Trans. of the Society of Instrument and Control Engineers
Vol.E-1, No.1, 27/33 (2001)

Fault Diagnosis of Linear Dynamical Systems†

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This paper considers a fault diagnosis problem of linear dynamical systems. Specifically, we derive conditions for fault detectability and fault distinguishability, and also we derive a fault diagnosis algorithm which decides the faulty element from the system observation.

As the main result of the paper, graphical necessary and sufficient conditions for fault distinguishability are derived. The conditions are given in terms of the system representation graphs which depict the system structures. The conditions are essential in carrying out the fault diagnosis algorithm at two stages: to check the distinguishability assumed in the algorithm; to find the covering set defined in the algorithm. An example is given to see how the algorithm works.

Key Words: fault diagnosis, linear dynamical system, graph theory, system structure

1. Introduction

This paper studies the fault diagnosis problem of linear dynamical systems. Specifically, we derive conditions for fault detectability and fault distinguishability, and also we derive a fault diagnosis algorithm which decides the faulty element from the system observation.

This paper employs an analytical method that uses system models. If we were to identify the system parameters such as elements of system matrices then we need complicated nonlinear calculation. The fault diagnosis method in this paper avoids parameter identification and uses linear calculation alone.

The method is originally proposed for the diagnosis of linear electrical circuits. If the circuits has a faulty element, then the observation vector is confined to a subspace corresponding to the fault. Fault detectability and distinguishability conditions are stated in terms of these subspaces [1)]. In 2), 3), these conditions are successfully translated into graph conditions that reflect electrical circuit structure. In 4)∼7), the study was extended to linear systems that are not necessarily electrical circuits. In 7) in particular, a fault diagnosis algorithm was proposed. If the system is described by linear algebraic equations such as static linear systems, graph distinguishability condition can be exploited in carrying out the algorithm.

In this paper, we apply the algorithm in 7) to linear dynamical systems. For this purpose we derive fault diagnosis conditions that are essential in (i) verifying the fault distinguishability condition required in the algorithm, and (ii) deciding a cover set in the algorithm. Some examples are included to show how to apply the conditions.

This paper is organized as follows. In Section 2, we review three kinds of system description and examine system behavior when a fault occurs. In Section 3, we show that the difference of observation vectors of faulty and normal systems are confined to a subspace corresponding the fault. A fault distinguishability condition is derived using the subspace. In Section 4, we briefly explain the system structure and representation graphs. Then we give graphical fault distinguishability conditions as a main result of the paper. In Section 5, the fault diagnosis algorithm in 7) is applied to linear dynamical systems. Some examples are included.

2. Fault diagnosis problem

In this section, we formulate a fault diagnosis problem of linear dynamical systems. We first describe the system description, and study the behavior of the faulty system. Then we introduce the concept of detectability and distinguishability of faults, and discuss what is required for a fault diagnosis algorithm.

2.1 System description

In this paper, we use three forms of system description; namely, the state space form, the descriptor form, and the interconnected form.

(s) State space form:

$$\dot{x} = Ax + Bu, \tag{1.s}$$

$$y = Cx, \tag{2.s}$$

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where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times r}, C \in \mathbb{R}^{m\times n}$ are the system matrices, $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^r$ is the input, and $y \in \mathbb{R}^m$ is the output.

(d) Descriptor form:

$$E\dot{x} = Ax + Bu, \tag{1.d}$$

$$y = Cx, \tag{2.d}$$

where $E \in \mathbb{R}^{n\times n}, A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times r}, C \in \mathbb{R}^{m\times n}$ are the system matrices, $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^r$ is the input, and $y \in \mathbb{R}^m$ is the output. We assume that $\det(sE - A) \not\equiv 0$.

