Trans. of the Society of Instrument and Control Engineers
Vol.E-1, No.1, 312/321 (2001)

System Identification Experiments of Large Space Structures †

—Algorithm in the Time Domain for a Rectangular Input—

Isao YAMAGUCHI* and Takashi KIDA**

This paper describes the theory and applications of a method of system identification of space structures using Hankel matrices in the time domain. The ERA(Eigensystem Realization Algorithm) treated herein is a typical system realization method utilizing Hankel matrices. This method, however, assumes ideal impulse response being available. In this paper, new iterative algorithm by least square method is developed for practical applications in the space environment. Ground-based experiments on air-bearing table attached by the flexible isogrid panel are carried out to demonstrate the validity of the new method.

Key Words: system identification, Hankel matrix, ERA(eigensystem realization algorithm), iterative least-square method

1. Introduction

Frequency-domain analysis techniques such as FFT (Fast Fourier Transform) and MEM (Maximum Entropy Method) have been mainly applied to the system identification for flexible structures including spectrum analysis, frequency response analysis and modal parameter identification test.

As the typical modal vibration test, resonance method in which frequency of input sinusoidal signal is swept in order to seek each modal frequency, and random vibration method in which all the modes are excited at once due to the wide band of input signal are utilized for the frequency-domain algorithm.

These methods are objected to such small structures as airplanes or automobiles for which ground based vibration test is applicable. However, in the case of large flexible space structure, it is impossible to conduct ground test due to the effects of 1-G gravity field and air drag. For this reason, we are planning the on-orbit system identification experiment of the large space structure [1]. However, in the identification experiment under the space environment, many restrictions occur to the examining method. First, since it is generally difficult to attach the vibrator at a desired position and the actuators for orbit control or attitude control, such as the existing reaction wheels and RCS (Reaction Control System) thrusters, should be used to generates an input required for identification. Moreover, only a position sensor or an accelerometer can be used as a measurement device. Finally, compared with the case of the identification experiment in the ground, sampling interval and sampling data number are limited [1),2)]. From such reasons, the identification methods in the time domain attract attentions in recent years [3)~8)].

Among these methods, ITD [6),7)] (Ibrahim's Time Domain) and ERA [3)~5)] (Eigensystem Realization Algorithm) have been developed as a system identification method in the time domain based on Hankel matrices. Both of the techniques identify system parameters using the impulse response sequence of a system. However, as stated previously, it is not easy to apply both algorithms without any modifications to an actual system under many restrictions. Viewing this, an identification algorithm effective in system identification of a large space structure on-orbit is proposed in this paper. In the second section, mathematical background of realization algorithm using Hankel matrix and ERA are outlined. In the third and forth sections, ERA is modified and improved so that a useful function is added to the algorithm for the on-orbit system identification. Then a new identification algorithm applicable for on-orbit experiment is proposed. In the fifth section, its validity is shown by the experiments using single-axis air bearing table with flexible isogrid panel.

2. System Realization

2.1 System Realization by Hankel matrix

In this section, we briefly describe system realization algorithm using Hankel matrix, which is called Ho-Kalman's algorithm [9)]. The following single-input

† Presented at The 6th Symposium on Guidance and Control for Aerospace (1989.11)

* National Aerospace Laboratory
** National Aerospace Laboratory (Currently, Dept. of Mechanical Engineering and Intelligent Systems, University of Electro-communications)

multi-output time-invariant state space model in time-continuous system is considered

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

where system matrix $A \in \boldsymbol{R}^{n\times n}$, input matrix $B \in \boldsymbol{R}^{n\times 1}$ and output matrix $C \in \boldsymbol{R}^{p\times n}$. In case of initial value of state $x(0) = 0$, impulse response with the sampling interval Δt becomes

$$y_k = Ce^{A(k-1)\Delta t}B, \quad k = 1, 2, \ldots \tag{2}$$

when ideal input $u = \delta(t)$ is applied. The pulse transfer function from input to output of Eq. (1) in the discrete time system using sift operator z^{-1} becomes

$$F(z^{-1}) = C(zI - e^{A\Delta t})^{-1}B = \sum_{k=1}^{\infty} y_k z^{-k}. \tag{3}$$

We construct the following matrix for system identification

$$H_{k-1} = \begin{bmatrix} y_k & y_{k+1} & \cdots & y_{k+r-1} \\ y_{k+1} & y_{k+2} & \cdots & \\ \vdots & \vdots & \ddots & \\ y_{k+q-1} & & & y_{k+q+r-2} \end{bmatrix} \tag{4}$$

where H_{k-1} is called Hankel matrix of size q. Then the Hankel matrix H_0 at $k = 1$ becomes

$$\begin{aligned} H_0 &= \begin{bmatrix} y_1 & y_2 & \cdots & y_r \\ y_2 & y_3 & \cdots & \\ \vdots & \vdots & \ddots & \\ y_q & & & y_{q+r-1} \end{bmatrix} \\ &= \begin{bmatrix} C \\ Ce^{A\Delta t} \\ \vdots \\ Ce^{A(q-1)\Delta t} \end{bmatrix} \times \\ &\quad \begin{bmatrix} B & e^{A\Delta t}B & \cdots & e^{A(r-1)\Delta t}B \end{bmatrix} \\ &= OG \end{aligned} \tag{5}$$

where O,G are observability and controllability matrices, respectively. If H_0 is able to be decomposed into O and G, the system realization A, B and C are given as follows:

