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Adaptive Control for Nonlinear Mechanical Systems†

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In this paper, a model reference adaptive contorl system(MRACS) for nonlinear mechanical systems is proposed.

In general, dynamic model of mechanical systems, such as manipulators, are highly nonlinear. This is the reason why control of such systems is difficult. Though the mechanical systems are nonlinear as mentioned above, the form of nonlinear terms in the models can be determined by the structure of the mechanical systems. So, we can know the terms of nonlinear terms perfectly. Moreover, because of the special feature of mechanical systems, the position, the velocity, and the acceleration of each degree of freedom are measurable.

First, I will show that for such systems, MRACS, which has particular adaptive mechanism and is assured global asymptotic stability, can be constructed, using actively the information of the nonlinearity and the special feature of mechanical systems. Then I will show the effectiveness of this MRACS using DARM-I(Direct Drive Arm with two degree of freedom).

Key Words: adaptive control, mechanical system, direct drive arm, global stability

1. Introduction

There are many researches concerned with adaptive control for linear systems. In recently, adaptive control methods for nonlinear systems have been proposed[1)2)].

On the other hand, since needs for high speed and high performance nonlinear mechanical systems such as manipulators are gradually increasing, adaptive control methods for such systems are proposed. However, since the adaptive control methods in the paper 3)-6) were based on an adaptive control theory for linear systems, there is a problem that the global asymptotic stability of the controlled system is not ensured. Since the control law proposed in the paper 7) is discontinuous one, mechanical vibration may be caused. It seems that there are at least the following two reasons why such problems have been arisen.

(i) a linear approximation has been used.

(ii) the special properties of mechanical systems have not been considered.

The author has already proposed a model referenced adaptive control method for n degrees of freedom of nonlinear mechanical systems[9)10)]. Using the special properties of such systems and without using any linear approximation, we designed the adaptive control system that can be ensured the global asymptotic stability. However, we could not ensure the boundedness of the internal signals in the control system.

In this paper, based on the result in the paper 10), we show a new design method of adaptive control system for nonlinear mechanical systems. As the results, the asymptotic stability and the boundedness of the internal signals in the system can be ensured. Then we show the effectiveness of the proposed method through experiment using a simple manipulator.

The contents of this paper are as follows. In Chapter 2, we clarify a class of mechanical systems and show the problem that we want to solve here. In Chapter 3, we show a design method of our adaptive control law. In Chapter 4, some experimental results are shown.

2. Problem Statement

We consider a design method of MRCAS for the following nonlinear mechanical systems.Here Eq.(1) is a controlled object, Eq.(2) is a reference model and Eq.(3) is a error signal.

$$\begin{bmatrix} I & 0 \\ 0 & J(x) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_2 \\ F(x)+u \end{bmatrix} \tag{1}$$

$$\dot{x}_M = A_M x_M + BU \tag{2}$$

$$e = x_M - x \tag{3}$$

Where,

$$A_M = \begin{bmatrix} 0 & I \\ k_1 I & k_2 I \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix},$$

$$x = (x_1^T, x_2^T)^T = (x_{11}, \dots, x_{1n}, x_{21}, \dots, x_{2n})^T,$$

$$x_M = (x_{M1}^T, x_{M2}^T)^T = (x_{M11}, \dots, x_{M1n}, x_{M21}, \dots, x_{M2n})^T,$$

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and $J(x) \in R^{n\times n}$ and $F(x) \in R^n$ are nonlinear matrices of the variable x. The matrix $I \in R^{n\times n}$is an unit matrix and $0 \in R^{n\times n}$ is a zero matrix, the constant parameters k_1 and k_2 are the design parameters.

Now, because of the system (1) is a mechanical system, we can assume the following assumptions.

Assumption (i) The nonlinear matrices $J(x)$ and $F(x)$ are linear with respect to physical parameters. Namely,

$$\begin{array}{l} J(x) = \sum_{k=1}^{m} a_k J_k(x) \\ F(x) = \sum_{k=1}^{q} a_k F_k(x), \end{array} \tag{4}$$

where $a_k (k = 1, \ldots, q)$ represent the uncertain physical parameters. The matrices $J_k(x) \in R^{n\times n}$ and $F_k(x) \in R^{n\times n}$ are nonlinear but do not contain any uncertain parameters.

