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A Robust Tuning Method for I-PD Controller Incorporating a Constraint on Manipulated Variable

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A PID settings formula is presented to provide a critically damped response to a setpoint change for a first-order lag process with dead time. It is shown that the optimized PID settings based on the ISE (Integral of Squared Error) has a significant disadvantage in that it tends to be more sensitive to model errors. To reduce the sensitivity, a method of robust PID tuning is developed by incorporating a constraint on the manipulated variable. It is found that control performance against model uncertainty is remarkably improved.

Key Words: process control, I-PD controller, first-order lag process with dead time, critically damped response, manipulated variable constraint

1. Introduction

The PID controller is indispensable for the process control system. Here, the PID controller is described as of how it is applied to real plants, how it is tuned, how its controllability is utilized and what problems remain to be resolved. Applications of PID controller in a certain plant are surveyed to understand its present status and remained problems. From this survey, it is revealed that most of the PID controllers are tuned by trial and error, and that a practical PID tuning method utilizing a process model is sometimes required.

To establish a practical model-based robust PID tuning method, following two problems must be resolved. First, uncertainty of the process model has to be estimated quantitatively. Second, a PID settings formula for a PID controller with I-PD type algorithm (called as "I-PD controller") has to be established for practical purpose to be utilized in the real plants. In this paper, first, a practical procedure to identify the process model is described with an example. Then, the uncertainty of process model is classified into three sets of process models, the nominal model, the severe model and the insensitive model. A robustness of control performance can be evaluated using these process models by numerical simulation. Secondly, a PID settings formula is derived for the I-PD controller. In the third place, a robust PID tuning method should be developed. For the severe model, it is obvious that the optimization of a tuning parameter in the PID settings formula to minimize the ISE results in a poor control performance. Therefore, a constraint on manipulated variable is introduced to reduce the sensitivity of control performance to the model uncertainty. It is presented that the appropriate setting of this constraint provides a greatly improved control performance.

In this work, the model-based PID settings formula and the robust PID tuning method for the feedback control system using I-PD controller are developed for the practical applications.

2. Applications and its Problems of the PID control systems

2.1 Applications of PID Control Systems

We have surveyed the present status of PID controller application in Mizushima Plant of Mitsubishi Chemical Corporation. The main results of this survey are summarized as follows;

(1) Ratios of the control methodologies are; 90% for the PID control, 9% for the conventional advanced control such as the feed forward control, and 1% for the model predictive control.

(2) PID Controllers with 5,000 loops are utilized in 24 production units.

(3) Ratio of the controlled variables of PID control loops are; 44% for flow, 21% for level, 17% for pressure, 16% for temperature, and 2% for composition.

(4) I-PD type is often adopted for the algorithm of PID control.

(5) The running rate of "auto mode control" is only 70%, upper than that is strongly expected by plant operators.

(6) Re-tuning is required for many PID control loops.

2.2 Actual Conditions of PID Tuning

The PID tuning is usually carried out according to following three steps. First, three parameters of the PID

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controller, proportional gain, reset time (integral time) and derivative time, are set to be initial settings. For the initial PID settings, some process model is needed to express the dynamic behavior of controlled process. This initialized PID controller is employed to the controlled process to observe its control performance. Then, the control performance is gradually improved by fine PID tuning.

How the PID controllers are tuned actually? Based on worldwide survey of over 60 plants, Stanton [6] pointed out the actual conditions of controller tuning. He stated that "Rarely do engineers tune controllers; the mechanics who do the tuning have never read any of the papers or books. Most of these have had simple training using the Ziegler-Nichols methods which they have forgotten, did not understand or do not want to take the time to apply. The major point is that tuning of most single loop control system is relatively easy and settings can normally be predetermined." In the meantime, McMillan [4] suggested a guideline of the initial PID settings for several processes by his rich experience.

We also have some simple rules based on our experience to succeed the PID tuning. For example, for the flow control system, "Wide Band Fast Reset," wide proportional band (weak proportional gain) and fast reset time (strong integral gain), has to be applied. On the contrary, for the liquid level control system, it has to be set as "Narrow Band Slow Reset." With appropriate rules of thumb and trial-and-errors, the PID controller can be tuned efficiently and the control performance of it will be satisfied. However, from the above-mentioned survey, 15% of the PID control loops are not controlled well merely applying these empirical rules. For these control loops, a model-based robust PID tuning method that can determine the suitable PID parameters based on the process model considering the robustness of control performance is required to achieve a good control performance. The control loops to apply this method will be temperature control with long time lag, composition control, flow averaging level control, and so on.