A : $\frac{1}{\Delta t}\log(O_b^+ O_b^{\#})$
B : first m columns of G
C : first p rows of O

where O_b, $O_b^{\#}$, O_b^+ are

O_b : submatrix of O with rank n
$O_b^{\#}$: submatrix of O_b constructing by after $p+1$ rows from top of O_b
O_b^+ : left side pseudo-inverse matrix of O_b i.e. $O_b^+ O_b = I_n$

where $I_n \in \boldsymbol{R}^{n\times n}$ is unit matrix. The SVD (Singular Value Decomposition) method is used as one of the techniques of decomposing Hankel matrix H_0 into O and G. The SVD of H_0 yields

$$\begin{aligned} H_0 &= \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & \\ & S_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \\ &= U_1 S_1 V_1^T + U_2 S_2 V_2^T \end{aligned} \tag{6}$$

where $S_1 \in \boldsymbol{R}^{n\times n}$, $S_2 \in \boldsymbol{R}^{(q-n)\times(q-n)}$ are diagonal matrices having positive singular values as their diagonal elements, and where $U_1 \in \boldsymbol{R}^{pq\times n}$, $U_2 \in \boldsymbol{R}^{pq\times(q-n)}$, $V_1^T \in \boldsymbol{R}^{n\times r}$, $V_2^T \in \boldsymbol{R}^{(q-n)\times r}$ are unitary matrices. Theoretically, the size of S_1 should be set to n of the system for identification, and S_2 should become null matrix of the size of $(q-n)$. However, in the actual numerical analysis, it may not be clearly divided into S_1 and a complete null matrix. Especially, for the system that has the infinite oscillating modes like a space structure, the criteria to determine the order of the system is not clear. If we suppose it is obtained as,

$$OG = U_1 S_1 V_1^T \tag{7}$$

then following three expressions can be considered as a candidate of O and G in Eq. (7).

(a) $O^{(1)} = U_1 S_1$, $G^{(1)} = V_1^T$
(b) $O^{(2)} = U_1 S_1^{1/2}$, $G^{(2)} = S_1^{1/2} V_1^T$
(c) $O^{(3)} = U_1$, $G^{(3)} = S_1 V_1^T$

For example, if O and G are realized in type (b), system matrix A, input matrix B, and output matrix C yield

$$\begin{aligned} \widehat{A} &= \frac{1}{\Delta t}\log(O_1^+ O_1^{\#}) \\ &= \frac{1}{\Delta t}\log(S_1^{-1/2} U_1^T (U_1 S_1^{1/2})^{\#}) \end{aligned} \tag{8}$$

$$\widehat{B} = S_1^{1/2} V_1^T E_m \tag{9}$$

$$\widehat{C} = E_p^T U_1 S_1^{1/2} \tag{10}$$

where $E_m \in \boldsymbol{R}^{r\times m}$ and $E_p^T \in \boldsymbol{R}^{p\times pq}$ are

$$E_m = \begin{bmatrix} I_m \\ O_{(r-m)\times m} \end{bmatrix} \tag{11}$$

$$E_p^T = \begin{bmatrix} I_p & O_{p\times(pq-p)} \end{bmatrix}, \tag{12}$$

respectively.

2.2 ERA

As an application of the system identification by the Hankel matrix, Juang et. al. proposed ERA method using H_0 with H_1 in case of identification of system matrix A [3)~5)]. The Hankel matrix H_1 as $k = 1$ is decomposed to the from of product of observability matrix O and controllability matrix G as follows:

$$H_0 = U_1 S_1 V_1^T$$

$$= OG. \tag{13}$$

And Hankel matrix H_1 at $k = 2$ is defied as follows:

$$H_1 = \begin{bmatrix} y_2 & y_3 & \cdots & y_{r+1} \\ y_3 & y_4 & & \\ \vdots & & \ddots & \\ y_{q+1} & & & y_{q+r} \end{bmatrix}. \tag{14}$$

Then, the Hankel matrix H_1 is decomposed as follows:

$$H_1 = \begin{bmatrix} C \\ Ce^{A\Delta t} \\ \vdots \\ Ce^{A(q-1)\Delta t} \end{bmatrix} e^{A\Delta t} \times \begin{bmatrix} B & e^{A\Delta t}B & \cdots & e^{A(r-1)\Delta t}B \end{bmatrix} = Oe^{A\Delta t}G. \tag{15}$$

Supposing the case of (b) adopted as expression of SVD of H_0

$$O = U_1 S_1^{1/2} \tag{16}$$

$$G = S_1^{1/2} V_1^T \tag{17}$$

yields following expression of H_1:

$$H_1 = U_1 S_1^{1/2} e^{A\Delta t} S_1^{1/2} V_1^T. \tag{18}$$

Such a derivation is applicable also to the case of (a) or (c). Therefore, the identification result of system matrix $\hat{A}$ becomes following;

$$\widehat{A} = \frac{1}{\Delta t}\log(S_1^{-1/2} U_1^T H_1 V_1 S_1^{-1/2}). \tag{19}$$

In addition, $\hat{B}$ and $\hat{C}$ are obtained by Eqs. (9) and (10).

It is noted that the ERA method uses another Hankel matrix H_1, instead of calculating $O_b^{\#}$ required in the state realization by the Hankel matrix. Therefore, it has the advantage of brief expression in formulation and it does not need a pseudo-inverse matrix calculation like O_b^+.

The ERA algorithm can be summarized as follows.

Step 1 : Acquisition of sampled observation data:

$$y_k = y(k\Delta t)\ ;\ k = 1 \sim (q + r) \tag{20}$$

Step 2 : Construction of Hankel matrices H_0 and H_1:

$$H_0 = \begin{bmatrix} y_1 & y_2 & \cdots & y_r \\ y_2 & y_3 & & \\ \vdots & & \ddots & \\ y_q & & & y_{q+r-1} \end{bmatrix} \tag{21}$$

$$H_1 = \begin{bmatrix} y_2 & y_3 & \cdots & y_{r+1} \\ y_3 & y_4 & & \\ \vdots & & \ddots & \\ y_{q+1} & & & y_{q+r} \end{bmatrix} \tag{22}$$

Step 3 : SVD of H_0:

$$H_0 = U_1 S_1 V_1^T \tag{23}$$

Step 4 : Identification of system matrix A:

$$\widehat{A} = \frac{1}{\Delta t}\log(S_1^{-1/2} U_1^T H_1 V_1 S_1^{-1/2}) \tag{24}$$

Step 5 : Identification of input and output matrices, B and C:

$$\widehat{B} = S_1^{1/2} V_1^T E_m \tag{25}$$

$$\widehat{C} = E_p^T U_1 S_1^{1/2} \tag{26}$$

Although the size of Hankel matrix should be theoretically just larger than the size of the system to be identified, its accuracy is limited by the capability of SVD computer algorithm.

It is known that the modal frequency up to the twice of a sampling frequency can be identified by the Nyquist's theorem. However, in the engineering practice, the oscillation modes in the frequency range of 1-decade lower than the sampling frequency seem to be identified in the best accuracy.

