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A Model of Evaluating the Effectiveness of Carbon Tax for Regulating Total Emission of Carbon Dioxide

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This paper deals with a method of evaluating the effectiveness of carbon tax for regulating total emission of carbon dioxide (CO_2) . For this purpose, we analyze the input-output model of Leontief as a primal linear programming model and its dual problem to deal with carbon tax for CO_2 emission. A numerical example is included by using inter-industry table obtained in 1985 and CO_2 emission table obtained in 1990.

Key Words: primal and dual problem of linear progarmming model, global warming, input-output model, total emission control, carbon tax

1. Introduction

The main source of carbon dioxide emission is the use of fossil fuel which amounts to 88% of total energy consumption. For solving global warming problem we need long range research and development in the area from energy acquisition to energy consumption. But, since as discussed in IPCC and many other meetings the time scale of reducing carbon dioxide emission is to be short like 10 to 20 years, we need to find feasible effective policies in the existing engineering-economic systems[1].

In this paper we propose a model to evaluate the effectiveness of carbon tax to reduce the total amount of carbon dioxide emission. Carbon tax is set up as surcharge to be paid for unit carbon dioxide emission, and thus the carbon tax plays a fundamental role as a market mechanism to reduce carbon dioxide emission. Hence, the carbon tax is expected to be an effective means to reduce the total amount of carbon dioxide emission[2)3)].

In this paper we show under what condition is carbon tax effective to reduce the carbon dioxide emission in the existing engineering-economic systems. In order to answer this question we formulate and analyze a primal problem of a linear programming model, which is an input-output model of Leontief type, and its dual problem.

2. Carbon Tax Model

Input-output analysis can be used to analyze relationship between the economic system and the environment[4]. If we formulate this input-output model as a primal problem of a linear programming model, we could formulate the relationship among total output (amount of production), environmental standard for total amount of carbon dioxide emissions, price of commodities and carbon tax as a dual problem. The primal problem based on the input-output model and its dual problem can be described as follows:

Primal Problem :

$$\underset{X_1, X_2}{\text{minimize}} \quad V_1X_1 + V_2X_2 \tag{1}$$

subject to

$$\left[\begin{array}{c|c} I - A_{11} + M & -A_{12} \\ \hline -A_{21} & I - A_{22} \end{array}\right] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \geq \begin{bmatrix} Y_1 \\ -Y_2 \end{bmatrix} \tag{2}$$

$$(X_1, X_2) \geq 0$$

Dual Problem :

$$\underset{P, R}{\text{maximize}} \quad Y_1P - Y_2R \tag{3}$$

subject to

$$[P \quad R] \left[\begin{array}{c|c} I - A_{11} + M & -A_{12} \\ \hline -A_{21} & I - A_{22} \end{array}\right] \leq [V_1 \quad V_2] \tag{4}$$

$$(P, R) \geq 0$$

where

$$V_1 = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}$$

$$V_2 = [v_{n+1}]$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$X_2 = \begin{bmatrix} x_{n+1} \end{bmatrix}$$

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$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} a_{1,n+1} \\ a_{2,n+1} \\ \vdots \\ a_{n,n+1} \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} a_{n+1,1} & a_{n+1,2} & \cdots & a_{n+1,n} \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} a_{n+1,n+1} \end{bmatrix}$$

$$M = \begin{bmatrix} m_1 & & & 0 \\ & m_2 & & \\ & & \ddots & \\ 0 & & & m_n \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} y_{n+1} \end{bmatrix}$$

$$P = \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix}$$

$$R = \begin{bmatrix} r \end{bmatrix}$$

and

v_i : value added per unit level of industrial production of commodity in sector i

v_{n+1} : value added per unit level of eliminating carbon dioxide which is assumed to be 0

x_i : total output in sector i

x_{n+1} : amount of carbon dioxide eliminated (fixed)

a_{ij} : input-output coefficient per unit level of industrial production from secter j to sector i

$a_{i,n+1}$: economic input of commodity per unit level of eliminating carbon dioxide in sector i

$a_{n+1,j}$: generation of carbon dioxide from unit level of industrial production

$a_{n+1,n+1}$: generation of carbon dioxide from unit level of eliminating carbon dioxide

m_i : import coefficient in sector i

y_i : final demand in sector i

y_{n+1} : environmental standard for total amount of carbon dioxide emissions

p_i : price of commodity with respect to a unit of value-added in sector i

r : carbon tax imposed on the carbon dioxide emission

It is assumed that $1 \leq i, j \leq n$.