2.3 Problems in Model-based Robust PID Tuning Method

There are three problems in the model-based robust PID tuning method. First is the uncertainty in the process model, the model errors have to be estimated quantitatively. Secondly, a PID settings formula for the I-PD controller is required. In the third place, a robust PID tuning method is strongly desired. A practical method to resolve these problems is described in detail in the following sections.

3. Process Modeling and Estimating Process Model Errors

3.1 Present Status in Process Modeling

The process model for the PID tuning is often built experimentally. Based on first principles such as a material and heat balance, it is possible to build a nonlinear process model. However, the process model for linear control systems should be linear model, and linearization and parameter estimation of the process model are usually complex and time consuming. For these difficulties, the process models are built on experiments with plant test rather than the analytical method.

In a plant test, the process input (manipulated variable) is changed, and the response of process output (controlled variable) is observed. In order to avoid large disturbance caused by the plant test, the magnitude of process input change is limited and some step response test are applied only a few times.

A model structure of stable process is typically assumed to be "first-order lag process with dead time." Based on the plant test result, the parameters of this process model are usually determined by eye, called "Eyeball Fitting" [3], to fit for the assumed process model structure.

3.2 Estimation of Process Model Errors

As described above, the uncertainty is inevitably included in the process model structure and its parameters. A nominal model as a result of process modeling, is just one of representatives of the process. If the model error is quantified, the process model could be widened around its nominal model. This concept is illustrated in **Fig. 1**.

The procedure for quantifying the process model error is presented with an example of the temperature control system of cracking furnace shown in **Fig. 2**. The fuel flow into the furnace is the manipulated variable u, and the

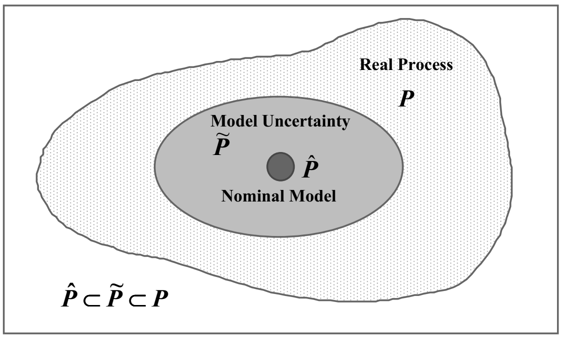

Fig. 1 Process model uncertainty

temperature at the outlet of the thermally cracked fluid is the controlled variable y. **Fig. 3** shows the comparison of the plant test data with the response of process model. It is very difficult in the real plant to keep all the conditions, such as the fuel composition, to be constant. In Fig. 3, the steady state gain seems to change after 70 minutes, but this was caused by fluctuations in the fuel system.

Utilizing an ARX function of MATLAB System Identification Toolbox, the following process model in discrete time system $\hat{P}(z)$ can be obtained.

$$\hat{P}(z) = \frac{b_0^{-(L+1)}}{1 + a_1 z^{-1}} \tag{1}$$

The model parameters $\{a_1, b_0, L\}$ and the standard deviation of the model parameters σ_a, σ_b are given as

$$a_1 = -0.9832, \quad b_0 = 0.7726 \times 10^{-2}, \quad L = 6$$
$$\sigma_a = 0.1127 \times 10^{-2}, \quad \sigma_b = 0.3721 \times 10^{-3}$$

where the sampling period is 10 seconds ($\tau = 10/60$min). This model can be transformed to the continuous time system process model $\hat{P}(s)$.

$$\hat{P}(s) = \frac{\hat{K}_p}{1 + \hat{T}_p s} e^{-\hat{T}_L s} \tag{2}$$

By the relations between the two models, the model parameters; the steady state gain $\hat{K}_p$, the time constant $\hat{T}_p$, and the dead time $\hat{T}_L$ are derived.

$$\hat{K}_p = \frac{b_0}{1 + a_1} = 0.431\%/\% \tag{3}$$

$$\hat{T}_p = \frac{\tau}{\log(-1/a_1)} = 9.85\text{min} \tag{4}$$

$$\hat{T}_L = L\tau = 1.00\text{min} \tag{5}$$

Error bounds of these model parameters will be estimated. The standard deviation of the steady state gain σ_{K_p} and the time constant σ_{T_p} are calculated by applying the Gauss' law.

$$\sigma_{K_p} = \frac{1}{1 + a_1} \sqrt{\frac{b_0^2}{(1 + a_1)^2} \sigma_a^2 + \sigma_b^2} = 0.0365 \tag{6}$$

$$\sigma_{T_p} = \left| \frac{\tau}{a_1 [\log(-a_1)]^2} \right| \sigma_a = 0.668 \tag{7}$$

Then, 3σ errors, e_{K_p} and e_{T_p} are estimated as follows.