3. Practical Extension of ERA

In this section, we describe the problems in applying the ERA to the on-orbit identification experiment for flexible space structure.

Compared with the ground, the following two features pose a significant problems in on-orbit identification in the space environment.

(i) Any ideal impulse input cannot be realized.

(ii) Space structure is a second-order oscillating system containing the rigid body mode.

Since the ground-use vibrator cannot be used in space, space structure must be excited by existing actuators such as RCS thrusters and reaction wheels for attitude or orbit control. Therefore, an ideal impulse input cannot be generated and only a rectangular input can be realized. Furthermore, even an ideal rectangular input may be unrealizable by RCS because the influence of the mechanical delay by valve operation cannot be disregarded.

On the other hand, in the space environment, since any modal test in the boundary conditions of cantilever or both-ends support cannot be performed, the vibration mode with the natural frequency of 0 [Hz] called rigid-body mode exists. As shown later, it is hard to treat this rigid-body mode by the conventional analysis technique. To this end, identification algorithm must be improved. Moreover, since object plant such as flexible space structure is a second-order vibration system, it is desirable that such information can be taken into analysis in advance.

Henceforth, an improved identification algorithm is derived for the following continuation systems, taking the item of (i) and (ii) into consideration.

3.1 Extension to a rectangular wave input

The expression as time-invariant continuous multi-input multi-output system is considered for large space structure as well as Eq. (1) as follows:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx. \end{cases} \tag{27}$$

Supposing the initial value of the system to be $x(0) = 0$, measurement y is obtained as

$$y(t) = C\int_0^t e^{A(t-\tau)}Bu(t)d\tau. \tag{28}$$

Now we consider the rectangular input that gives unit value during the first ϵ seconds,

$$u(t) = \begin{cases} 1 & 0 \le t \le \epsilon \\ 0 & \epsilon < t. \end{cases} \tag{29}$$

Consider the sampled measurement $y_k = y(\epsilon + \delta + (k-1)\Delta t)$ which is obtained at every Δt second after an interval of δ second after rectangular input is applied. The Hankel matrices H_0 and H_1 with the size of q rows and r columns are composed as

$$H_0 = \begin{bmatrix} y_1 & y_2 & \cdots & y_r \\ y_2 & y_3 & \cdots & \\ \vdots & \vdots & \ddots & \\ y_q & & & y_{q+r-1} \end{bmatrix} = \begin{bmatrix} C \\ Ce^{A\Delta t} \\ \vdots \\ Ce^{A(q-1)\Delta t} \end{bmatrix} \times \begin{bmatrix} B^* & e^{A\Delta t}B^* & \cdots & e^{A(r-1)\Delta t}B^* \end{bmatrix} \tag{30}$$

$$H_1 = \begin{bmatrix} y_2 & y_3 & \cdots & y_{r+1} \\ y_3 & y_4 & \cdots & \\ \vdots & \vdots & \ddots & \\ y_{q+1} & & \cdots & y_{q+r} \end{bmatrix} = \begin{bmatrix} C \\ Ce^{A\Delta t} \\ \vdots \\ Ce^{A(q-1)\Delta t} \end{bmatrix} e^{A\Delta t} \times \begin{bmatrix} B^* & e^{A\Delta t}B^* & \cdots & e^{A(r-1)\Delta t}B^* \end{bmatrix} \tag{31}$$

where $B^* = e^{A(\epsilon+\delta)}\int_0^\epsilon e^{-A\tau}d\tau B$. By decomposing the Hankel matrix H_0 by SVD, $\widehat{A}$, $\widehat{B}$ and $\widehat{C}$ are realized as

$$\widehat{A} = \frac{1}{\Delta t}\log(S_1^{-1/2}U_1^T H_1 V_1 S_1^{-1/2}) \tag{32}$$

$$\widehat{B} = (e^{\widehat{A}(\epsilon+\delta)}\int_0^\epsilon e^{-\widehat{A}\tau}d\tau)^{-1}S_1^{1/2}V_1^T E_m \tag{33}$$

$$\widehat{C} = E_p^T U_1 S_1^{1/2}. \tag{34}$$

In case of realizing observability and controllability matrices in another form by SVD of Hankel matrix H_0, following expression are obtained

$$\widehat{B} = S_1^{1/2}V_1^T E_m \tag{35}$$

$$\widehat{C} = E_p^T U_1 S_1^{1/2}(e^{\widehat{A}(\epsilon+\delta)}\int_0^\epsilon e^{-\widehat{A}\tau}d\tau)^{-1}. \tag{36}$$

When the rigid-body mode does not exist in identified $\widehat{A}$, the part of $(e^{\widehat{A}(\epsilon+\delta)}\int_0^\epsilon e^{-\widehat{A}\tau}d\tau)^{-1}$ is reduced as follows:

$$e^{\widehat{A}(\epsilon+\delta)}\int_0^\epsilon e^{-\widehat{A}\tau}d\tau = \widehat{A}^{-1}e^{\widehat{A}\delta}(e^{\widehat{A}\epsilon} - I). \tag{37}$$

In this case, Eq. (35) and (36) are replaced as

$$\widehat{B} = (e^{\widehat{A}\epsilon} - I)^{-1}e^{-\widehat{A}\delta}\widehat{A}S_1^{1/2}V_1^T E_m \tag{38}$$

$$\widehat{C} = E_p^T U_1 S_1^{1/2}(e^{\widehat{A}\epsilon} - I)^{-1}e^{-\widehat{A}\delta}\widehat{A} \tag{39}$$

respectively.