(ii) The structure of the matrices $J_k(x)$ and $F_k(x)$ are known and they have the following properties.

(a) The all of elements of the each matrix are nonlinear piecewise continuous functions.

(b) The all elements of $J_k(x)$ (J_k^{ij})are bounded for arbitrary x, and their upper and lower bounds can be calculated in advance, namely,

$$\text{For } \forall x, \max_{\substack{1\le k\le m \\ 1\le i,j\le n}} \{\mid J_k^{ij}(x) \mid\} \le M_J. \tag{5}$$

(c) The all elements of $F_k(x)$ are bounded for arbitrary x.

(iii) The upper and lower bounds of uncertain parameters a_k can be estimated in advance, namely,

$$\max_{1\le k\le p} \{\mid a_k \mid\} \le a_{max}. \tag{6}$$

(iv) The state variable x, $\dot{x}$ and x_M can be detectable.

(v) The input U is bounded.

Here, we explain the meaning and the appropriateness of the above problem statement using a manipulator.

The kinetic energy T, the potential energy V, and the dissipation energy E of manipulators with n d.o.f. can be written as the following.

$$T = \frac{1}{2}\dot{x}_1^T J(x_1)\dot{x}_1, \tag{7}$$

$$V = v(x_1), \tag{8}$$

$$E = \frac{1}{2}\dot{x}_2^T H \dot{x}_2. \tag{9}$$

Where, $x_1 \in R^n$ is a generalized coordinate vector, $J(x_1) \in R^{n\times n}$ is a nonlinear matrix, $v(x_1) \in R$ is a nonlinear function, and $H = \text{diag}(h_1, \ldots, h_n)$ is a coefficient of viscosity matrix of the joint. Here, set $x_2 = \dot{x}_1$, and including the input torque vector $u \in R^n$ and the friction term $C = \text{diag}(f_1\text{sgn}x_{21}, \ldots, f_n\text{sgn}x_{2n})$, we can have the following dynamical equation of the system via Lagrange's method.

$$J(x_1)\dot{x_2} + \left\{\frac{d}{dt}J(x_1) - \frac{1}{2}\tilde{x}_2^T\left(\frac{\partial J(x_1)}{\partial x_1}\right)\right\}x_2 + \frac{\partial V}{\partial x_1} + Hx_2 + C = u, \tag{10}$$

where, $\tilde{x}_2$ is a matrix whose size is $n^2 \times n$. Here, if we set

$$J(x) = J(x_1) \tag{11}$$

$$F(x) = -\left\{\frac{d}{dt}J(x_1) - \frac{1}{2}\tilde{x}_2^T\left(\frac{\partial J(x_1)}{\partial x_1}\right)\right\}x_2 - \frac{\partial V}{\partial x_1} - Hx_2 - C, \tag{12}$$

then we have the system(1). Now, in general, as shown in Eqs.(10)-(12), the dynamical equation of manipulator is a nonlinear differential equation system with respect to the variable x. But, it is well known that the equation is linear with respect to special parameters[6]. Therefore, if we set

$$P_1 = \{\text{parameter set of all independent parameters in } J(x)\}, \tag{13}$$

$$P_2 = \{\text{parameter set of all independent parameters in } F(x)\}, \tag{14}$$

then form Eqs.(11) and (12), we have

$$P_1 \subseteq P_2. \tag{15}$$

[8]

Therefore, if we set

$$P_1 = \{a_1, a_2, \cdots, a_m\} \tag{16}$$

$$P_2 = \{a_1, a_2, \cdots, a_m, a_{m+1}, \cdots, a_p\} \tag{17}$$

this implies that Eq.(4) can represents the nonlinear system (1).

From the above discussions, we can say that the assumption (i) can be satisfied for manipulators. Since the structure of the nonlinear function in dynamical equation of manipulators are decided from the structure of the manipulators, the assumption (ii) can be satisfied in general. The assumption (iii) says that we should identify the parameters in the dynamical equation roughly in advance. Since when we control a system, we may know some about the system in advance, this assumption seems reasonable. Since we can have high quality sensors in recently, the assumption (v) also can be satisfied.

The similar analysis can be carried out for other mechanical systems. And we think that there are many systems which satisfy the assumptions.