$$e_{K_p} = (3\sigma_{K_p}/\hat{K}_p) \times 100 = 25.4\% \tag{8}$$

$$e_{T_p} = (3\sigma_{T_p}/\hat{T}_p) \times 100 = 20.3\% \tag{9}$$

This result shows that the 3σ errors of the process model are 25% for the steady state gain, and 20% for the time constant. For the remaining parameter, the dead time, the 3σ error cannot be evaluated as above, therefore, it is assumed to be 20%, the same as that for the time constant. Under this condition, process model parameters are in the ranges below.

$$\begin{cases} 0.75\hat{K}_p \le K_p \le 1.25\hat{K}_p \\ 0.80\hat{T}_p \le T_p \le 1.20\hat{T}_p \\ 0.80\hat{T}_L \le T_L \le 1.20\hat{T}_L \end{cases} \tag{10}$$

3.3 Process Models for Robust PID Tuning

From the parameter range of process model, model uncertainty can be imaginarily represented a cubic shown as in **Fig. 4**. The process models corresponded to the apices of cubic respectively and the nominal model (the center of the gravity) are considered. When an I-PD controller whose PID parameters are tuned appropriately is applied to these 9 process models, the control performances are evaluated by ISE with the numerical simulation. The simulated ISEs are normalized by the ISE of the nominal process model and these normalized ISEs are shown in the corresponding apices in Fig. 4. The best control performance is of Sensitive Model (highest steady state gain, fastest time constant and shortest dead time), and the worst is of Insensitive Model (lowest steady state gain, slowest time constant and longest dead time). However, when the PID parameters are changed to control more aggressively (the proportional gain is increased), Severe Model (highest steady state gain, fastest time constant

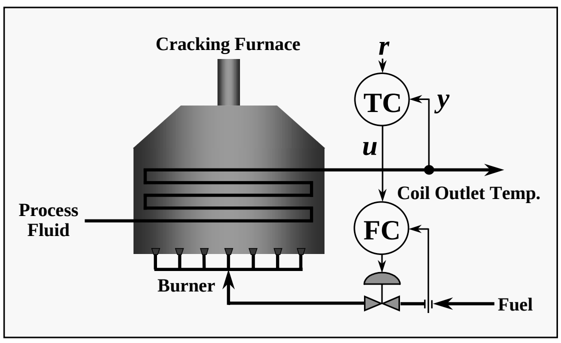

Fig. 2 Temperature control system of cracking furnace

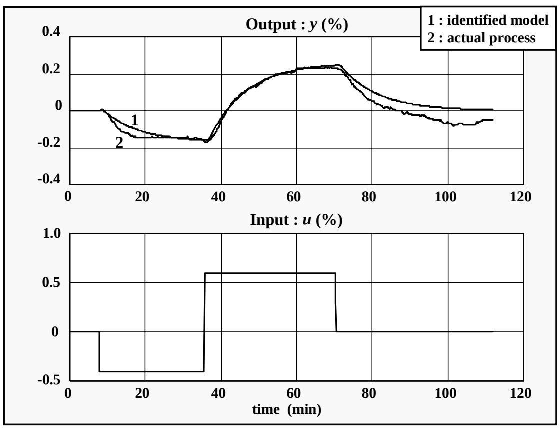

Fig. 3 Comparison of model response with actual response

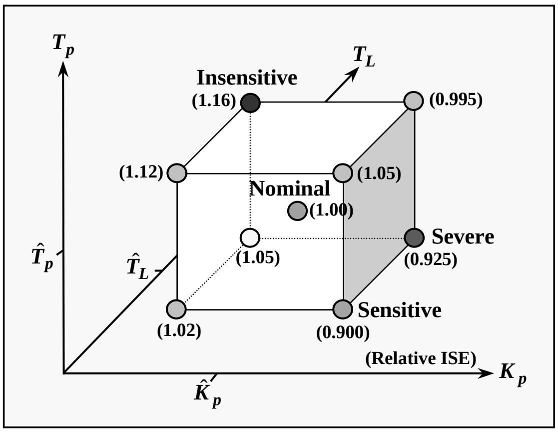

Fig. 4 Process model uncertainty cube

and longest dead time) is revealed to be the first to become unstable among others.

From this result, the uncertainty of process model is represented in the following three sets of models. As shown in **Fig. 5**, the actual response in the plant test is inside of the responses of each model, if the data after 70 minutes when external disturbance affected are excluded.