Furthermore, when matrix $\widehat{A}$ has the duplicate zero natural frequencies corresponding to the rigid-body mode, it is necessary to divide $\widehat{A}$ as follows:

$$\widehat{A} = T\begin{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} & \\ & \widehat{A}_f \end{bmatrix}T^{-1} \tag{40}$$

where T is a transformation matrix dividing $\widehat{A}$ into rigid-body mode and flexible modes $\widehat{A}_f$. By the similar transform, we can express Eq. (37) as follows:

$$\begin{aligned} & e^{\widehat{A}(\epsilon+\delta)}\int_0^\epsilon e^{-\widehat{A}\tau}d\tau \\ &= T\begin{bmatrix} \begin{bmatrix} 1 & \epsilon+\delta \\ 0 & 1 \end{bmatrix} & 0 \\ 0 & e^{\widehat{A}_f(\epsilon+\delta)} \end{bmatrix} \times \begin{bmatrix} \begin{bmatrix} \epsilon & -\epsilon^2/2 \\ 0 & \epsilon \end{bmatrix} & 0 \\ 0 & \widehat{A}_f^{-1}(I - e^{-\widehat{A}_f\epsilon}) \end{bmatrix}T^{-1} \\ &= T\begin{bmatrix} \begin{bmatrix} \epsilon & \epsilon^2/2 + \epsilon\delta \\ 0 & \epsilon \end{bmatrix} & 0 \\ 0 & \widehat{A}_f^{-1}e^{\widehat{A}_f\delta}(e^{-\widehat{A}_f\epsilon} - I) \end{bmatrix} \\ &\times T^{-1}. \end{aligned} \tag{41}$$

Then $\widehat{B}$ is obtained by substituting backward Eq. (41) into (38) as

$$\widehat{B} = T\times \begin{bmatrix} \begin{bmatrix} 1/\epsilon & -1/2 - \delta\epsilon \\ 0 & 1/\epsilon \end{bmatrix} & \\ & (e^{\widehat{A}_f\epsilon} - I)^{-1}e^{-\widehat{A}_f\delta}\widehat{A}_f \end{bmatrix}$$

$$\times T^{-1} S_1^{-1/2} V_1^T E_m \quad (42)$$

and $\widehat{C}$ of Eq. (39) is also

$$\widehat{C} = E_p U_1 S_1^{1/2} T \times \left[\begin{bmatrix} 1/\epsilon & -1/2-\delta\epsilon \\ 0 & 1/\epsilon \end{bmatrix} \quad (e^{\widehat{A}_f \epsilon} - I)^{-1} e^{-\widehat{A}_f \delta} \widehat{A}_f \right] \times T^{-1}. \quad (43)$$

Thus, even if the system input is a rectangular signal and the rigid-body mode exists in the system, it is theoretically possible to identify and realize system matrix A, input-and-output matrices B and C exactly.

However, practically, even if the system matrix A is quite correctly identified, some substantial error will be remain in the input and output matrices B and C due to the measurement errors, such as the influence of the quantization error of A/D conversion and nonlinear characteristic of damping ratio depending the amplitude of vibration.

The errors mainly affect the identification of an anti-resonant point in the frequency domain power spectrum. This problem can be improved by applying the iterative least-square method as described in the next subsection.

3.2 Identification for rigid-body mode

System identification method is discussed for a general case that includes the rigid-body mode in the previous section. In case of the rigid-body mode existing in the plant, we can employ another method different from the ERA. In fact, since the frequency of the rigid-body mode is zero and there is no damping in the mode, the only parameter to be identified is modal shape of rigid-body, i.e. the element corresponding to the rigid-body mode of input and output matrix. It is known that the modal shape of the rigid-body mode is equivalent to the inverse of the square root of a moment of inertia of the spacecraft. For example, impulse response of SISO system is described as:

$$y_k = c e^{A(\epsilon+\delta+(k-1)\Delta t)} \int_0^{\epsilon} e^{-A\tau} d\tau b \quad (44)$$

Assuming that the only rigid-body mode exists in the system, Eq. (44) is expanded as follows:

$$y_k = c \begin{bmatrix} \epsilon & \epsilon^2 + \epsilon(\delta + (k-1)\Delta t) \\ 0 & \epsilon \end{bmatrix} b. \quad (45)$$

Furthermore, when the system is collocation, we obtain

$$b = \begin{bmatrix} 0 \\ \psi_r \end{bmatrix}, \qquad c = \begin{bmatrix} \psi_r & 0 \end{bmatrix}. \quad (46)$$

Substituting Eq.(46) into (45) yields

$$y_k = \psi_r^2 [\epsilon^2/2 + \epsilon(\delta + (k-1)\Delta t)]. \quad (47)$$

In order to estimate ψ_r, we consider the cost function J_2:

$$J_2 = \sum (y_k - \psi_r^2 [\epsilon^2/2 + \epsilon(\delta + (k-1)\Delta t)])^2 \quad (48)$$

made by the square sum of the difference between the estimation and actual measurement containing several vibration modes with rigid body mode. Then the modal shape ψ_r of the rigid-body mode which minimizes cost function (48) is derived by the least-squares method as

$$\widehat{\psi}_r^2 = \frac{\sum y_k [\epsilon^2/2 + \epsilon(\delta + (k-1)\Delta t)]}{\sum [\epsilon^2/2 + \epsilon(\delta + (k-1)\Delta t)]^2}. \quad (49)$$

4. Iterative algorithm

When ERA method is applied to the system identification for large space structure, system matrix A is rather easily identified accurately in almost all cases. However, it is difficult to identify input and output matrices B and C. This means the estimation of modal frequencies and modal damping ratio from the system poles are accurate but that of modal shapes are inaccurate. In order to overcome the difficulty, we consider applying iterative algorithm with least-square method to identify input and output matrices after accurate system matrix A is obtained by the ERA.

4.1 Collocated SISO system

The collocated single-input single-output system whose input and output points are same position is considered

$$\begin{cases} \dot{x} = Ax + bu \\ y = cx \end{cases} \quad (50)$$

where A, b, c are in $\boldsymbol{R}^{n\times n}$, $\boldsymbol{R}^{n\times 1}$, $\boldsymbol{R}^{1\times n}$, respectively. By the condition of collocation system, input and output matrices b and c is expressed using $\psi \in \boldsymbol{R}^{1\times n/2}$:

$$b = \begin{bmatrix} 0 \\ \psi^T \end{bmatrix}, \qquad c = \begin{bmatrix} \psi & 0 \end{bmatrix}. \quad (51)$$

Hence, identification of input and output matrices means that of modal shape matrix ψ.

