measurement using Isothermal Chamber

2.1 Principle

The principle of the proposed method is as follows: the state equation of compressible fluids in a chamber can be written as

$$PV = WR\bar{\theta} \quad (1)$$

Then, following equation is derived by differentiating Eq.(1).

$$V\frac{dP}{dt} = GR\bar{\theta} + WR\frac{d\bar{\theta}}{dt} \quad (2)$$

If the state of air in the chamber while charge or discharge remains isothermal, next equation is obtained from Eq.(2).

$$G = \frac{V}{R\theta_a}\frac{dP}{dt} \quad (3)$$

It is clear from Eq.(3) that if the volume of the isothermal chamber V and the room temperature θ_a is known, we can obtain the mass flow rate G by measuring the pressure and differentiated the pressure. Then, the mass flow rate G is converted to the volumetric flow rate Q on the standard condition by the conversion factor k.

$$Q = kG \quad (4)$$

3. Characteristics of Isothermal Chamber

3.1 Experimental Apparatus and Procedure

It is very important to realize the isothermal condition on the proposed method. Therefore, we investigated characteristics of the isothermal chamber at first. We measured pressure and temperature responses of the chamber while discharge and charge. Dehumidified air was used in the experiments.

The average temperature while discharge or charge was measured as follows [6]: stop the solenoid valve at the time we want to know the temperature $\bar{\theta}(t)$. Measure the pressure at that time $P(t)$ and the pressure when it becomes stable P_∞ using a pressure gauge. When the pressure becomes stable, the temperature in the chamber recovers to the room temperature. Hence, we can measure the average temperature at the time t using the Law of Charles.

$$\bar{\theta}(t) = \frac{P(t)}{P_\infty}\theta_a \quad (5)$$

By changing the time to stop discharge or charge, the average temperature at any time could be measured. This method is called a "stop method." The errors of this method is considered to be less than 0.3[K].

The experimental apparatus of the stop method is shown in Fig.1 and Fig.2. Fig.1 shows the case when air is charged into the chamber and Fig.2 shows the case when air is discharge from the chamber. The charge or discharge was done through cylindrical restrictions. To investigate the relation between the speed of the pressure change and the temperature change, two cylindrical restrictions were used whose diameters are 1.0[mm] and 1.5[mm]. Moreover, to investigate the effect of the mass of the stuffed material and the shape of isothermal chambers, experiments were done using chambers shown in Tab.1. Tank0 is a normal chamber and other chambers are isothermal chambers. Here, the material stuffed in the chamber might become flow resistance which causes pressure distribution in the chamber. However, the volume of material is less than 4% compared with that of the chamber. Measuring the pressure at both sides of the chamber, we confirmed that there is no pressure distribution in the chambers.

3.2 Results and Discussion

3.2.1 Comparison between normal and Isothermal chamber

Fig.3 shows experimental responses of pressure and temperature while air was discharged from tank0 and tank1. The initial pressure was set at 592[kPa] and

Table 1 Specification of the isothermal chamber

	$V_0 \times 10^{-3}$[m^3]	m[kg]	A_r	material	l_s[μm]
tank0	1.02	-	0.7	-	-
tank1	1.02	0.31	0.7	steel	25
tank2	1.02	0.37	0.7	copper	100
tank3	3.02	0.45	0.8	steel	50
tank4	3.02	0.45	2.0	steel	50

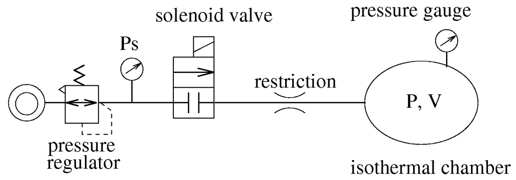


Fig. 1 Experimental apparatus for the temperature measurement (the case of charge)

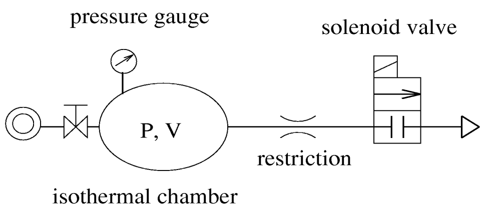


Fig. 2 Experimental apparatus for the temperature measurement (the case of discharge)