Interpretation of Primal Problem :

The primal problem described by eqns. (1) and (2) shows to get equilibrium among the environmental standard for total amount of carbon dioxide emissions and the amount of commodities. In eqn.(2)

$$(I - A_{11} + M)X_1 - A_{12}X_2 \geq Y_1$$

is interpreted that sum of total output and import minus input for production and for eliminating carbon dioxide must be greater than or equal to final demand. In eqn. (2)

$$A_{21}X_1 + (A_{12} - I)X_2 \leq Y_2$$

is interpreted that total amount of carbon dioxide emission minus eliminated (fixed) carbon dioxide must be less than or equal to environmental standard. The primal problem described by eqns. (1) and (2) is to find an optimal solution to minimize total cost subject to these two constraints.

Interpretation of Dual Problem :

The dual problem described by eqns. (3) and (4) shows to get equilibrium between carbon tax and the price of commodities. In eqn. (4)

$$P(I - A_{11} + M) - RA_{21} \leq V$$

is interpreted that the price of commodities per unit amount of products must be less than or equal to the cost of production plus carbon tax to be paid imposed on the carbon dioxide emission for unit level of production[5].

$$-PA_{12} + R(I - A_{22}) \leq 0$$

is interpreted that carbon tax imposed on the carbon dioxide emission for unit level of antipollution activity must be less than or equal to the price of commodities used for unit level of antipollution activity.

Complementary slackness condition

$$\{-PA_{12} + R(I - A_{22})\}X_2 = 0$$

says that if

$$R(I - A_{22}) < PA_{12}$$

that is, if carbon tax is cheaper than the cost of antipollution activity, antipollution activity would not be realized ($X_2 = 0$). On the other hand, if $X_2 > 0$, that is antipollution activity was performed, the carbon tax imposed on the unit carbon dioxide emission is equal to the cost of eliminating (fixing) unit amount of carbon dioxide. The dual problem of eqns. (3) and (4) is to find an optimal solution to maximize the profit subject to these two constraints.

3. Nonlinearity between Elimination (Fixing) Rate of Carbon Dioxide and Cost of Elimination

In general, it is recognized that by increasing the elimination rate of carbon dioxide the cost of eliminating a unit amount of carbon dioxide would be increased[6]. We take into account this nonlinearity between the elimination rate α_j and marginal input coefficient $a_{i,n+1}$ for eliminating carbon dioxide as follows:

$$a_{i,n+1} = \eta a^0_{i,n+1}, \quad \alpha_j = f(\eta)\alpha_{0j} \tag{5}$$

where

- $a^0_{i,n+1}$: marginal input coefficient for eliminating carbon dioxide at the time when the input-output table was obtained
- η : multiplier coefficient for $a^0_{i,n+1}$
- α_{0j} : elimination rate of carbon dioxide at the time when the input-output table was obtained
- $f_j(\eta)$: multiplier coefficient for α_{0j} as a function of η

Equation (5) implies that if the input of commodities for carbon dioxide elimination was multiplied by η, the elimination rate of carbon dioxide would be multiplied by $f_j(\eta)$. For modeling this relationship we need to realize the properties as follows:

- $\alpha_j = 0$ if $\eta = 0$
- $\alpha_j = \alpha_{0j}$ if $\eta = 1$
- $\alpha_j \to 1$ if $\eta \to \infty$
- $\eta > f_i(\eta)$ for $\eta > 1$

For obtaining these properties we postulate that $f_j(\eta)$ is written as[7]

$$f_j(\eta) = \frac{\eta}{1 + \alpha_{0j}(\eta - 1)} \tag{6}$$

Applying eqns. (5) and (6) to A_{12} in eqns. (1)-(4), we could incorporate the nonlinearity between the elimination rate of carbon dioxide and cost of elimination in our linear programming model described by eqns. (1) and (2) and eqns. (3) and (4). Then we could model the phenomena that by increasing the elimination rate of carbon dioxide the cost of eliminating a unit amount of carbon dioxide would be increased.