- Nominal Model $P_n \in \{\hat{K}_p, \hat{T}_p, \hat{T}_L\}$
- Severe Model $P_s \in \{1.25\hat{K}_p, 0.80\hat{T}_p, 1.20\hat{T}_L\}$
- Insensitive Model $P_i \in \{0.75\hat{K}_p, 1.20\hat{T}_p, 1.20T_L\}$

Therefore, the robust PID tuning is then pursued with these three sets of models. These models that represent the error range of the nominal model do not thoroughly express the uncertainty of the model. However, as far as our experience of the PID tuning at the plant based on this concept, this procedure is proven to be practically effective enough.

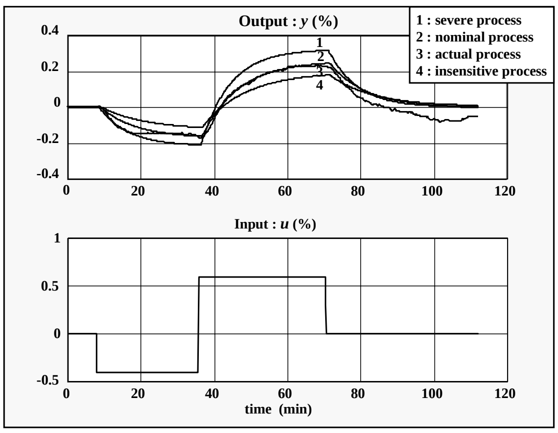

Fig. 5 Comparison of model responses with actual response

4. Model-based Robust PID Tuning for I-PD Controller

The second problem is how to establish a model-based robust PID tuning method, where the uncertainty of process model is included for the I-PD controller.

4.1 Feedback Control System Using I-PD controller

The feed back control system with an I-PD controller is shown in Fig. 6. $r(s)$ is the setpoint, $y(s)$ is the controlled variable, $u(s)$ is the manipulated variable, $d(s)$ is the disturbance, $e(s)$ is the control error and $P(s)$ is the controlled process. In this control system, the response of controlled variable to the setpoint change and the disturbance is written as

$$y(s) = \frac{C(s)P(s)}{1+C(s)P(s)}F(s)r(s) + \frac{1}{1+C(s)P(s)}d(s) \qquad (11)$$

where the I-PD controller has a setpoint filter $F(s)$. This is the difference from the PID controller $C(s)$, although the closed-loop transfer function for the disturbance is the same in both controllers. The PID controller and the setpoint filter are defined in following equations, where the PID parameters are the proportional gain K_c, the reset time T_i, and the derivative time T_d. $1/\gamma$ is the derivative gain.

$$C(s) \equiv \frac{u(s)}{e(s)} = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1+\gamma T_d s}\right) \qquad (12)$$

$$F(s) \equiv \frac{1}{1 + T_i s + T_i T_d s^2/(1+\gamma T_d s)} \qquad (13)$$

An important objective of the feedback control system is to reject the disturbance for the most part. Meanwhile, when the setpoint is changed, for modest effects over the descending process, sluggish change in the manipulated variable with less overshoot of the controlled variable is preferred. This is the reason why the I-PD controller is widely employed in real applications.

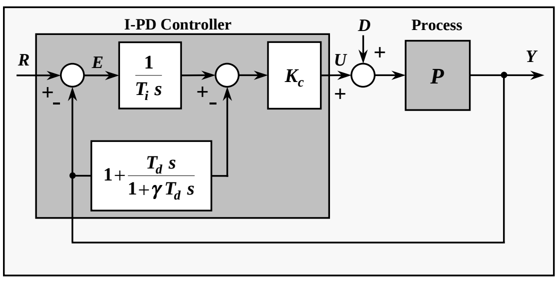

Fig. 6 Feedback control system using I-PD controller

4.2 PID Settings for I-PD Controller

As in the feedback control system shown in **Fig. 6**, when the disturbance acts on the same point as the manipulated variable, its effect on the controlled variable will appear with a time lag through the controlled process. Despite the existence of the setpoint filter, this response of controlled variable to the disturbance is usually slower than that to the setpoint change. In other words, the setpoint change effect is more intense than the effect of disturbance.

For this reason, as the PID parameters are tuned for more aggressive control, the response to setpoint change reaches faster to the unstable region. Therefore, in order to satisfy the control performances both for the setpoint change and the disturbance rejection with a single set of PID parameters, the I-PD controller has to be tuned to optimize the control performance for setpoint change.

Kitmori [2] developed a unique method of PID settings for the I-PD controller. A desired response to the step change in setpoint is found of selecting a little overshoot response. The transfer function of the desired response is expressed with a denominator polynomial such as the transfer function of Butterworth filter. Then, the PID parameters are determined to fit the closed-loop response to the desired one. In order to adjust the closed-loop response speed, a normalizing factor of the time scale is introduced as the only tuning parameter.