(c) Interconnected form. Consider the interconnected system composed of n single-input single-output systems. The input-output characteristic of the i-th component is described as

$$x_i(s) = g_i(s)u_i(s),$$

where x_i is the output, u_i is the input, and g_i is the transfer function of the i-th subsystem ($i \in \mathbf{n} := \{1, \cdots, n\}$). The overall system description is

$$x(s) = G(s)u(s), \tag{1.c}$$

$$u(s) = Lx(s) + Bv(s), \tag{1.c$'$}$$

$$y(s) = Cx(s), \tag{2.c}$$

where

$$x(s) = \begin{bmatrix} x_1(s) \\ \vdots \\ x_n(s) \end{bmatrix}, \quad u(s) = \begin{bmatrix} u_1(s) \\ \vdots \\ u_n(s) \end{bmatrix},$$

$$G(s) = \begin{bmatrix} g_1(s) & & 0 \\ & \ddots & \\ 0 & & g_n(s) \end{bmatrix},$$

and $L \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, $C \in \mathbb{R}^{m\times n}$ are the system matrices. The matrix L denotes the interconnection between subsystems. We assume that $\det(I - G(s)L) \not\equiv 0$.

2.2 Fault diagnosis problem

Suppose that the system (1) has a faulty component which results in a failure in one of the equation, where (1) denotes one of (1.s), (1.d), and (1.c). Hence when the system is faulty,

$$\dot{x} = Ax + Bu + \epsilon, \tag{3.s}$$

$$E\dot{x} = Ax + Bu + \epsilon, \tag{3.d}$$

$$x = Gu + \epsilon, \tag{3.c}$$

holds for some ϵ which have zero elements but for the i-th element. We call ϵ a fault vector. One of the advantage of defining such a fault vector is that a simple algorithm exists to distinguish which equation has failed as is shown in the rest of the paper. This does not mean that we can specify the reason of the equation failure. For example, there might be actuator errors, unexpected disturbances, and parameter change of the system matrices.

The fault diagnosis problem studied in the rest of the paper is stated as follows. Given the observation by (2), decide whether the faulty vector ϵ is non-zero (fault detectability), or decide which element of ϵ is non-zero (fault distinguishability), and construct an algorithm to make such decisions.

3. Fault distinguishability condition

In this section, we show that the difference of observation vectors of the faulty and normal systems is confined to a subspace corresponding to the fault (we shall call such a subspace as fault observation subspace), and that this fact can be used to diagnose faults. Then we mention a distinguishability condition for faults.

Let us see the behavior of the observation vectors when a fault occurs in the systems. To do this, we take the Laplace transform for (s) and (d). Then (1), (2) and (3) become

$$(sI - A)\,x(s) = Bu(s) + \epsilon(s) + x_0, \tag{4.s}$$

$$y(s) = Cx(s), \tag{5.s}$$

$$(sE - A)\,x(s) = Bu(s) + \epsilon(s) + Ex_0, \tag{4.d}$$

$$y(s) = Cx(s), \tag{5.d}$$

$$(I - G(s)L)\,x(s) = G(s)Bv(s) + \epsilon(s), \tag{4.c}$$

$$y(s) = Cx(s). \tag{5.c}$$

From (4) and (5), it follows that the difference $\Delta y(s) = y_f(s) - y_n(s)$ of the observation $y_f(s)$ of the faulty system and the observation $y_n(s)$ of the normal system, *i.e.* $\epsilon = 0$, satisfies

$$\Delta y(s) = C\,(sI - A)^{-1}\,\epsilon(s), \tag{6.s}$$

$$\Delta y(s) = C\,(sE - A)^{-1}\,\epsilon(s), \tag{6.d}$$

$$\Delta y(s) = C\,(I - G(s)L)^{-1}\,\epsilon(s), \tag{6.c}$$

respectively.

Hence if we take $(sI - A)$, $(sE - A)$, and $(I - G(s)L)$ as the system matrix in 7) and unit vectors $e_1, \cdots, e_n$ as the fault element vectors, we can apply the results in 7).