3. MRACS for Mechanical Systems

In this chapter, we construct the adaptive control system for the system (1),(2) and (3). To do this, we define the following observe matrix and the generalized error signal.

$$-\lambda z = \Big[J_1(x)\dot{x}_2 - F_1(x)\vdots,\cdots,\vdots J_m(x)\dot{x}_2 - F_m(x)\vdots$$
$$-F_{m+1}(x)\vdots,\cdots,\vdots - F_p(x)\Big], \quad (18)$$
$$v = De, \quad (19)$$

where, $D = [D_1 I \vdots D_2 I]$, z_{jk} denotes the $j-k$ element of the matrix z, and v_j denotes the j element of the vector v.

The main result of this paper is the following.

Theorem 1. Consider the nonlinear mechanical system (1) and the input

$$\begin{aligned} u &= -\lambda\dot{x}_2 + \lambda[k_1x_1 + k_2x_2 + U] \\ &+ \sum_{k=1}^{m} \hat{a}_k[J_k(x)\dot{x}_2 - F_k(x)] \\ &+ \sum_{k=m+1}^{p} \hat{a}_k[-F_k(x)], \end{aligned} \quad (20)$$

$$\begin{aligned} \hat{a}_k &= -\Gamma_k \int_0^t \sum_{k=1}^{n} z_{jk}v_j d\tau + \hat{a}_k(0), \\ &(\Gamma_k > 0, \max_{1\le k\le p}\{|\ \hat{a}_k(0)\ |\} \le a_{max}. \end{aligned} \quad (21)$$

If the matrix D of Eq.(19) is chosen to make the transfer function $W(s) = D(sI - A)^{-1}B$ to be strictly positive real, then we can have $\lim_{t\to\infty} e(t) = 0$. Furthermore, we can prove the existence of a constant value λ which assures the boundedness of the all internal signals in the hole system and can calculate the value.

proof. Substituting Eqs.(4) , (20) and (21) into Eq.(1), we get

$$\dot{x}_1 = \dot{x}_2 \quad (22)$$

$$\begin{aligned} \dot{x}_2 &= k_1x_1 + k_2x_2 + U + \sum_{k=1}^{m}(\hat{a}_k - a_k) \\ &\cdot[J_k(x)\dot{x}_2 - F_k(x)]/\lambda \\ &+ \sum_{k=m+1}^{p} (\hat{a}_k - a_k)[-F_k(x)]/\lambda \end{aligned} \quad (23)$$

Further, subtracting Eqs.(22) and (23) from Eq.(2) and using Eq.(3), we have the following error equation,

$$\begin{aligned} \dot{e} &= Ae + B[Z\phi], \\ y &= De, \end{aligned} \quad (24)$$

where $\phi = (\phi_1, \cdots, \phi_k)^T = (\hat{a}_1 - a_1, \cdots, \hat{a}_q - a_q)^T$. Since the transfer function $W(s)$ is chosen to be strictly positive real, using the Kalman-Yakubovich-Popov Lemma, we can say that there exists a positive definite matrix P which satisfies the equations,

$$\begin{aligned} &PA + A^TP = -Q \quad (\forall Q > 0) \\ &B^TP = D. \end{aligned} \quad (25)$$

Now, consider the positive definitie function

$$V(t) = e^TPe + \phi^T\Gamma^{-1}\phi \ge 0, \quad (26)$$

where $\Gamma = diag[\Gamma_1, \cdots, \Gamma_n]$. After differentiation of $V(t)$ along the solution of Eq.(24) yields

$$\dot{V}(t) = -e^TQe \le 0. \quad (27)$$

From Eqs.(24) and (25), the following properties can be obtained.

(i) e is bounded.

(ii) ϕ is bounded and $\lim_{t\to\infty}\phi = \phi^*$ (constant).

(iii) $V(t) \le V(0)$.

To assure $lim_{t\to\infty}e = 0$, we must prove that the signal e is continuous. On the other hand, the continuity of e can be proved by assuring the boundedness of the internal signals in the whole system.