Sampled measurement data y_k at time t_k after the interval δ second following the rectangular input of ϵ second applied is

$$\begin{aligned} y_k &= cx(\epsilon + \delta + (k-1)\Delta t) \\ &= c e^{A(\epsilon+\delta+(k-1)\Delta t)} \int_0^{\epsilon} e^{-A\tau} d\tau b \\ &= c S^k b \end{aligned} \quad (52)$$

where S^k is a known value uniquely determined by already identified system matrix A as follows:

$$S^k \equiv e^{A(\epsilon+\delta+(k-1)\Delta t)} \int_0^{\epsilon} e^{-A\tau} d\tau \quad (53)$$

Substituting Eq. (53) into Eq. (52), measurement data y_k yields

$$y_k = \begin{bmatrix} \psi & 0 \end{bmatrix} S^k \begin{bmatrix} 0 \\ \psi^T \end{bmatrix} = \psi S^k(1,2)\psi^T \quad (54)$$

where $S^k(1,2)$is defined as a submatrix constructed by (1,2) block of S^k as:

$$S^k(1,2) = \begin{bmatrix} I_{n/2\times n/2} & O \end{bmatrix} S^k \begin{bmatrix} O \\ I_{n/2\times n/2} \end{bmatrix}. \quad (55)$$

By the iterative calculation, we attempt to revise input and output matrices based on the criteria of the least-square sense:

$$J = \sum_{k=1}^{m} (y_k - \widehat{\psi} S^k(1,2)\widehat{\psi})^2 \quad (56)$$

The minimization of the cost by optimal $\widehat{\psi}$ is achieved by the following procedure.

Step 1 : Assume the initial value of $\widehat{\psi}$ as $\widehat{\psi_0}$. For example, this may be referred as the value $\widehat{b}$ or $\widehat{c}$ obtained by previous section. Updating the modal shape $\delta\widehat{\psi}$ using error from the true $\widehat{\psi_0}$:

$$\widehat{\psi} = \widehat{\psi_0} + \delta\widehat{\psi} \quad (57)$$

Step 2 : Substituting Eq.(57) into Eq.(54), we obtain scalar measurement $\delta\widehat{\psi}$ as

$$\begin{aligned} y_k &= (\widehat{\psi}_0 + \delta\widehat{\psi}) S^k(1,2)(\widehat{\psi}_0^T + \delta\widehat{\psi}^T) \\ &\simeq \widehat{\psi}_0 S^k(1,2)\widehat{\psi}_0^T + \\ &\quad \delta\widehat{\psi}\psi_0(S^k(1,2) + S^{k^T}(1,2))\widehat{\psi}_0^T \end{aligned} \quad (58)$$

where it is assumed that second order terms are negligibly small.

Then, the measurement error Δy_k between current estimated value and actual measurement becomes

$$\Delta y_k = y_k - \widehat{\psi}_0 S^k(1,2)\widehat{\psi}_0 \quad (59)$$

Substituting Eq. (58) into Eq. (59) yields

$$\Delta y_k = \delta\widehat{\psi}(S^k(1,2) + S^{k^T}(1,2))\widehat{\psi}_0^T = \delta\widehat{\psi} P^k \quad (60)$$

where P^k is defined as

$$P^k = (S^k(1,2) + S^{k^T}(1,2))\widehat{\psi}_0^T \quad (61)$$

Step 3 : Gathering ℓ measurement errors of Eq. (61), we get following relation:

$$\begin{aligned} &\begin{bmatrix} \Delta y_1 & \Delta y_2 & \cdots & \Delta y_\ell \end{bmatrix} \\ &= \delta\widehat{\psi} \begin{bmatrix} P^1 & P^2 & \cdots & P^\ell \end{bmatrix}. \end{aligned} \quad (62)$$

Finally, applying the least square method into Eq. (62), $\delta\widehat{\psi}$ is obtained as:

$$\begin{aligned} \delta\widehat{\psi} = &\begin{bmatrix} \Delta y_1 & \Delta y_2 & \cdots & \Delta y_\ell \end{bmatrix} \times \\ &\begin{bmatrix} P^1 & P^2 & \cdots & P^\ell \end{bmatrix}^+. \end{aligned} \quad (63)$$

We modify the error $\delta\widehat{\psi}$ by repeating this procedure until the residual error becomes sufficiently small.

4.2 Non-collocated SIMO system

Next, the non-collocated single-input multi-output system whose input and output points are different position is considered

$$\begin{cases} \dot{x} = Ax + bu \\ y = Cx \end{cases} \quad (64)$$

where the size of matrices A, b, C are $\boldsymbol{R}^{n\times n}$, $\boldsymbol{R}^{n\times 1}$ and $\boldsymbol{R}^{p\times n}$, respectively.

Input and output matrices b and C are constructed by the modal shape matrices of input and output points $\phi \in \boldsymbol{R}^{1\times n/2}$ and $\Psi \in \boldsymbol{R}^{p\times n/2}$ as follows:

$$b = \begin{bmatrix} 0 \\ \phi^T \end{bmatrix}, \qquad C = \begin{bmatrix} \Psi & 0 \end{bmatrix} \quad (65)$$

System matrix A for second-order vibration system such as flexible space structures has the following structure:

$$A = \begin{bmatrix} O & I \\ -\omega^2 & -2\zeta\omega \end{bmatrix}. \quad (66)$$

Hence, transfer function in s plane between input and output of the system is expressed as

$$\frac{y}{u} = \sum \frac{\psi\phi^T}{s^2 + 2\zeta_j\omega_j s + \omega_j^2} \quad (67)$$

where ζ_j and ω_j are the j-th element of diagonal matrices of ζ and ω in Eq. (66). From Eq. (67), it is clear that the only the product value of ϕ and Ψ is able to be identified and it is impossible to distinguish input and output matrices b and C.

Therefore, assuming that the non-zero element ϕ of input matrix b be all unit, we obtain

$$b = \begin{bmatrix} 0 \\ U_{n/2\times 1} \end{bmatrix} \quad (68)$$

where $U_{n/2\times 1} \in \boldsymbol{R}^{n/2\times 1}$ is a column matrix whose entries are all unit. In this case, sampled measurement y_k is obtained as:

$$y_k = \widehat{\Psi}_0 S^k(1,2) U_{n/2\times 1}. \quad (69)$$

Based on the above assumption, we propose new high-performance algorithm identifying the modal shapes Ψ at the non-collocated multi-output points as elements of output matrix C by iterative procedure.