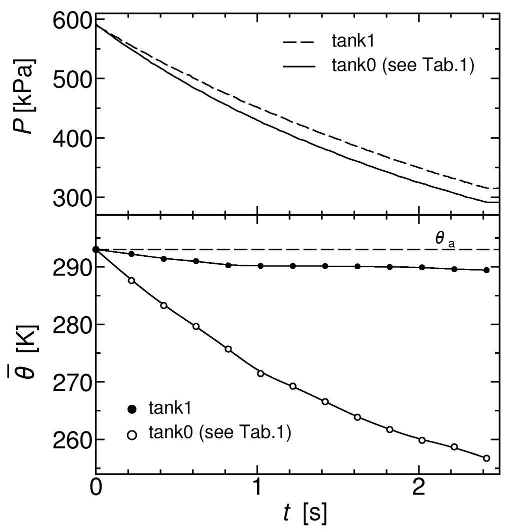


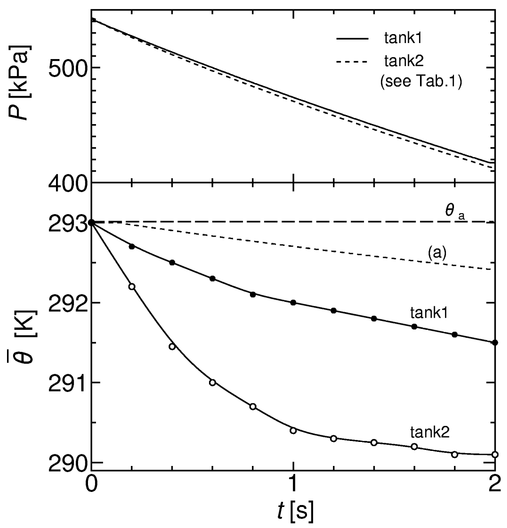
Fig. 3 Temperature changes during discharge

the cylindrical restriction whose diameter is 1.5[mm] was used.

The upper figure shows pressure curves and the lower figure shows temperature curves. It is clear that by stuffing the steel wool, isothermal condition is almostly realized. The temperature drop is 35[K] in the normal chamber, but it is only 3[K] in the isothermal chamber.

3. 2. 2 Effect of diameters of materials

We investigated the effect of diameters of materials stuffed in the chamber. The average temperature was measured while air was discharged from tank1 and tank2. The initial pressure was set at 542[kPa] and the cylindrical restriction whose diameter is 1.5[mm] was used. The heat capacity of the material stuffed in tank1 and tank2 is almost same.



(a)temperature response assuming that the whole heat generated by the expansion of air was transfered to the material

Fig. 4 Temperature changes during discharge

Experimental results are shown in Fig. 4. The dotted line in the lower figure of Fig. 4 shows temperature response assuming that the whole heat generated by the expansion of air was transfered to the material in the chamber. Even the pressure response and the heat capacity is almost same in tank1 and tank2, the temperature drop is smaller in tank1. The temperature drop of both chambers is larger than the dotted line, which means the heat capacity of the material has not been made full used of. Therefore, to realize the isothermal condition, it is most important to make heat transfer area larger.

Stuffed materials are different between tank1 and tank2. Though, the difference of the temperature response is not considered to be the difference in materials. Because, the thermal conductivity of both materials are faster enough compared with that of air.

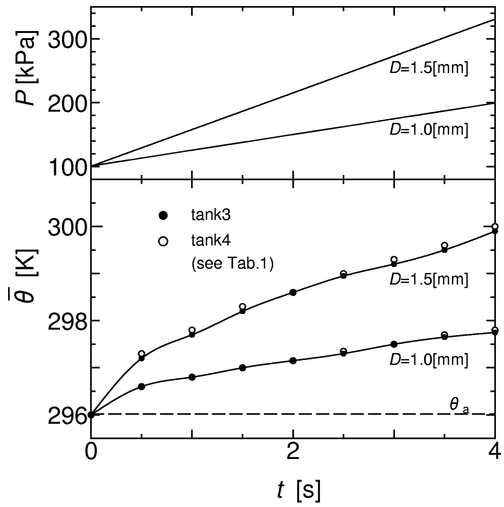


Fig. 5 Relation between the shape of the chamber and the temperature change

3.2.3 Effect of aspect ratio of chambers

The relation between the shape of chambers and temperature responses were examined while charge using tank3 and tank4. The initial pressure was set at 542[kPa] and the cylindrical restriction whose diameter is 1.0[mm] and 1.5[mm] was used. Results were shown in Fig.5. It became clear from Fig.5 that the temperature change become larger as the pressure change become faster, but the shape of the chamber makes no effect on the condition. We confirmed that the same phenomena could also be seen while discharge.

From the experimental results, it became clear that the isothermal condition could almost be realized by stuffing steel wool in a normal chamber. As the condition in isothermal chambers are governed by heat transfer areas and the shape of chambers make no effect on the condition, we consider that the heat conductivity rules the phenomena. Therefore, if the diameter of the steel wool is given, the characteristic of the isothermal chamber could be evaluated by the mass of the steel wool per volume of the chamber.