(Boundedness of the internal signals) Rewrite the system (22) and (23) as the following.

$$\dot{x}_1 = x_2 \quad (28)$$

$$\left[\lambda I - \sum_{k=1}^{m}\phi_k J_k(x)\right]\dot{x}_2 = \lambda[k_1x_1 + k_2x_2 + r] + \sum_{k=m+1}^{p}\phi_k[-F_k(x)]. \quad (29)$$

Since r, k_1 and k_2 are bounded, it is clear that x_{M1} and x_{M2} are bounded. And so, from Eq.(3) and the property (i), x_1 and x_2 are bounded. Therefore, from the assumption (iii), the all elements of $J_k(x)$ and $F_k(x)$ are bounded. Moreover, from the property (ii), ϕ_k are bounded.

In this way, we can prove that the all elements of the right hand side of Eq.(29) and the matrix defined as

$$R(x) = \lambda I - \sum_{k=1}^{m}\phi_k J_k(x) \quad (30)$$

are bounded. Therefore, the boundedness of the internal signals in Eq.(20) can be ensure the boundedness of the signal $\dot{x}_2$. To do so, we prove that, for some constant ϵ and for arbitrary $t \ge 0$, the following equation holds.

$$R_{ii} \ge \epsilon, \quad (31)$$

where R_{ii} denotes the i-i element of $R(x)$. This claim can be proved by choosing the constant λ as the following way. At first, from Eqs. (6) and (21), for all $k(1 \le k \le p)$, we have

$$\theta_k(0)^2 = (\hat{a}_k(0) - a_k)^2 \le 4a_{max}^2. \quad (32)$$

Then, from Eqs. (26) and (32), and the property (iii), we have the following.

$$\theta^T\Gamma^{-1}\theta = \sum_{k=1}^{p}\theta_k^2/\Gamma_k$$
$$\leq e(0)^T Pe(0) + \sum_{k=1}^{p}\theta_k(0)/\Gamma_k$$
$$\leq e(0)^T Pe(0) + \sum_{k=1}^{p} 4a_{max}^2/\Gamma_k =: M_\theta. \quad (33)$$

Eq.(33) implies that, for arbitrary $t \geq 0$, θ_k does not go outside of a certain p dimensional ellipsoid. From this result, if we set $\Gamma_{max} = \max_{1\leq k\leq p}\Gamma_k$, then, for all $k(1 \leq k \leq p)$, we have

$$|\theta_k| \leq \sqrt{\Gamma_{max}M_\theta}. \quad (34)$$

Therefore, since we have

$$R_{ii} = \lambda - \sum_{k=1}^{m}\theta_k J_k^{ii}(x)$$
$$\geq \lambda - mM_J\sqrt{\Gamma_{max}M_\theta}(1 \leq I \leq n), \quad (35)$$

if we set λ as

$$\lambda \geq \epsilon + mM_J\sqrt{\Gamma_{max}M_\theta}, \quad (36)$$

we have Eq.(31).

From the above discussions, we could prove the existence of a value of λ which ensure the boundedness of the whole control system. And we can calculate the concrete value via Eq.(36).

(proof of $\lim_{t\to\infty} e = 0$) Since the boundedness of the internal signals in the control system have been proved, we can show that the error signal $\dot{e}$ is bounded. This implies that the signal e is continuous signal. Then, using Eqn.(26) and (27), we have $\lim_{t\to\infty} e = 0$. ∎

The structure of the proposed adaptive control system is shown in Fig.1.

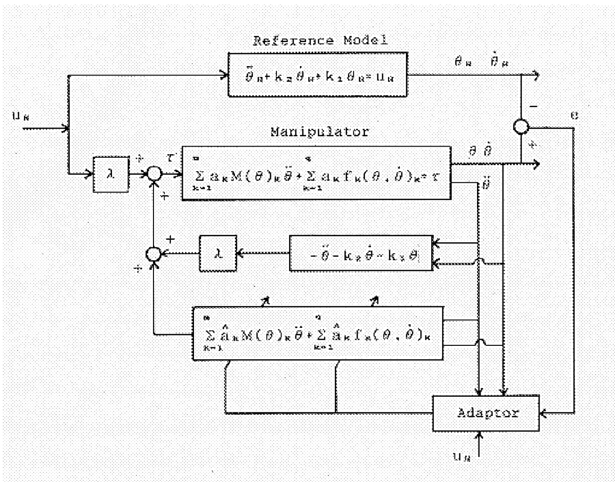

Fig. 1 Adaptive control for nonlinear mechanical systems

As you can see in Fig.1, the control system is designed based on nonlinear compensation method with adaptive functions. That is, the each parameter in Eqs.(20) and (21) has a physical meaning. Therefore, if we set $\Gamma_k(1 \leq k \leq p) = 0$, this control law realizes a nonlinear compensation method with acceleration signals.