Let $J \subset \mathbf{n}$. If the fault vector ϵ in (3) lies in the subspace spanned by $\{e_i, i \in J\}$, we say the fault J occurs. Let $|J|$ denote the number of elements in J. If $|J| = k$, we say J is a k-th order fault. When the fault J occurs, form (6) we see that Δy lies in the following fault observation subspace $\mathcal{S}(J)$:

$$\mathcal{S}(J) = \mathrm{span}_{\Lambda\mathbb{R}} \left\{ C\,(sI - A)^{-1}\, e_i, i \in J \right\}, \tag{7.s}$$

$$\mathcal{S}(J) = \mathrm{span}_{\Lambda\mathbb{R}} \left\{ C\,(sE - A)^{-1}\, e_i, i \in J \right\}, \tag{7.d}$$

$$\mathcal{S}(J) = \mathrm{span}_{\Lambda\mathbb{R}} \left\{ C\,(I - G(s)L)^{-1}\, e_i, i \in J \right\}, \tag{7.c}$$

where $\mathrm{span}_{\mathcal{K}} X$ is the linear span of the vectors in X with the field $\mathcal{K}$, and $\Lambda\mathbb{R}$ denote the field of formal Laurent series at $s = \infty$ having the maximal power. If $z', z'' \in \Lambda\mathbb{R}$, and $z' = \sum_{t=t'_0}^{\infty} z'_t s^{-t}$, $z'' = \sum_{t=t''_0}^{\infty} z''_t s^{-t}$, then the sum is power-wise addition, $z = z' + z'' = \sum_{t=t_0}^{\infty} (z'_t + z''_t)\, s^{-t}$, $t_0 = \max\{t'_0, t''_0\}$, and the product id the convolution, $z = z'z'' = \sum_{t=t_0}^{\infty} z_t s^{-t}$, $t_0 = t'_0 + t''_0$, $z_t = \sum_{j=t'_0}^{t-t''_0} z'_j z''_{t-j}$.

We will determine which fault has occurred based on which fault observation subspace contains the observation vector Δy. For this purpose, we have to consider the situation where for $J_1 \neq J_2$ (i) $\Delta y \in \mathcal{S}(J_1) \cap \mathcal{S}(J_2)$, and (ii) $\mathcal{S}(J_1) = \mathcal{S}(J_2)$.

We shall assume the following in view of (i), which effectively says that the elements of the fault vector ϵ are independent.

Assumption H. If the fault J_1 occurs and $\mathcal{S}(J_2) \not\supset \mathcal{S}(J_1)$, then $\Delta y \notin \mathcal{S}(J_2)$.

As for (ii), notice that the fault vector ϵ is an element of the n-dimensional space $\Lambda\mathbb{R}^n$, and the fault observation subspace is in the m-dimensional space $\Lambda\mathbb{R}^m$ ($m < n$). Hence $J_1 \neq J_2$ does not imply that $\mathcal{S}(J_1) \neq \mathcal{S}(J_2)$ in general. However, if the following condition (k-distingusihability) is satisfied and Assumption H holds, then it is possible to diagnose from the observation vector Δy whether the system has a fault of order less than or equal to k, and, if this is affirmative, which fault has occurred [7].

Definition (k-distinguishability).
The system (1),(2) is called k-distinguishable if J_1 and J_2 are distinct faults whose orders are less than or equal to k.

A condition for the k-distinguishability is stated in terms of the fault observation subspaces.

Proposition 1 (k-distinguishability condition [2]).
The system (1),(2) is k-distinguishable ($k < n$) if and only if $\dim \mathcal{S}(J) = k + 1$ for any fault J of order $k + 1$.

4. Graph conditions for distinguishability

In this section, we first mention the notion of interconnection between subsystems (system structure), and introduce system representation graphs for the system structure. Then we derive graphical distinguishability conditions. These conditions are useful when we apply the diagnosis algorithm in 7) at the following stages: (i) to check the distinguishability, and (ii) to derive so-called cover sets.

4.1 System structure and representation graph

The three types of dynamical systems have been discussed in Section 2.1. If we regard each equation in (1) as a subsystem, then we define system structure by interconnection between subsystems.