Step 1 : Assume the initial value of $\widehat{\Psi}$ as $\widehat{\Psi_0}$. Update the modal shape $\delta\widehat{\Psi} \in \boldsymbol{R}^{p\times n/2}$ using error from the true $\widehat{\Psi_0}$:

$$\widehat{\Psi} = \widehat{\Psi_0} + \delta\widehat{\Psi} \quad (70)$$

Step 2 : Substituting Eq. (70) into Eq. (69), we get multiple measurements $y_k \in \boldsymbol{R}^{p\times 1}$ as

$$\begin{aligned} y_k &= (\widehat{\Psi}_0 + \delta\widehat{\Psi})S^k(1,2)U_{n/2\times 1} \\ &= \widehat{\Psi}_0 S^k(1,2)U_{n/2\times 1} + \delta\widehat{\Psi}_0 S^k(1,2)U_{n/2\times 1} \end{aligned} \quad (71)$$

Measurement error Δy_k between actual measurement and current estimation is defined as:

$$\Delta y_k = y_k - \widehat{\Psi}_0 S^k(1,2)U_{n/2\times 1} \quad (72)$$

From Eq. (71) we obtain

$$\Delta y_k = \delta\widehat{\Psi} S^k(1,2)U_{n/2\times 1} = \delta\widehat{\Psi} R^k \quad (73)$$

where $R^k = S^k(1,2)U_{n/2\times 1}$ is defined.

Step 3 : Gathering ℓ measurement errors in rows, we get following relation:

$$\begin{aligned} &\begin{bmatrix} \Delta y_1 & \Delta y_2 & \cdots & \Delta y_\ell \end{bmatrix} \\ &= \delta\widehat{\Psi} \begin{bmatrix} R^1 & R^2 & \cdots & R^\ell \end{bmatrix} \end{aligned} \quad (74)$$

Finally, applying the least square method into Eq. (74), $\delta\widehat{\Psi}$ is obtained as:

$$\begin{aligned} \delta\widehat{\Psi} = &\begin{bmatrix} \Delta y_1 & \Delta y_2 & \cdots & \Delta y_\ell \end{bmatrix} \times \\ &\begin{bmatrix} R^1 & R^2 & \cdots & R^\ell \end{bmatrix}^+ \end{aligned} \quad (75)$$

We modify the error $\delta\widehat{\Psi}$ by repeating this procedure until the residual error becomes sufficiently small.

5. Experimental Demonstration using Air-bearing Table

Before performing on-orbit identification experiments, the proposed identification method was evaluated through the ground-based air-bearing table test assembly shown in **Fig. 1**. The table is supported by high pressure air and has a single degree of freedom in vertical direction. Two flexible isogrid panels, 1.2 [m] in length and 0.5 [kg] in weight per each, are mounted on the table by rigid joints. Total length from center of table to the tip of the panel is about 3.1 [m].

As an actuator, uncontact blushless DC motor (AEROFLEX: model V34Y-27) is used, whose capability of maximum peak torque is 0.381 [Nm] and maximum continuous torque is 0.268 [Nm] in the flat torque range of ± 60 degree bounds. As sensors, magnetic rotary encoder with 0.005 degree resolution is set to vertical axis of the table to measure attitude angle of the table.

In the identification experiment, 100 [msec] rectangular pulse signal is applied into the motor and output signals

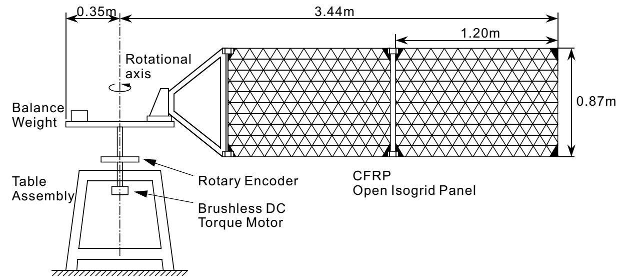

Fig. 1 Flexible CFRP panel mounted on single axis air-bearing table test assembly

from the encoder and accelerometers sampled in 50 [msec] interval are analyzed by ERA.

Dynamics equation for rotational motion of the assembly is derived in the form of non-constrained modal model [10] as follows:

$$\begin{cases} M\ddot{\eta} + D\dot{\eta} + K\eta = Fu \\ y = E\eta. \end{cases} \quad (76)$$

Mass matrix M, damping matrix D, stiffness matrix K, modal shape matrices for input and output points F, E are defined as:

$$M = U_{5\times 5} \quad (77)$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 2\zeta\omega \end{bmatrix} \quad (78)$$

$$K = \begin{bmatrix} 0 & 0 \\ 0 & \omega^2 \end{bmatrix} \quad (79)$$

$$F = \begin{bmatrix} \psi_r \\ \psi_e \end{bmatrix} \quad (80)$$

$$E = F^T. \quad (81)$$

where $\zeta$is modal damping ratio, ω modal frequency, ψ_r,ψ_e rigid-body mode shape and vibration mode shape, respectively.

Numerical values of each variables are obtained by FEM modal analysis as:

$$\zeta = diag\{\ 0.005 \quad 0.005 \quad 0.005 \quad 0.005\ \} \quad (82)$$

$$\omega = diag\{\ 7.929 \quad 25.12 \quad 26.17 \quad 74.33\ \} \quad (83)$$

$$\psi_r = 0.3088 \quad (84)$$

$$\psi_e = \begin{bmatrix} -0.3455 & -0.07978 & 0.02855 & 0.003984 \end{bmatrix}^T. \quad (85)$$

The assembly has a degree-of-freedom of rotation along the vertical axis and is a collocated system since input and output points are both about the rigid rotational axis.

In the experiment 0.1-second rectangular input is applied for excitation. For the reference, identification results are presented using the conventional ERA technique without considering rectangular input nor performing iterative least square method in **Fig. 2** and that with considering rectangular input only in **Fig. 3** by thick green line. In both cases, sampling frequency is 20 [Hz] with controlling rigid-body mode in order to stabilize the free motion of the table. This controller is made up by a PD controller with narrow band designed not to influence the flexible modes. It is clear that resonance frequencies are well in agreement, however, total level gain curve and phase curve in Bode plots are not identified correctly in Fig. 2 and Fig. 3.