The parameter λ which is introduced for ensuring the internal signals makes the value of the moment of inertia of Eq.(30) big. That is, the parameter λ works as a kind of a fly-wheel. Furthermore, from Eq.(26) and the property (iii) in the proof, we can say that the control performance of the proposed control system arises if the initial values of the adaptive parameters θ_k are chosen to be near the real values of the parameters.

4. Application to Control of a Direct Drive Arm

In order to show the effectiveness of the proposed control system (MRACS), we applied the controller to solve the control problem of a certain mechanical system.

4.1 Experimental System

Photo 1 shows the photograph of the direct drive arm. This arm is a SCARA-Type manipulator which has two degrees of freedom. The rare earth D.C. torque motors are used for joint actuators and the links are made of aluminum. The each joint has a photo-encoder for measuring the joint angle and a tach-generator for measuring the angular velocity of the joint. Moreover, an accelerometer is mounted on the edge of the each link as shown in Fig.2. We can obtain the joint acceleration using the signals from these accelerometers as follows;

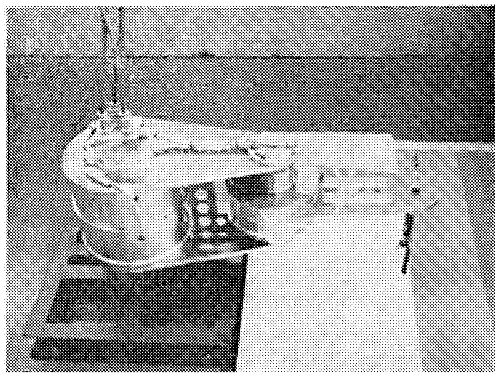

Photo. 1 Direct Drive Arm

$$\ddot{\theta}_1 = \alpha_1/d_1 \quad (37)$$
$$\ddot{\theta}_2 = [\alpha_2 - \alpha_1(\cos\theta_2 - d_2/d_1) - d_1\dot{\theta}_1^2\sin\theta_2]/d_2. \quad (38)$$

Here, you can see θ_1, θ_2, d_1 and d_2 in Fig.2, and α_1 and α_2 are the signals obtained from the accelerometers.

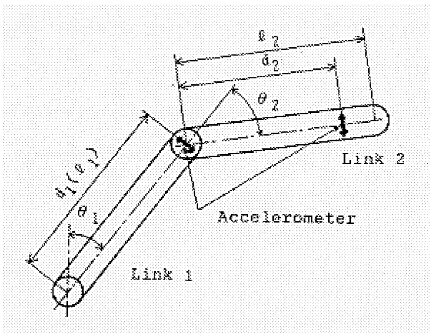

Fig. 2 Model of D.D. Arm

Thus, we can obtain the joint angles, the joint angular velocities and the joint angular accelerations of the arm. And, obtaining the dynamical model of this arm, we can see easily understand that the assumptions (i)-(v) are satisfied for this arm.

Fig.3 shows the structure of the experimental system. The CPU of the system is 80286+8087. The sampling period is 7[msec]. The main datas of the arm are tabulated in Table 1.

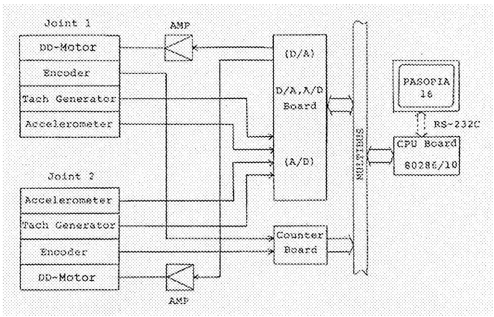

Fig. 3 System Configuration

Table 1 Parameters

	1	2
l_i	0.3m	0.3m
d_i	0.3m	0.3m
Weight of link i	3.7 kg	2.2 kg
Peak torque of joint i	1,647 kgm	304 kgm