Rivera et al. [5] established the IMC (Internal Model Control) method, which is widely used in the process industry. This method is to determine the PID parameters to ensure a desired closed-loop response to the step change in setpoint for the PID type algorithm. In that sense, the concept of IMC method is similar to that of Kitamori's method, but the IMC method employs the desired closed-loop response as "first-order lag system with dead time" without overshooting. If the dead time exists in the process model, the desired closed-loop response is delayed by the dead time. The response speed to the setpoint change is tuned by the time constant of desired closed-loop response.

The reasons why IMC method is preferred in real plants are as follows. First, the process model includes errors, so if the response in the nominal model is determined to be first-order lag, there are much time to reach the unstable condition though the real plant response is subtly quicker. Second, because it can be tuned intuitively described as "the time constant of the desirable closed-loop response to be set half as that of the process," the IMC method is now readily accepted by the plant operators.

4.3 PID Settings Formula to Obtain Critically Damped Closed-loop Response

The IMC method concept can be extended easily into the feedback control system using I-PD controller to establish a new PID settings formula. For a desired closed-loop response, a high order critically damped response with dead time is chosen, and the PID settings formula is derived with the Kitamori's method modified to the dead time system. The controlled process is assumed to be the stable system, and its characteristic is assumed to be "first-order lag process with dead time."

(1) Desired Closed-loop Response

A "second-order lag with dead time" characteristic is selected to be the desired closed-loop response by considering the setpoint filter involved in the I-PD controller. The time constant of the desired closed-loop response is defined to be T_F, and its dead time is assumed to be the process dead time T_L. By using the first-order Padé approximation of the dead time, the transfer function of desired closed-loop response $W_r(s)$ can be written

$$\begin{aligned} W_r(s) &\equiv \frac{1}{(1+T_F s)^2} e^{-T_L s} \\ &\approx \frac{1 - T_L s/2}{(1+T_F s)^2 (1+T_L s/2)} \end{aligned} \tag{14}$$

Parameters p and q are defined as Eq. (15). By normalizing the process dead time and the time constant of desired closed response by the process time constant, these values can be obtained. The former, process parameter p represents a difficulty of the controlled process, and the latter, tuning parameter q is used to optimize the control performance.

$$p \equiv T_L/T_p, \qquad q \equiv T_F/T_p \tag{15}$$

$W_r(s)$ can be expressed in the following denominator polynomial form.

$$W_r(s) = \frac{1}{1 + \alpha_1 s + \alpha_2 s^2 + \alpha_3 s^3 + \cdots} \tag{16}$$

$$\begin{cases} \alpha_1 = (p + 2q) T_p \\ \alpha_2 = (p^2/2 + 2pq + q^2) T_p^2 \\ \alpha_3 = (p^3/4 + p^2 q + pq^2) T_p^3 \\ \cdots \end{cases} \tag{17}$$

(2) Closed-loop Response of Feedback Control System

The transfer function $W_c(s)$ that represents the response from the setpoint change to the controlled variable in the feedback control system using the I-PD controller in Fig. 6, is expressed by neglecting the derivative gain.

$$W_c(s) = \frac{1}{1 + T_i s + T_i T_d s^2 + T_i s / K_c P(s)} \tag{18}$$

Here, the denominator polynomial form of the process model is obtained by applying the first-order Padé approximation of the dead time.

$$\begin{aligned} P(s) &= \frac{K_p}{1+T_p s} e^{-T_L s} \\ &\approx \frac{K_p(1-T_L s/2)}{(1+T_p s)(1+T_L s/2)} \\ &= \frac{1}{\beta_0+\beta_1 s+\beta_2 s^2+\cdots} \end{aligned} \tag{19}$$

$$\begin{cases} \beta_0 = 1/K_p \\ \beta_1 = (1+p)T_p/K_p \\ \beta_2 = p(1+p/2)T_p^2 K_p \\ \cdots \end{cases} \tag{20}$$

Then, the denominator polynomial form of $W_c(s)$ is obtained.

$$W_c(s) = \frac{1}{1+\sigma_1 s+\sigma_2 s^2+\sigma_3 s^3+\cdots} \tag{21}$$

$$\begin{cases} \sigma_1 = (1+1/K_p K_c)T_i \\ \sigma_2 = T_i T_d + (1+p)T_p T_i/K_p K_c \\ \sigma_3 = p(1+p/2)T_p^2 T_i/K_p K_c \\ \cdots \end{cases} \tag{22}$$

(3) PID Settings Formula

The PID setting formula can be obtained by coinciding the corresponding coefficients of both denominator polynomials expressed by Eq. (17) and (22).