On the other hand, in the following examples of Case 1∼4, the input-and-output matrices are reconfigured by

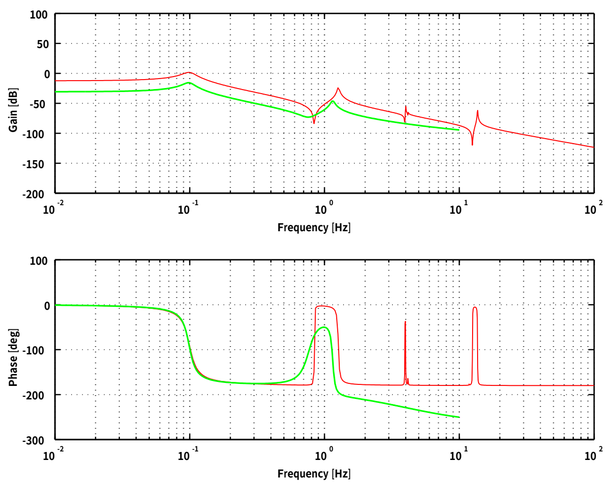

Fig. 2 System transfer function identified by conventional ERA (20 [Hz] sampling) by thick green line, nominal model by thin red line

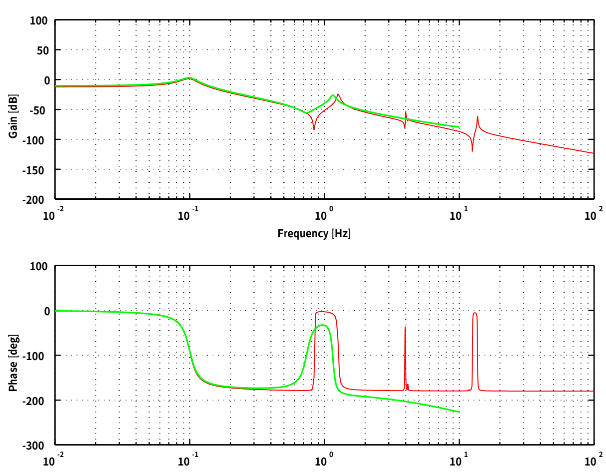

Fig. 3 System transfer function identified by ERA considering rectangular input (20 [Hz] sampling) by thick green line, nominal model by thin red line

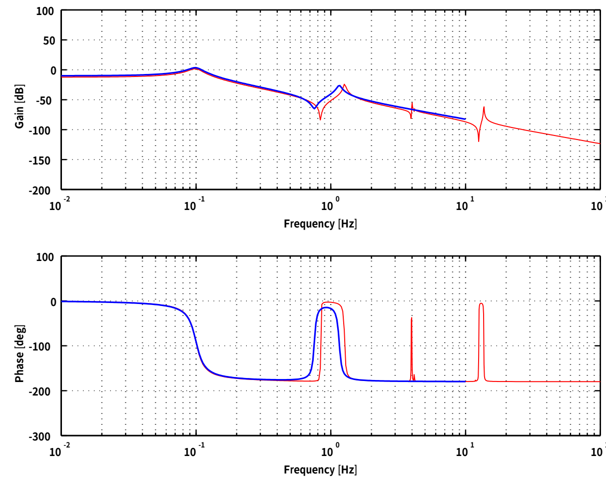

Fig. 4 Case1: system transfer function including controller identified by the proposed method (20 [Hz] sampling) by thick blue line, nominal model by thin red line

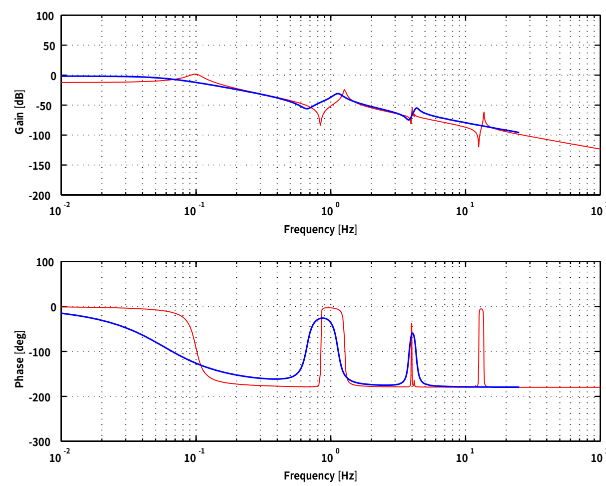

Fig. 5 Case 2: system transfer function including controller identified by the proposed method (50 [Hz] sampling) by thick blue line, nominal model by thin red line

the iterative least square method proposed in the previous section. In Case 1(**Fig. 4**) and Case 2 (**Fig. 5**), rigid-body mode is stabilized by weak PD controller and their sampling frequencies are 20 [Hz], 50 [Hz], respectively. In Case 3(**Fig. 6**) and Case 4 (**Fig. 7**), rigid-body mode is not controlled so that angle of air-table is moving in one direction with vibration and their sampling frequencies are 20 [Hz], 50 [Hz], respectively. In each plots, thick blue lines show the frequency response function of the identification results and thin red lines show that of nominal plant model which resonance frequencies are 1.262, 3.998, 4.165, 11.83, 13.65 [Hz] from lower. **Table 1** and **Table 2** summarize the identified frequencies, damping ratios and

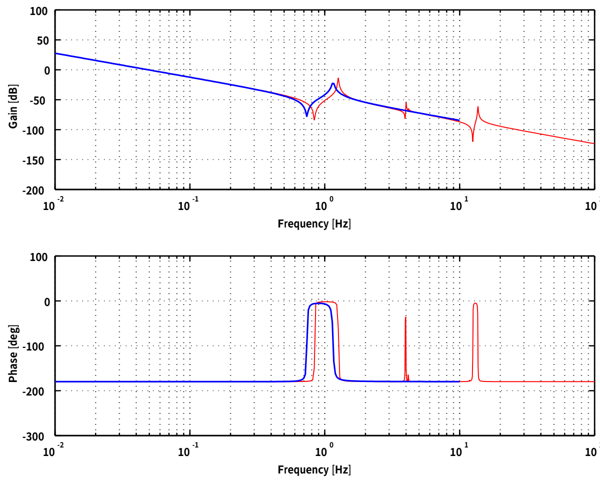

Fig. 6 Case 3: system transfer function without controller identified by the proposed method (20 [Hz] sampling) by thick blue line, nominal model by thin red line