If we regard the i-th equation of the state space form (1.s) as the i-th system component, then $a_{ij} \neq 0$ ($A = (a_{ij})$) means that the i-th component is affected by the j-th component. If we regard the i-th equation of the descriptor form (1.d) as the i-th system component, then $e_{ij} \neq 0$ ($E = (e_{ij})$) or $a_{ij} \neq 0$ ($A = (a_{ij})$) means that the i-th component is affected by the j-th component. If we regard the i-th equation of the interconnected form (1.c) as the i-th system component, then $l_{ij} \neq 0$ ($L = (l_{ij})$) means that the i-th component is affected by the j-th component. Hence non-zero elements of the matrix A (E and A, or L) represent interconnection between system components, and therefore we call the pattern of non-zero elements as the system structure.

The matrix C in the observation equation (2) represents the relation between the variable (or state) x and the observation y. We observe m out of n elements of x, which we shall call the observation variables. Then the rows of C consist of unit vectors and the matrix C is of full row rank. We call the pattern of non-zero elements of C as the system (observation) structure.

Graphs are useful to represent the system structure as follows. For the state space form (1.s), (2.s), we use the Coates graph $G_A(N, B_A)$ and the modified bipartite graph $G_b(N_r, N_c, B_b)$. The node set N of the Coates graph G_A has one-to-one correspondence with the variables $x_1, \cdots, x_n$. The branch set B_A is defined as $(i, j) \in B_A$ if and only if $a_{ij} \neq 0$. Note that G_A is a directed graph. The node sets N_r and N_c of the bipartite graph G_b have one-to-one correspondence with the rows and columns of the matrix A, respectively. The branch set B_b is defined as $(i_r, j_c) \in B_b$ if and only if $a_{ij} \neq 0$ or $i_c = j_c$. We shall use the subscript r and c to represent rows and columns of matrices. Note that G_b is an undirected graph. Let $J \subset \mathbf{n}$ be a fault. Let us slightly abuse the notation and denote $J \subset N$. Also corresponding to the observation matrix C, we shall define $C \subset N$ as the set $\{i : \text{the } i\text{-th row of the matrix } C \text{ is non-zero}\}$. We refer C as the observation node set. We shall use similar notations for subsets of N_r and N_c: for example, $J_r \subset N_r$ and $C_c \subset N_c$. Let $\overline{C}_c = N_c \setminus C_c$ be the complement of

C_c.

For the descriptor form (1.d), (2.d), we use the bipartite graph $G_b(N_r, N_c, B_b)$ associated with the matrices E and A. The node sets N_r and N_c of the bipartite graph G_b have one-to-one correspondence with the rows and columns, respectively, of the matrices A and E. Note that the rows of the matrices correspond to the equations in (1.d) and the columns to the variables of x. The branch set B_b is defined as $(i_r, j_c) \in B_b$ if and only if $e_{ij} \neq 0$ or $a_{ij} \neq 0$. Notations such as J_r for a fault J are defined as in the state space form.

For the interconnected form (1.c), (1.c'), (2.c), we use the Coates graph $G_L(N, B_L)$ for the matrix L. The node and branch sets are defined similarly as in the state space form.

4. 2 g-distinguishability condition

In this section, the k-distinguishability condition (Proposition 1) is paraphrased as conditions on the representation graphs defined in Section 4. 1. Advantages of using the graphs are the following: (i) the distinguishability of faults is determined by the system structure, (ii) the decision on which variables to observe is possible in view of (i), and (iii) techniques of graph theory can be applied both in the verification of the distinguishability and in the decision of the so-called cover set in the fault diagnosis algorithm proposed in 7).

The graphical distinguishability conditions are given as a generic [8] property of the non-zero elements of A for the state space form and the non-zero elements of E and A for the descriptor form. A generic property is a property which holds for almost all parameter values. Henceforce, we shall use the suffix "g-" to denote things which hold generically.