4. Model of Isothermal Chamber

4.1 Proposal of a model

Considering the experimental results, we simulated the temperature change in the isothermal chamber with the simplified model as follows: The model of the isothermal chamber is considered to be shown in Fig.6.

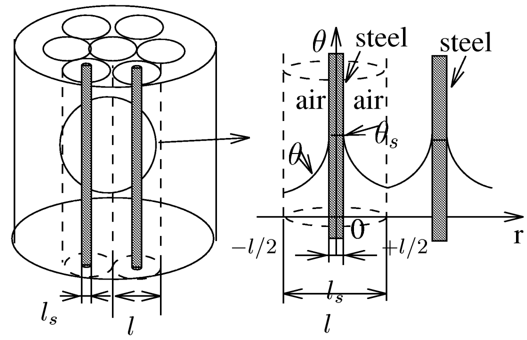


Fig. 6 Simplified model of the isothermal chamber

The following assumption is given from the experimental results.

1) Heat tranfer is governed by heat conductivity. 2) As shown in Fig.6, the stuffed material is put at r=0 and the cylindrical area is given around the material(diameter$l$) which mass par volume is constant. 3) Heat conductivity is considered only the vertical axis to the side wall of the material. 4)Temperature distribution in the material is negligible.

4.2 Fundamental equations

The equations [7] used in the simulation is as follows:

Unsteady heat conductivity equation of air

$$\frac{\partial \theta}{\partial t} = a\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right) + q_g. \tag{6}$$

here q_g which shows the effect of compression and expansion of air is given by

$$\int_{l_s/2}^{l/2} q_g dr = \frac{d\bar{\theta}}{dt}. \tag{7}$$

Energy equation (charge)

$$C_v W \frac{d\bar{\theta}}{dt} = C_v G(\theta_a - \bar{\theta}) + GR\bar{\theta} + S_r \lambda_a \left(\frac{\partial \theta}{\partial r}\right). \tag{8}$$

Temperature change of the stuffed material

$$-\lambda_a \left(\frac{\partial \theta}{\partial r}\right) S_r = Cm\left(\frac{d\theta_s}{dt}\right). \tag{9}$$

Flow equation (at choke condition)

$$G = S_e P_s \sqrt{\frac{\kappa}{R\bar{\theta}}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}. \tag{10}$$

In the isothermal chamber steel wool is touched with each other. Therefore, the real heat transfer area S_r is given by the following equation using the compensation factor c.

$$S_r = cS. \tag{11}$$

Here S is the side area of the cylindrical material. The heat conductivity λ_a is given from the Sutherland's equation with the Prandle number 0.72.

4.3 Simulation method

We used numerical differentiation on the simulation. Against Eq.(6), we used the implicit method to make the stability higher and other equations are solved by the explicit method. On Eq.(6),$(l-l_s)/2$ is divided to 20. Mass of the unit volume m/V is 310[kg/m^3] and the average diameter of the stuffed material l_s is 25[μm] on tank1. The heat transfer distance r became 0.13[mm] from the next equation.

$$\frac{m}{V} = \frac{\rho_s l_s^2}{l^2}. \tag{12}$$

Effective heat transfer area S_r became 0.12[m^2] par 1[m^3] when c=0.15. The time interval dt was set at 0.1[ms].

The procedure of the simulation is as follows: At first, using the Eq.(10), we obtain flow G from the pressure and the effective area of the nozzle. At this time, we assume the condition is adiabatic. That means the last term of the Eq.(8) is neglected and obtain the average temperature in the chamber. Then we substitute it to Eq.(6) and Eq. (7) and get the temperature distribution of the air in the chamber. Hence, the thermal boundary layer is given. By substituting the temperature slope on the boundary layer to the last term of the energy equation, the average temperature at time dt later is given. From Eq.(9), the temperature of the material is obtained and from Eq.(??), the pressure in the chamber is given. After time dt, obtain the flow G and calculate average temperature from Eq.(8). Temperature distribution of the chamber is given by Eq. (6) and Eq.(7). Repeat the procedure shown above, the average temperature during charge and discharge is calculated.