4. Numerical Examples

As a numerical example an aggregated input-output model with 4 sectors is shown. This 4-sector model is obtained by aggregating the 13-sector model obtained in 1985[8]. Each sector in the 4-sector model includes following items:

Sector 1: Agriculture, mining and constructions
Sector 2: Manufacturing
Sector 3: Transportation
Sector 4: Public and others

In this case the input-output table can be summarized as

$$A_{11} = \begin{bmatrix} 0.0413 & 0.341 & 0.00446 & 0.0662 \\ 0.0816 & 0.429 & 0.0305 & 0.0823 \\ 0.127 & 0.218 & 0.103 & 0.216 \\ 0.0344 & 0.125 & 0.0214 & 0.156 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.226 & & & 0 \\ & 0.0525 & & \\ & & 0.0484 & \\ 0 & & & 0.0135 \end{bmatrix}$$

When we solve the dual problem, it is assumed that the value added (cost) for one unit production is 1 million yen as

$$V_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

(million yen/unit)

Based on this value of V_1 the value of commodities P without imposing carbon tax is obtained as

$$P = \begin{bmatrix} 1.239 & 3.076 & 1.205 & 1.861 \end{bmatrix}$$

(million yen/unit)

Using the input-output table and the value of P we obtain the final demand Y_1 and the total output X_1 as

$$Y_1 = \begin{bmatrix} 4.471 \times 10^7 \\ 3.999 \times 10^7 \\ 1.078 \times 10^7 \\ 9.134 \times 10^7 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 7.188 \times 10^7 \\ 9.351 \times 10^7 \\ 7.114 \times 10^7 \\ 1.2476 \times 10^8 \end{bmatrix}$$

(unit)

Input coefficient for eliminating carbon dioxide is postulated as[6]

$$A_{12} = \begin{bmatrix} 3 \times 10^{-7} \\ 3 \times 10^{-7} \\ 3 \times 10^{-7} \\ 3 \times 10^{-7} \end{bmatrix} \text{(unit/kgC)}$$

for carbon dioxide elimination rate of 1% (α_{0j}=0.01), where kgC denotes one kilogram of equivalent carbon. The environmental standard for the total emission of carbon dioxide is assumed to be 0.28 billion ton carbon (tC) and the generation rate of carbon dioxide in each sector is assumed to be

$$A_{21} = \begin{bmatrix} 198.0 & 1460 & 822.3 & 573.8 \end{bmatrix}$$
$$\text{(kgC/unit)}$$

It is assumed that no carbon dioxide is emitted in the elimination process of carbon dioxide as

$$A_{22} = 0$$

In **Fig. 1** nonlinearity between elimination (fixing) rate of carbon dioxide and cost of elimination is shown. In **Figs. 2, 3** and **4** relationship between the environmental standard for total carbon dioxide emission and the price change ratio, carbon tax and total output is shown, respectively. In this example we obtained the result that if the environmental standard for total carbon dioxide emissions is more than or equal to 0.28 billion tC, it is not necessary to impose carbon tax. If we postulate more severe environmental standard for the total carbon dioxide emission than 0.28 billion tC, we will get higher price, carbon tax and total output.

From Fig. 3 if we impose carbon tax of 25 yen/kgC, which is equivalent to 15 yen/liter increase of the price of gasoline, we could reduce 28 million tC which is 10 % of the total carbon dioxide emission. On the other hand if we set the environmental standard of the total carbon dioxide emission as 0.26 billion tC, then we would need the carbon tax of 18 yen/kgC, which is equivalent to 10.8 yen/liter increase of the price of gasoline.

Next, we analyze the case when the cost for eliminating carbon dioxide is decreased by some technology innovation. If we postulate that the input coefficient for eliminating carbon dioxide is decreased to one half of the previous value, we would obtain **Fig. 5** as the nonlinear relationship between the elimination rate of carbon dioxide and the cost of elimination. In **Figs. 6, 7** and **8** relationship between the environmental standard of the total carbon dioxide emission and the price change ratio, carbon tax and total output is shown, respectively.