We shall use the following terminologies for the graphs. A directed path in the directed graph $G(N, B_A)$ is an alternate sequence of nodes and branches of the form $(n_0, b_1, \cdots, b_k, n_k)$, $n_i \in N$, $0 \leq i \leq k$ and $b_i = (n_{i-1}, n_i) \in B_A$, $1 \leq i \leq k$, $k \geq 0$, where n_0 is the initial node and n_k is the terminal node. Note that the path consisting of a single node (n_0) is a directed path. Two directed paths are disjoint if they share no nodes in common. A set of directed paths are disjoint if any two directed paths in the set are disjoint. Let $J, C, D \subset N$. We say that D separates C from J if there is no directed path whose nodes are in $\overline{D}$ with the initial node in J and the terminal node in C. A subset $M \subset B_b$ in the bipartite graph $G_b(N_r, N_c, B_b)$ is called a matching if any two branches in M share no node. A subset $X \subset N_r \cup N_c$ is said to be saturated by a matching M if for any node in X there is a branch in M connecting to the node.

Theorem 1-s (state space form, g-k-distinguishability). The following four statements are equivalent.

(i) The sate space form (1.s), (2.s) is g-k distinguishable.

(ii) For any $J \subset N$ with $|J| = k+1$, there is a set of $k+1$ disjoint paths from J to C in G_A.

(iii) For any $J \subset N$ with $|J| = k+1$, the minimum cardinality of a subset of N which separates J form C in G_A is $k+1$.

(iv) For any $J_r \subset N_r$ with $|J_r| = k+1$, there is a matching of $\overline{C}_c$ and $\overline{J}_r$ in G_b which saturates $\overline{C}_c$.

Proof. Note that (ii), (iii), and (iv) are necessary and sufficient conditions for g-$\dim \mathcal{S}(J) = k+1$ [9]. This and Proposition 1 prove the theorem. □

Theorem 1-d (descriptor form, g-k-distinguishability). The descriptor form (1.d), (2.d) is g-k distinguishable if and only if for any $J_r \subset N_r$ with $|J_r| = k+1$ there is a matching of $\overline{C}_c$ and $\overline{J}_r$ in G_b which saturates $\overline{C}_c$.

Proof. Notice that

$$\operatorname{rank}\begin{bmatrix} sE-A & J \\ C & 0 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} sE-A & J \\ 0 & -C\,(sE-A)^{-1}\,J \end{bmatrix}$$
$$= n + \dim \mathcal{S}(J),$$

where J in the matrices represents the $n \times (k+1)$ matrix consisting of column unit vectors corresponding to the set J_r. On the other hand,

$$\operatorname{rank}\begin{bmatrix} sE-A & J \\ C & 0 \end{bmatrix}$$
$$= \operatorname{rank}\begin{bmatrix} (I-JJ^{\mathrm{T}})(sE-A)(I-C^{\mathrm{T}}C) & J \\ C & 0 \end{bmatrix}$$
$$= m + k + 1 + \operatorname{rank}\left\{(sE-A)(\overline{J}_r, \overline{C}_c)\right\},$$

where $(sE-A)(\overline{J}_r, \overline{C}_c)$ is the submatrix of $(sE-A)$ consisting of rows in $\overline{J}_r$ and columns in $\overline{C}_c$. Hence $\dim \mathcal{S}(J) = k+1$ if and only if the matrix $(sE-A)(\overline{J}_r, \overline{C}_c)$ has non-zero $(n-m)$-th order minor. The necessity is obvious from the definition of determinant. For the sufficiency, let M be a matching satisfying the condition. Substitute 0 into the elements of E and A which do not correspond to the branches in M. Let $X_r \subset \overline{J}_r$ be a maximal subset which is saturated by M. Then from the definition of determinant $\det(sE-A)(X_r, \overline{C}_c) \neq 0$. Hence $(sE-A)(\overline{J}_r, \overline{C}_c)$ is of full column rank generically. □

Theorem 1-c (interconnected form, k-distinguishability). If the interconnected form (1.c), (1.c'), (2.c) k distinguishable, then the following statements hold.