4.4 Appropriate of the model

Fig.7 shows an example of the compared results of the experiment and the simulation. Tank1 in Tab.1 is used for the isothermal chamber. Two types of the nozzle that diameter is 0.7[mm] and 1.0[mm] is used. Supply pressure is set at 542[kPa]. The upper figure shows the pressure curves, the middle figure shows the average temperature in the chamber. On the upper figure, dotted line shows the experimental results and the solid line shows the calculated result. On the middle figure, the black dots shows the experimental results and the solid line shows the calculated results. Even in the different charging speed, the experimental results and the calculated results show good agreement. The appropriate of the model was confirmed. It is clear even from the simulation that the heat conductivity ruled the heat transfer. The lower figure on Fig.7 will be discussed in chapter 5.3.

It is clear from the results that the characteristics of the

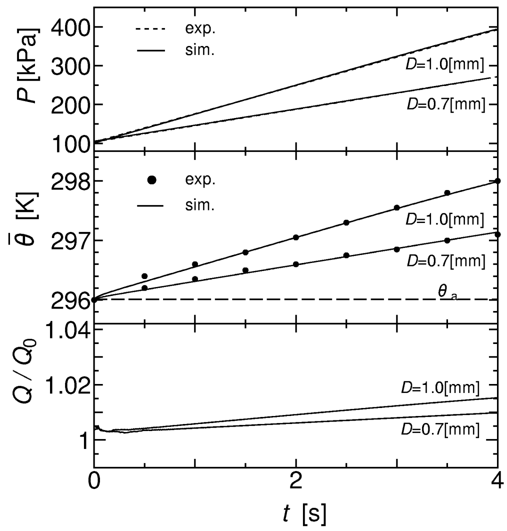


Fig. 7 Comparison between experiments and simulations

hear transfer in the isothermal chamber is ruled by heat conductivity. It is also clear from the simulation that the temperature change of the material is small which suggests the heat capacity is enough. Therefore, it became clear that to realize the isothermal condition to make the heat transfer area larger is most effective. To make the heat transfer area larger, it is important to stuff the material as much as possible and stuff the material which average diameter is very small. Therefore, on the flow rate measurement the tank1 in Tab.1 is used.

5. Steady Flow Rate Measurement

5.1 Apparatus and method

To confirm the effectiveness of the proposed method, steady flow is measured. The experimental apparatus is shown in Fig.1. The flow rate charged to the chamber through the nozzle is measured. On the choke condition, the flow rate through the nozzle depends only on the supply pressure. Therefore, the flow rate becomes steady. If we measure the flow rate on the choke condition, the steady flow can be measured.

The supply pressure was set by an accurate pressure regulator. A sub tank is installed to eliminate the pressure drop of the supply pressure. A sonic nozzle [5] which diameter is 0.7mm was used. Nylon tubes are used to connect the elements.

The procedure is as follows: at first, the pressure in the isothermal chamber was set at atmospheric pressure. Then, opening the solenoid valve, we started the pressure measurement. The measured pressure was taken into a personal computer through an AD converter. The sampling time of the measurement was 20[ms]. The measured pressure was smoothed by moving average at ten points, and the data was differentiated numerically with five points. Then, the flow rate Q could be obtained from Eq.(2).

On the other hand, the flow rate could be measured from the discharge coefficient of the restriction. If the discharge coefficient C_d is given, the flow rate on the choking condition is given by

$$Q_r = 0.685 C_d k \frac{P_s A}{\sqrt{R\theta_a}} \tag{13}$$

The restriction used in this experiment was a critical sonic venturi nozzle which discharge coefficient is given within 0.15% of errors by National Research Laboratory of Metrology of Japan. Thus, we confirmed the effectiveness of the proposed method by comparing Q with Q_r. By changing the supply pressure, we measured several flow rates.

5.2 Measurement errors

Source of measurement errors of the proposed method could be as follows: 1)Measurement error of the tank volume of the isothermal chamber. 2)Measurement error of the room temperature. 3)Measurement error due to resolution and accuracy of the pressure sensor. 4)Measurement error due to the temperature change in the isothermal chamber.

Since the volume of the chamber could be a source of error, it was measured carefully by charging a known volume air and measuring the rise of the pressure. The measurement error of this method is considered to be 0.3The measurement error of the room temperature is considered to be less than 0.1%.

On the pressure measurement, a semi conductive type pressure sensor which resolution is 0.05[kPa] is used. We calibrated the sensor with the accurate pressure gauge which accuracy is 0.1%. When the pressure change is very small, the measured value will become smaller than the resolution which means the measurement error owing to 3) becomes bigger. In this experiment, we consider at least pressure change of 18[kPa/s] is needed to make the error less than 0.3%. From the experiment, we found at least 3[s] of the pressure time constant $VP_s/(GR\theta_a)$ [8] is needed. Therefore, the maximum flow rate we can measure with a known volume chamber is given. From the above discussion, the total error due to 1)∼3) is assumed to be less than 0.4%.Next, we investigate the error due to 4).