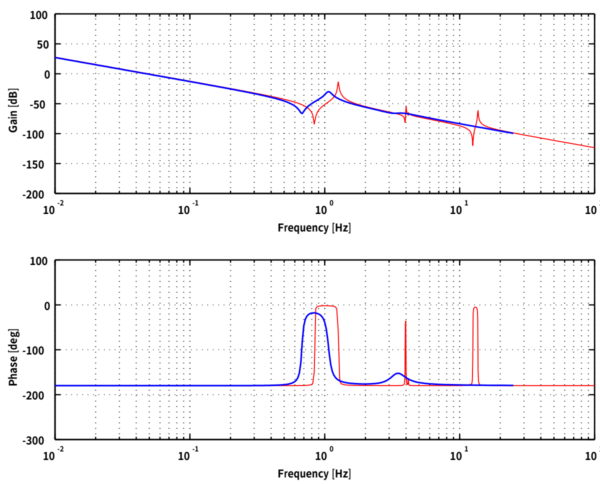

Fig. 7 Case 4: system transfer function without controller identified by the proposed method (50 [Hz] sampling) by thick blue line, nominal model by thin red line

modal shapes of air-table attitude angle. All the results show that the techniques considering rectangular input and performing iterative least square method proposed in previous sections are effective in identification of a space structure.

6. Conclusions

The theory and applications of a method of system identification using Hankel matrices in the time domain were summarized in this paper. The proposed new iterative algorithm by least square method is suitable for practical applications in the space environment. Prior to the actual on-orbit flight experiments, ground-based experiments on air-bearing table attached by the flexible isogrid panel are conducted to show the validity of the new algorithm. The proposed method were applied to the on-orbit system identification experiments on Engineering Test Satellite-VI(ETS-VI) that was launched in 1994 and the experiments were successfully completed in 1995.

Table 1 Identified modal frequencies (ω_i), damping ratios (ζ_i) and modal shapes (ϕ_i) including controller in Case 1 and 2

	controller mode		
case	ω_i [Hz]	ζ_i [%]	ϕ_i
nominal	0.100	0.100	0.309
Case 1	0.100	0.106	0.351
Case 2	0.058	0.764	0.328
	first mode		
case	ω_i [Hz]	ζ_i [%]	ϕ_i
nominal	1.262	0.004	-0.346
Case 1	1.153	0.032	-0.410
Case 2	1.127	0.069	-0.447
	second mode		
case	ω_i [Hz]	ζ_i [%]	ϕ_i
nominal	3.998	0.002	-0.080
Case 1	—	—	—
Case 2	4.296	0.036	-0.299

Table 2 Identified modal frequencies (ω_i), damping ratios (ζ_i) and modal shapes (ϕ_i) without controller in Case 3 and 4

	rigid-body mode		
case	ω_i [Hz]	ζ_i [%]	ϕ_i
nominal	0.000	0.000	0.309
Case 3	0.000	0.000	0.309
Case 4	0.000	0.000	0.300
	first mode		
case	ω_i [Hz]	ζ_i [%]	ϕ_i
nominal	1.262	0.004	-0.346
Case 3	1.151	0.013	-0.371
Case 4	1.066	0.045	-0.362
	second mode		
case	ω_i [Hz]	ζ_i [%]	ϕ_i
nominal	3.998	0.002	-0.080
Case 3	—	—	—
Case 4	3.583	0.171	-0.198

References

1) NAL and NASDA, "Final Report on Flexible Spacecraft ETS-VI On-orbit Experiment," NAL STR8-8911, NASDA SE-601017 (1989).
2) Komatsu, K. and Sano, M. "On Modal Testing for Structures," National aerospace Laboratory Technical Memorandum, TM-466 in Japanese (1982).
3) Juang, J.-N. and Pappa, R.S., "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction," J. Guidance, **8**-5, 620/627 (1985).
4) Juang, J.-N. and Pappa, R.S., "Effects of Noise on Modal Parameters Identified by Eigensystem Realization Algorithm," J. Guidance, **9**-3, 294/303 (1986).
5) Horta, L.G. and Juang, J.-N., "Identifying Approximate Linear Model for Simple Nonlinear Systems," J. Guidance, **9**-4, 385/390 (1986).
6) Ibrahim, S.R. and Mikulcik, E.C. "A Method for the Di-

rect Identification of Vibration Parameters from the Free Response," Shock and Vibration Bulletin, **47**-4, 183/198 (1977).
7) Ibrahim, S.R. and Pappa R.S., "Large Modal Survey Testing Using the Ibrahim Time Domain (ITD) Identification Technique," AIAA Paper 81-0528 (1981).
8) Adachi, S., Shigemasa, T., Yamaguchi, I. and Kida, T, "System Identification Experiment on Large Space Structure," Transaction on SICE, **26**-11 in Japanese (1990).
9) SICE editing, "Automatic Control Handbook, Basic Volume," Ohmsha (1983).
10) Likins, P.W., "Dynamics and Control of Flexible Space Vehicles," Jet Propulsion Laboratory TR 32-1329, Revision 1 (1970).

Isao Yamaguchi (Member)

He received B.S., M.S. and the Ph.D. degrees in aeronautical engineering in 1981, 1983 and 1997, respectively from the University of Tokyo. He became a researcher of National Aerospace Laboratory (NAL) in 1983. Now he is a leader of Flexible Structures Group of Structures and Materials Research Center in NAL. His research interests are dynamics, system identification, modeling and control of spacecraft and large space structures. He is a recipient of the 1993 and 1997 paper awards of The Society of Instrument and Control Engineers (SICE) and the 1997 paper award of The Japan Society of Mechanical Engineers (JSME). He is a member of SICE, JSME, The Japan Society for Aeronautical and Space Sciences, AIAA and IEEE.

Takashi Kida (Member)

He received his B.S and Ph.D. degrees in mathematical engineering in 1973 and 1994, respectively from the University of Tokyo. He joined National Aerospace Laboratory from 1973 to 1996. Since 1996, he is a professor of the Department of Mechanical Engineering and Intelligent Systems, University of Electro-Communication. His research interests are dynamics and control of spacecraft. He is a recipient of the 1993 and 1997 paper awards of The Society of Instrument and Control Engineers (SICE). He is a member of SICE, JSME, The Japan Society for Aeronautical and Space Sciences, AIAA and IEEE.

Translated from Trans. of the SICE, Vol. 28 No. 8, 982/990 1992