(i) For any $J \subset N$ with $|J| = k+1$, there is a set of $k+1$ disjoint paths from J to C in G_L.

(ii) For any $J \subset N$ with $|J| = k+1$, the minimum cardinality of a subset of N which separates J form C in G_L is $k+1$.

Proof. The proof is identical to the necessity part of Theorem 1-s. □

For $k = 0$, k-distinguishability is called detectability. This means that we can distinguish if the system is normal (*i.e.*, 0-th order fault) or the system has a fault.

The distinguishability graph conditions are exploited in determining a cover set required in the fault diagnosis algorithm. This is discussed in detail in 7, Lemma 1, Theorem 2).

The distinguishability graph conditions are similar to those for linear static systems [7)]. The reason for this is that we would like to distinguish which component of the fault vector is nonzero. In other words, we would like to distinguish faults up to which equation has an error.

5. Fault diagnosis algorithm

Since linear dynamical systems are a special case of linear systems studied in 7), the fault diagnosis algorithm studied in the paper is readily available. In order to determine which subspace contains the fault vector, the algorithm in the paper requires differentiation of Δy. However, as we shall show that an left inverse system will suffice to find the fault subspace.

In what follows we shall state the result for the state space form. Similar arguments hold for the descriptor form and the interconnected form.

Theorem 2. Suppose that the state space form (1), (2) is k-distinguishable. Assume that the difference of the normal and fault observation Δy satisfies Assumption H. Then the following algorithm determines whether the system has a fault of order less than or equal to k, and, if this is affirmative, which fault has occurred.

Algorithm:

(Step 1) Find a k-cover $\mathbf{K} \subset \mathcal{P}(\mathbf{n})$ which satisfies the following two conditions, where $\mathcal{P}(\mathbf{n})$ is the power set of $\mathbf{n} = \{1, \cdots, n\}$.

(i) For any $K \in \mathbf{K}$, $|K| = m$, and $\dim \mathcal{S}(K) = m$, where m is the number of measurements.

(ii) For any $J \in \mathbf{K}$ with $|J| \le m$, there is a $K \in \mathbf{K}$ such that $J \subset K$.

(Step 2) For each $K \in \mathbf{K}$, with a slight abuse of notation let K be the $n \times m$ matrix consisting of columns $e_i,\ i \in K$. Define an left inverse system $H_{L,K}(s)$ of $H_K(s) = C\,(sI - A)^{-1}\,K$ such that

$$H_{L,K}(s) H_K(s) = \Lambda_K(s),$$

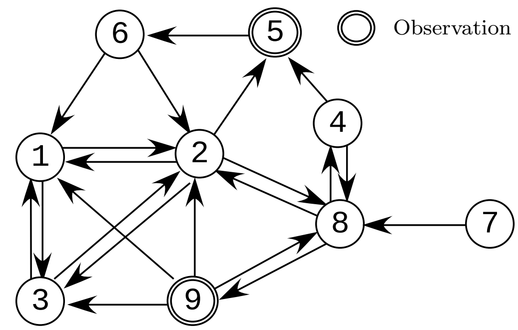

Fig. 1 The Coates graph of the example.

where $\Lambda_K(s)$ is a nonsingular diagonal transfer matrix. Define

$$\mu_K(s) = H_{L,K}(s)\Delta y(s),$$

and let $\#K$ be the number of nonzero elements of μ_K, and

$$J = \{i : \text{the } \rho(i)\text{-th element of } \mu_K \text{ is non-zero}\},$$

where the i-th column of the matrix K is the unit vector $e_{\rho(i)}$. If $\#K \le k$, then fault J has occurred.

(Step 3) If $\#K > k$ for any $K \in \mathbf{K}$ in Step 2, then a fault of order larger than k has occurred.

Note. A condition for the existence of a proper left inverse $H_{L,K}(s)$ in Step 2 using the relative degrees of $\Lambda_K(s)$ is discussed in 9).