$$\begin{cases} K_c \equiv f_p(p,q)/K_p = \left[\dfrac{p-2q+4}{p+2q}\right]\dfrac{1}{K_p} \\ T_i \equiv f_i(p,q)T_p = \left[\dfrac{(p+2q)(p-2q+4)}{2p+4}\right]T_p \\ T_d \equiv f_d(p,q)T_p = \left[\dfrac{p(p+4q-2q^2)}{(p+2q)(p-2q+4)}\right]T_p \end{cases} \tag{23}$$

Here, $K_c K_p$, T_i and T_d have to be greater than zero. Therefore, the necessary and sufficient condition for that is $p-2q+4>0$ and $p+4q-2q^2>0$. Then, the tuning parameter q has to satisfy the following condition.

$$0 < q < 1+\sqrt{1+p/2} \tag{24}$$

4.4 Optimum PID Tuning

In the PID tuning, the tuning parameter q is required to be guessed to minimize the ISE. If a perfect critically damped response can be realized, then q is set to be nearly zero to provide a quick response. However, in this PID settings formula as derived above, the dead time is approximated by the first-order Padé method, and the corresponding coefficients both of the desired closed-loop response and the control system response agree only by the third order term. Because of these approximations, if q is decreased toward zero, the feedback control system falls into unstable condition and the ISE gets larger. This means that the optimum q exists to minimize the ISE.

(1) Searching Optimum Tuning Parameter by Numerical Simulation

The ISE is employed as an index to evaluate the control performance for the setpoint change. The minimization problem of ISE that is dependent on tuning parameter q is stated as

$$\min_q J = \int_0^\infty e^2(t)dt \tag{25}$$

The simulator of the feedback control system is built by using the MATLAB Simulink Toolbox. The ISE for this system is calculated in the following conditions.

Process model: $K_p = 0.432\%/\%, T_p = 9.85\text{min}$, $T_L = 1.00\text{min}$
I-PD controller: $1/\gamma = 10$
Parameter ranges: $p \in [0.05, 1.00], q \in [0.01, 1.00]$
Control period: $\tau = 10\text{sec}$
Simulation time duration: 100min
Setpoint change: $r = 1$

The calculated results are shown in **Fig. 7**. It is clearly shown that the tuning parameter q exists to minimize the ISE. If the process dead time is smaller than the process time constant, then ISE will be drastically increased by decreasing parameter q. That is, if the process parameter p is small, the optimum point will close to the unstable region of the control system. The control error is expressed by the parameters p and q, and the process time constant. If the time scale is normalized by process time constant, then the control error depends only on parameters p and q (refer to the Appendix). Therefore, the optimum condition can be generally applied to any process time constant. The pairs of p and q to minimize the ISE are obtained by reading Fig. 7 graphically. The following equation gives reasonable approximation of optimum tuning parameter q_{opt}.

$$q_{opt} = -0.1902p^2 + 0.6974p + 0.007393 \tag{26}$$

(2) Evaluation

This optimum tuning rule is applied to the temperature control system of cracking furnace. The PID parameters are determined based on the nominal model; $p = 0.102, q = 0.0762; K_c = 36.0, T_i = 2.35\text{min}$, $T_d = 0.394\text{min}$. In the three process models, the control performances are simulated for the step setpoint change and the step disturbance. The setpoint is changed in $r = 1$ at 10 minute, and the disturbance $d = -1$ appeared at 50 minutes. From the results of simulation shown in **Fig. 8**, these problems arose.

1) Although the critically damped response is expected, the oscillatory response of the controlled variable is ap-

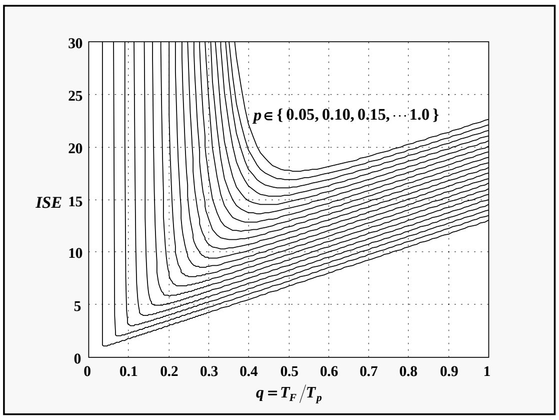

Fig. 7 ISE for step setpoint change

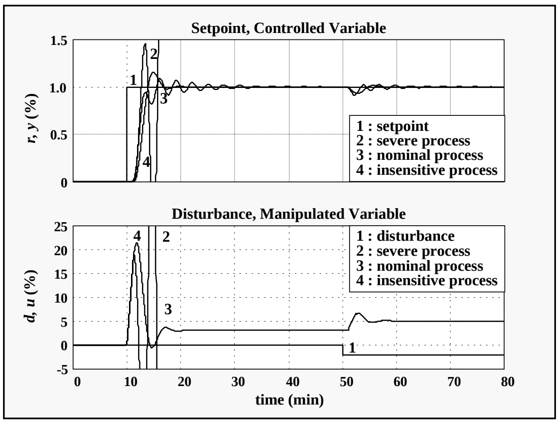

Fig. 8 Control performance using PID settings to minimize ISE

peared.