In this case if we impose carbon tax of 25 yen/kgC, which is equivalent to 15 yen/liter increase of the price of gasoline, we could reduce 50 million tC which is 18 % of the total carbon dioxide emission. On the other hand if we set the environmental standard of the total carbon

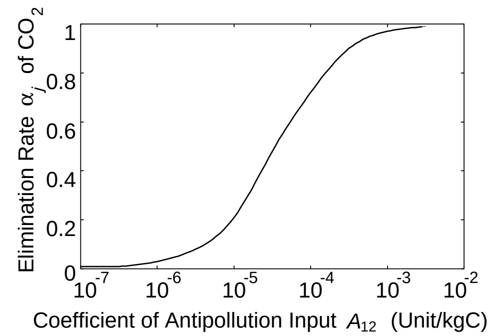

Fig. 1 Relationship between antipolltion input and the elimination rate of CO_2 at present

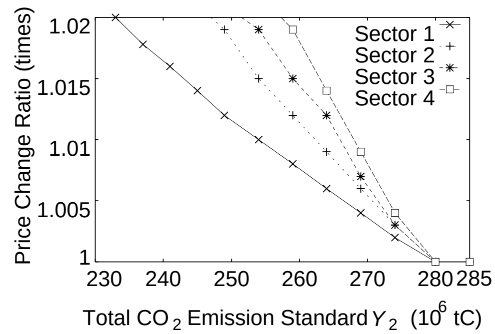

Fig. 2 Relationship between the total CO_2 emission control and price change ratio at present

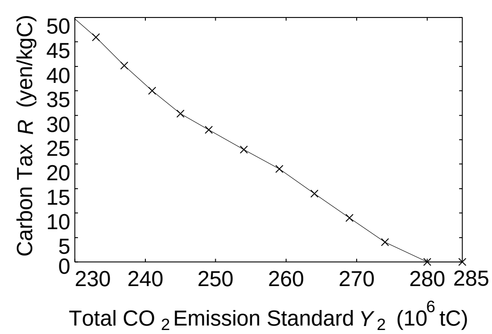

Fig. 3 Relationship between the total CO_2 emission standard and environmental tax at present

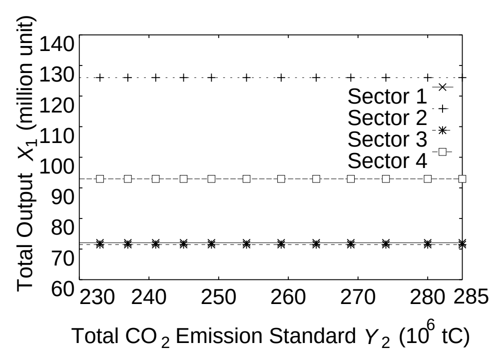

Fig. 4 Relationship between the total CO_2 emission standard and total output at present

dioxide emission as 0.26 billion tC, then we would need the carbon tax of 9 yen/kgC, which is equivalent to 5.3 yen/liter increase of the price of gasoline.

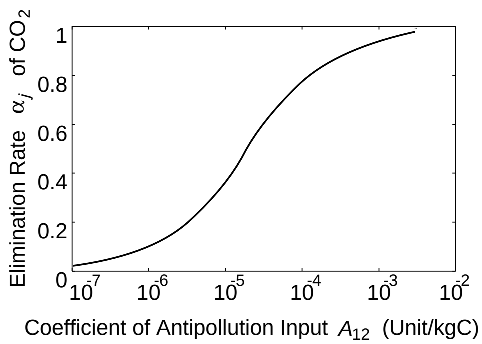

Fig. 5 Relationship between antipolltion input and the elimination rate of CO_2 after technology innovation

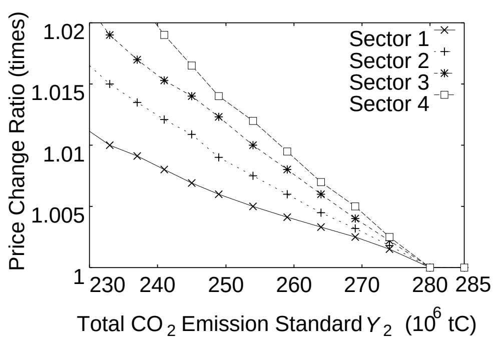

Fig. 6 Relationship between the total CO_2 emission standard and price change ratio after technology innovation