We shall show a numerical example. The system is represented by the state space form with the A matrix shown in **Table 1**. We apply an input to the variable 7 and measure the variables 5 and 9. The Coates graph of the system is shown in **Fig. 1**. Since the system is not g-1-distinguishable, we cannot directly apply the algorithm of Theorem 2 to distinguish faults of order 1. However, with an appropriate modification the algorithm can distinguish fault equivalent classes. Using the graph condition in 9), the fault equivalence classes are $\{1,2,3,6\}$, $\{4\}$, $\{5\}$, $\{7,8\}$ and $\{9\}$. Choose a g-1-cover as $\mathbf{K} = \{K_1, K_2, K_3\}$, $K_1 = \{1,4\}$, $K_2 = \{5,8\}$ and $K_3 = \{5,9\}$. Choose left inverses $\Lambda_{K_i}(s) = 1/(s+1)^{p_i} I$, where p_i is the nonnegative integer defined as Note.

Let Fault 3 be such that a_{31} is 50% of the normal, and Fault 5 be such that a_{54} is 60% of the normal. The impulse responses of the faulty system (Fault 3) and the normal system is shown in **Fig. 2**. The difference of the outputs is fed into the three left inverses H_{L,K_i}, $i = 1,2,3$. The outputs of H_{L,K_2} and H_{L,K_3} have two non-zero elements whereas the output of H_{L,K_1} has only one non-zero element which corresponds the variable 1 as is shown in **Fig. 3**. Thus we rightly conclude that the fault is in the equivalence class $\{1,2,3,6\}$.

Table 1 The system matrix A of the example.

		column								
		1	2	3	4	5	6	7	8	9
	1	-2.27	-3.01	2.98			1.0			-0.599
	2	26.8	-61.5	0.524			0.176		-0.0923	-32.1
	3	30.0	-15.5	-32.3						-15.6
r	4				-27.7				-0.0828	-5.32
o	5		44.0		-89.8	-100.0				
w	6					-3875.0	-100.3			
	7							-5.0		
	8		-223.0		-47.8			55.4	-0.35	-222.0
	9								1.0	

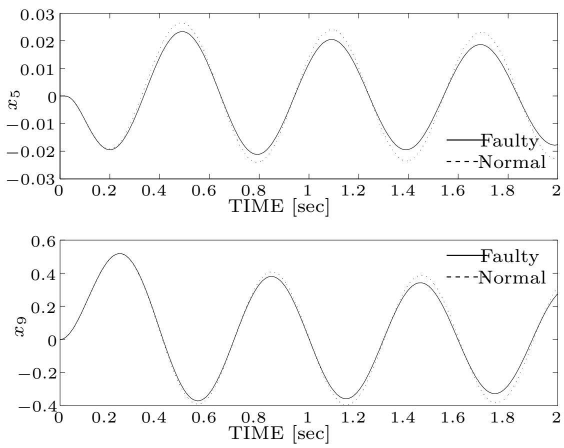

Fig. 2 The impulse responses of the faulty system and the normal system when the faulty a_{31} is 50% of the normal value.

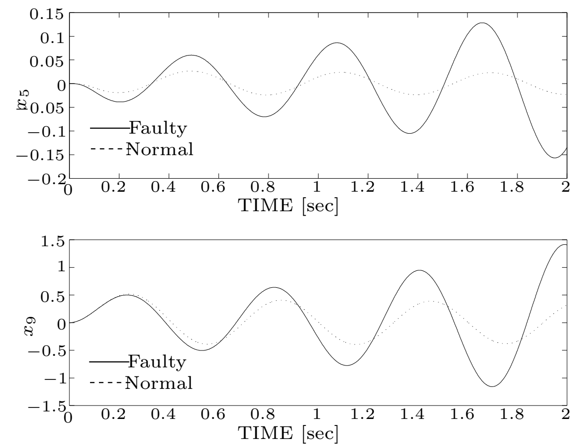

Fig. 4 The impulse responses of the faulty system and the normal system when the faulty a_{54} is 60% of the normal value.