2) The manipulated variable also fluctuates periodically and the amplitude of fluctuation is large.

3) The poor control performance is caused by the process model uncertainty.

4) In the severe model, the control system becomes an unstable system.

4.5 Robust PID Tuning Incorporating Constraint on Manipulated Variable

In a process whose process parameter p is too small, the optimum PID tuning sets the control system close to the unstable region. For this reason, the optimum PID tuning is unsuitable to apply for the practice. In order to improve the robustness of PID tuning, some constraints on the manipulated variable have to be considered. Kawabe and Katayama [1] have proposed novel method of robust PID tuning. The integral of the squared control error and the squared velocity of manipulated variable is stated as the objective function, and the optimization problem where the model uncertainty is considered is numerically solved as a Minimax problem expressed in Eq. (27).

$$\min_{S} \max_{P} J = \int_0^{\infty} [e^2(t) + \rho(du/dt)^2]dt \tag{27}$$

where S is the PID parameter set and P is the process model parameter set.

(1) Constraint on Manipulated Variable

A constraint on the manipulated variable has to be considered to obtain the robust PID tuning method. As a robustness index in case of the setpoint change, a ratio of the maximum value to the steady state value of manipulated variable is introduced

$$\delta_{max} = (u_{max}/u_{\infty}) \times 100(\%) \tag{28}$$

where u_{max} and u_{∞} are the maximum value and the steady state value of manipulated variable respectively. The robustness index δ_{max} can be easily calculated from the same simulation in the previous section and is shown in **Fig. 9**. A constraint on the robustness index Δ_{max} is employed as an upper limit of the robustness index. From Fig. 9, relations of process parameter p and tuning parameter q are known to satisfy the constraints: $\Delta_{max} = 125, 150, 200\%$. These relations of robust PID tuning are plotted together with the result of optimum PID tuning into **Fig. 10**. From this plot, the tuning parameter q can be determined by considering the constraint on manipulated variable to achieve an appropriate control performance.

(2) Evaluation

This robust PID tuning method is applied to the temperature control system of cracking furnace. The constraint on robustness index is $\Delta_{max} = 200\%$, and the tuning parameter $q = 0.248$ is chosen to avoid the oscillatory behavior in the control system and to equalize the control performances corresponding to the three process models. The PID parameters; $K_c = 14.0, T_i = 5.05\text{min}, T_d = 0.450\text{min}$ are applied. By comparing Fig. 8 with **Fig. 11**, it is evident that the robustness of control performance is remarkably improved.

Finally, Kawabe and Katayama method is compared with this method. The nominal model and the model errors are assumed as

Process model: $K_p = 1\%/\%, T_p = 7\,\text{min}, T_L = 1\,\text{min}$

Model error: $e_{K_p} = 20\%, e_{T_p} = 20\%, e_{T_L} = 20\%$

The PID parameters and control performance are compared in **Table 1**. The simulation is run in the same condition for the previous example. In Kawabe and Katayama method, the weighting factor on the squared velocity of manipulated variable is selected as $\rho = 0.5$.

Table 1 Comparison of robust PID tuning methods

tuning method	K_c	T_i (min)	T_d (min)	ISE
Kawabe & Katayama method	4.35	3.85	0.42	4.14
this method	5.49	3.91	0.43	3.83

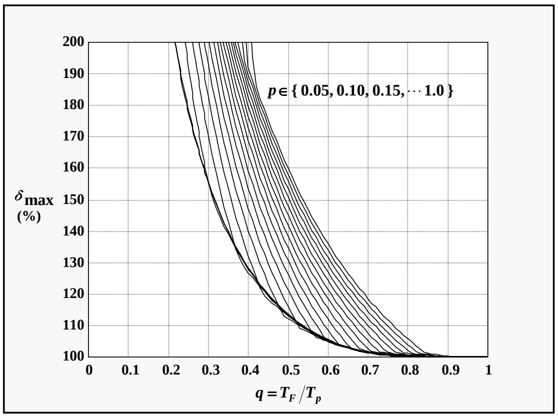

Fig. 9 Maximum MV for step setpoint change

In this method, the constraint on robustness index is $\Delta_{max} = 200\%$, and the PID parameters are determined for the nominal model. The control performance will be the worst ISE for the insensitive model. From these results, this method is proven to give the results close to Kawabe and Katayama method.