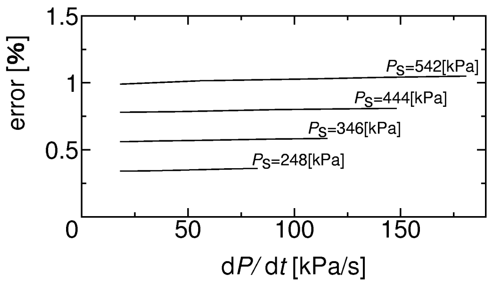


Fig. 8 Estimation of error according to the temperature change

5.3 Error due to temperature change

The ratio of flow rates obtained by the proposed method Q and the flow rate Q_0 which considered the temperature change are given as follows from Eq.(2),(3) and (4).

$$\frac{Q}{Q_0} = \frac{1}{\frac{\theta_a}{\theta}\left(1 - \frac{P}{\theta}\frac{d\theta}{dt} / \frac{dP}{dt}\right)} \tag{14}$$

It is clear from this equation that the temperature change becomes the error which makes the measured flow rate larger than the real value.

When we measure the flow rate charge into the isothermal chamber, the measurement error owing to the temperature change becomes as lower figure of Fig.7.

It is clear from Fig.7 that the temperature change becomes the error which makes the measured flow rate larger than the real value.
It is also clear that at the same time the error becomes larger as the pressure change becomes larger. The reason of the tendency is clear from Eq.(14).

When we use the isothermal chamber tank1 shown in Tab.1, the measurement error against pressure change becomes as shown in Fig.8. The measurement error was obtained using the calculated results. As the measurement range changes due to the supply pressure, in the figure four cases are shown. It is clear from Fig.8 that the measurement range becomes larger as the supply pressure becomes higher, the measurement error becomes larger. Even though, the measurement error is almost less than 1% which means the proposed method is useful.

5.4 Results and discussion

Fig.9 shows the pressure curves and the flow rates of the experimental results. The experiment was done against four flow rates. The case1 to 4 show the results when the supply pressures were set at 542,444,346 and 248[kPa] respectively. The upper figure shows the pressure curves and the lower figure shows the flow rates. As the pressure curves increase straightly, it became clear that the flow rates kept steady.

It became clear from the lower figure that the proposed method is useful and effective, because both flow rates Q and Q_r show good agreement. Though, little different could be seen in case1 and case2. This is consider to be due to the temperature change in the chamber. Since the temperature change becomes larger as the pressure change become faster, case1 and case2 showed little different. Even though the different is less than 1%. These results indicate that using the isothermal chamber which 0.25[μm] diameter steel wool was stuffed 300[kg/m^3], the flow rate could be measured within 1% of errors with the proposed method when the pressure change is less than 60[kPa/s].

6. Conclusions

(1)The isothermal chamber which could almost realize isothermal condition by stuffing steel wool was developed. It became clear from the experiment that the characteristic of the chamber is given by the mass of the steel wool per volume of the chamber.

(2)The characteristics of the heat transfer mechanism of the isothermal chamber was investigated both numerically and experimentally. It became clear that the heat conductivity rules the heat transfer in the chamber.

(3) The method to measure the gas flow rate from the pressure change in the isothermal chamber was proposed. It became clear that the steady flow rate could be measured within 1% of error.

Acknowledgements

We are grateful to Dr.Nakao.S. of National Research Laboratory of Metrology of Japan who provided the critical sonic venturi nozzle.

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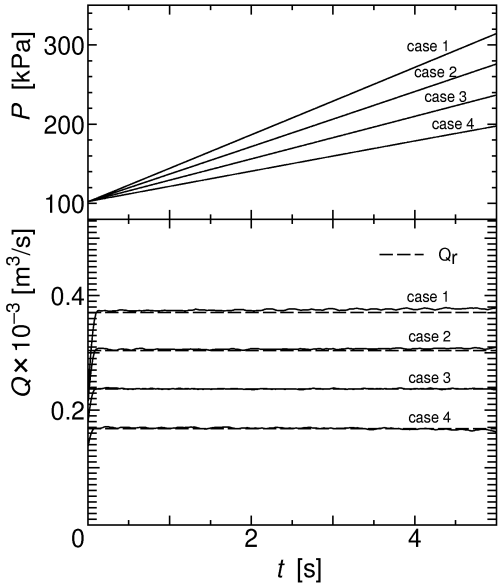

Fig. 9 Experimental results of the steady flow rate measurement

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Reprinted from Trans. of the SICE
Vol. 32 No. 11 1485/1492 1996