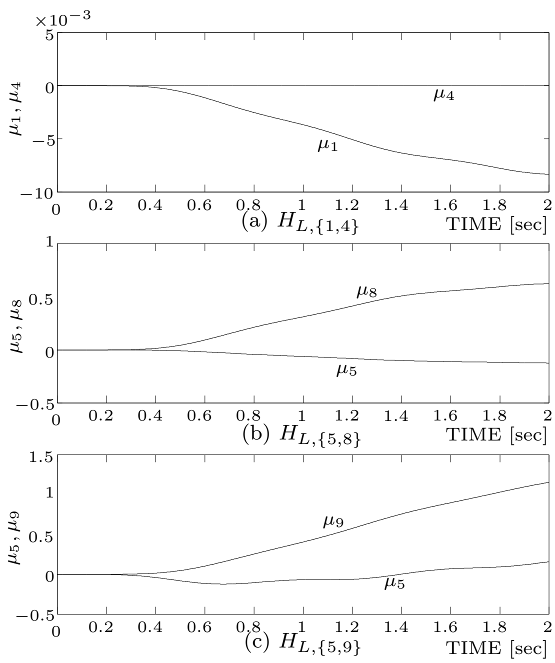

Fig. 3 The outputs of the left inverse systems when the faulty a_{31} is 50% of the normal value.

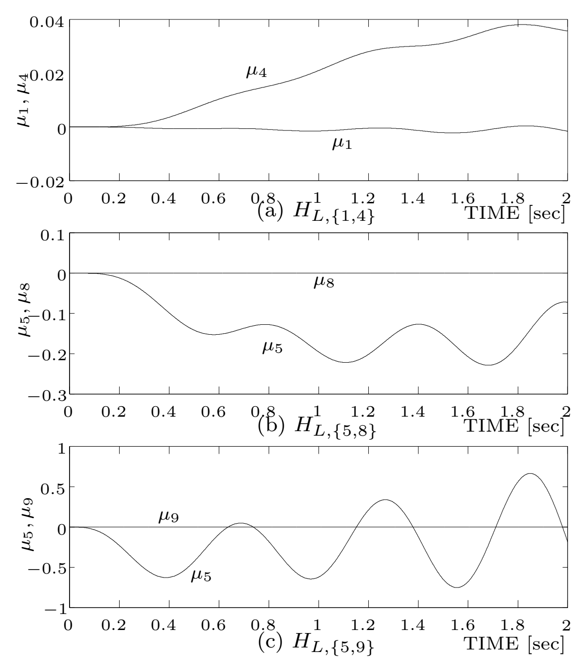

Fig. 5 The outputs of the left inverse systems when the faulty a_{54} is 60% of the normal value.

When Fault 5 occurs, the impulse responses of the faulty system (Fault 5) and the normal system is shown in **Fig. 4**. The difference of the outputs is fed into the three left inverses H_{L,K_i}, $i = 1, 2, 3$. The output of H_{L,K_1} has two non-zero elements whereas the outputs of H_{L,K_2} and H_{L,K_3} have only one non-zero element which corresponds the variable 5 as is shown in **Fig. 5**. Thus we rightly conclude that the fault is in the equivalence class $\{5\}$.

6. Conclusions

We derived a fault distinguishability condition and a fault diagnosis algorithm for linear dynamical systems in state space form, descriptor form and interconnected form.

Advantages of the graph distinguishability condition are:

(i) we can determine the distinguishability from the system structure,

(ii) we can design the observation variables using (i), and

(iii) we can apply various graph theoretic techniques to the fault diagnosis algorithm.

When we apply the algorithm, we need to choose

(i) a cover set, and

(ii) left inverses $H_{L,K}(s)$ (or equivalently diagonal transfer matrices $\Lambda_K(s)$,

appropriately. Such a choice needs further investigation.

This research was supported by Grant-in-aid for Scientific research A 59750318 (1984).

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Reprinted from Trans. of the SICE
Vol. 22 No. 4 391/397 1986