5. Conclusions

Applications of PID controller in the certain plant is surveyed to understand its present status and remained problems. This survey shows that the establishment of the model-based robust PID tuning method for the I-PD controller is obviously required in practical use. In this paper, the method for identifying the process model and

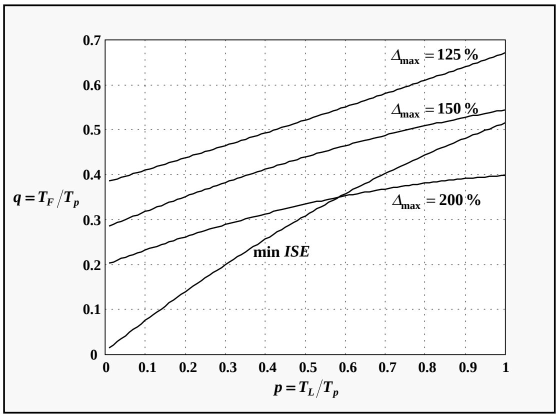

Fig. 10 Robust PID tuning parameter

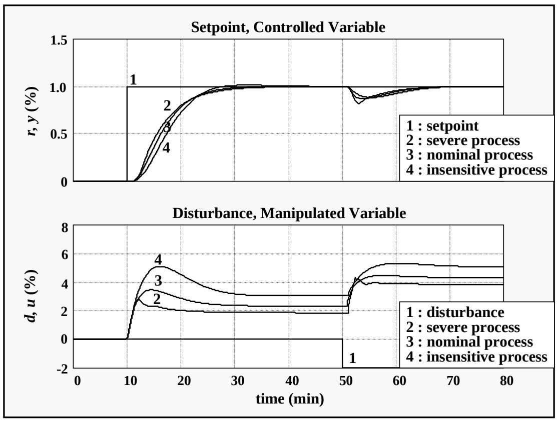

Fig. 11 Control performance of robust PID tuning

the error bounds of its parameters is explained with the simple example. The PID settings formula is consequently derived for the I-PD controller. The robust PID tuning method is finally devised by incorporating the intuitive robustness index.

Even if any rigorous process modeling and any precise PID tuning method are employed, these approaches will inevitably include model errors and approximations. In addition, there is a case where aggressive control cannot be implemented because of the measurement noise and/or the dead band of the final control element. In real applications, tuning of PID controller by a person is inevitable. The model-based robust PID tuning method proposed in this paper will be of great help for the person to determine the initial PID parameters.

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Appendix A. Control Error of Feedback Control System Using I-PD Controller

In the feedback control system using I-PD controller shown in Fig. 6, the control error for the step setpoint change is

$$e(s) \equiv \frac{N_e(s)}{D_e(s)} \qquad \text{(A. 1)}$$

where the numerator $N_e(s)$ and the denominator $D_e(s)$ are given as

$$\begin{aligned} N_e(s) &= (1+T_p s)T_i s - K_p K_c(1+T_d s)T_i s e^{-T_L s} \\ &= (1+T_p s) f_i(p,q) T_p s \\ &\quad - K_p K_c[1+f_d(p,q)T_p s] f_i(p,q) T_p s e^{-pT_p s} \\ D_e(s) &= (1+T_p s)T_i s^2 \\ &\quad + K_p K_c(1+T_i s+T_i T_d s^2) s e^{-T_L s} \\ &= (1/T_p s)\{(1+T_p s) f_i(p,q) T_p^2 s^2 \\ &\qquad + f_p(p,q)[1+f_i(p,q)T_p s \\ &\qquad\quad + f_i(p,q) f_d(p,q) T_p^2 s^2] T_p s e^{-pT_p s}\} \end{aligned}$$

Therefore, the control error can be expressed as a function of the process parameter p, the tuning parameter q, and the process time constant T_p.

If $e_1(s)$ and $e_2(s)$ are the control errors corresponding to the process time constants T_{p1} and T_{p2} respectively, then $e_2(s)$ is expressed from Eq. (A. 1).

$$e_2(s) = e_1(s/a)/a \qquad \text{(A. 2)}$$

where $a = T_{p1}/T_{p2}$. When the time shift theorem of inverse Laplace transformation is applied to Eq. (A. 2), the relation in time domain of $e_1(t)$ and $e_2(t)$ can be obtained.

$$e_2(t) = L^{-1}\{e_2(s)\} = L^{-1}\{e_1(s/a)/a\} = e_1(at) \text{(A. 3)}$$

It is proven that the control error is independent from process time constant, in which the time domain is normalized by the process time constant.

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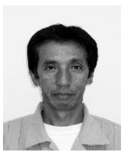

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