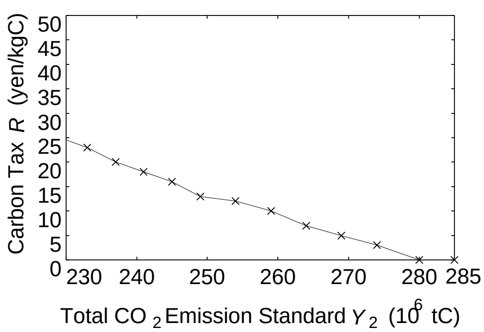

Fig. 7 Relationship between the total CO_2 emission standard and environmental tax after technology innovation

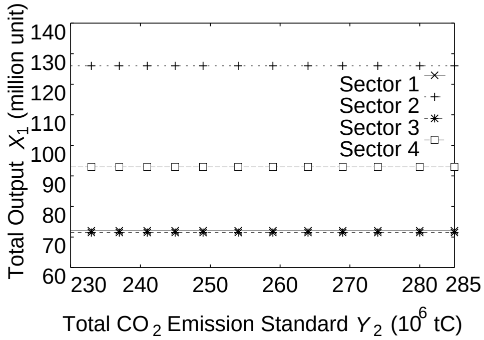

Fig. 8 Relationship between the total CO_2 emission standard and total output after technology innovation

5. Concluding Remarks

In this paper we developed a model for evaluating the effectiveness of carbon tax in order to decrease the total amount of carbon dioxide emission by using a static inter-industry input-output model and the primal problem and its dual problem of linear programming. We postulated nonlinearity between elimination (fixing) rate of carbon dioxide and cost of elimination. By using the model developed in this paper we could evaluate the items as follows:

- By assigning a carbon tax we could find the corresponding amount of decrease in the total carbon dioxide emission.
- By assigning the environmental standard for the total carbon dioxide emission we could find the necessary carbon tax for realizing this situation.

Furthermore, we took into account the case when some technology innovation would be achieved for decreasing (or fixing) carbon dioxide emission, and we tried to analyze the effectiveness of carbon tax. As the result we found the following interesting results.

- The effectiveness of carbon tax would become more remarkable to decrease the total amount of carbon dioxide emission, if the technology innovation for decreasing carbon dioxide emission would be achieved.
- Rise in prices of commodities due to carbon tax would be suppressed, if the technology innovation for decreasing carbon dioxide emission would be achieved.

These results show that we could expect multiplicative effect between environmental-economic policy and technology innovation to decrease the total amount of carbon dioxide emission.

For further research we need to develop dynamic environmental-economic models to evaluate various scenarios such as change of life style, development of renewable energy, and so forth.

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References

1) S. Ihara and S. Koyama: Structure of energy systems for controlling CO_2 emissions, *Systems, Control and Information*, Vol. 35, No. 9, pp. 555-562 (1991) (in Japanese)
2) R.F. Kosobud, T.A. Dary, D.W. South and K.G. Quinn : Tradable Cumulative CO_2 Permits and Global Warming Control, *The Energy Journal*, Vol. 15, No. 2, pp. 213–232 (1994)
3) F.W. Gaskins. Jr and J.P. Weyant : Model Comparisons of the Cost of Reducing CO_2 Emissions, *American Economic Review, Papers and Proceedings*, Vol. 83, No. 2, pp. 318–

323 (1993)
4) P.D.Lowe : Pricing Problems in an Input-Output Approach to Environment Protection, *The Review of Economics and Statistics*, Vol.61, No.1-2, pp. 110-117 (1979)
5) H. Siebert: *Economics of the Environment*, Springer, Berlin (1995)
6) Environment Agency: Environment White Paper for 1992 (1992) (in Japanese)
7) H. Tamura and T. Ishida: Environmental-economic models for total emission control of regional environmental pollution, *Ecological Modelling*, Vol. 30, pp. 162-173 (1985)
8) General Affair's Agency: 1985 Input-Output Report (1985) (in Japanese)

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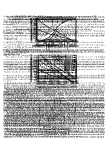

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