4.2 Experimental Results

Define a coordinate system as shown in Fig.4 in the work space of the arm according to the following way. At first, define the center of the joint one as the origin of the coordinate system. Define the direction, which can be obtained by setting $\theta_1 = \theta_2 = 0$, as the Y direction. Define the direction which crosses at right angles to the Y direction as the X direction. Then we designed the test motion as the following. Test motion: Wright a straight line by the tip of the second link of the

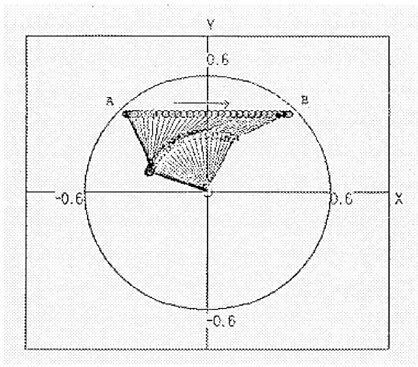

Fig. 4 Arm work space

arm from A-point ($X = -0.4m, Y = 0.4m$) to B-point ($X = 0.4m, Y = 0.4m$). The experiments were executed under the following cases.

Case1 The desired maximum speed of the edge of the arm is 0.5m/s, and no payload is mounted on the arm.

Case2 The desired maximum speed of the edge of the arm is 0.5m/s, and the payload is mounted on the edge of the arm is 2kg.

Case3 The desired maximum speed of the edge of the arm is 0.5m/s, and the payload is mounted on the edge of the arm is 4kg.

Case4 The desired maximum speed of the edge of the arm is 1.0m/s, and no payload is mounted on the arm.

Case5 The desired maximum speed of the edge of the arm is 1.0m/s, and the payload is mounted on the edge of the arm is 2kg.

Here, the maximum speed is the maximum velocity of the edge of the arm.

For each case, we compared with the experimental results due to the proposed control scheme and the results due to the conventional control scheme, i.e., nonlinear compensation method of computed torque method. The nonlinear compensation method can be obtained by setting $\Gamma_k (k = 1, \cdots, p)$ equal to zero. In all cases, we used the values of the physical parameters identified via our method proposed in 8) as the initial values of the adaptive parameters $\hat{a}_k(0)$. Here, it should be noticed that the parameters may not be identified perfectly. The identified parameters are tabulated in Table A.1.

Fig.5 and Fig.6 show the experimental results. We show only the data of the joint 2. In each figure, figure (a) shows the result due to the proposed control scheme, and figure (b) shows the result due to the conventional control scheme, i.e., nonlinear compensation method. In the all figures, the reference trajectory and the actual trajectory are shown.

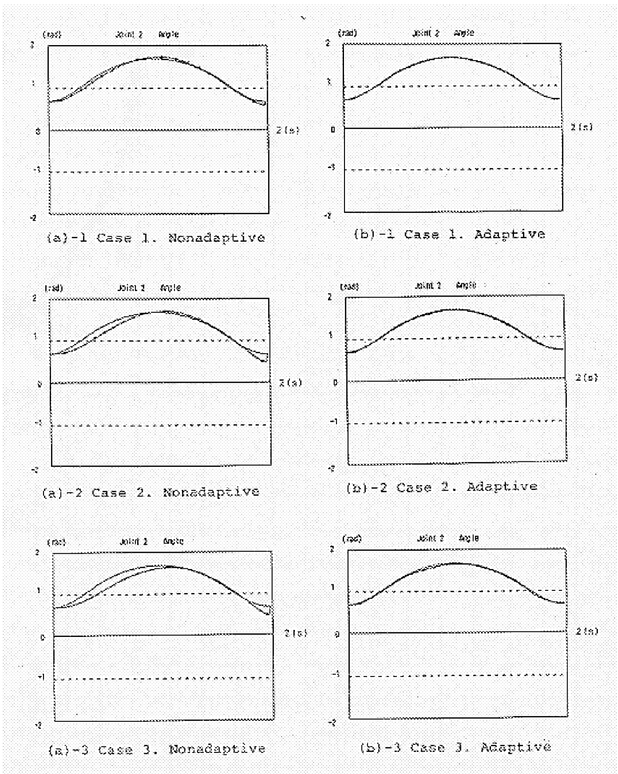

Fig. 5 Experiment results(case1 to case3)

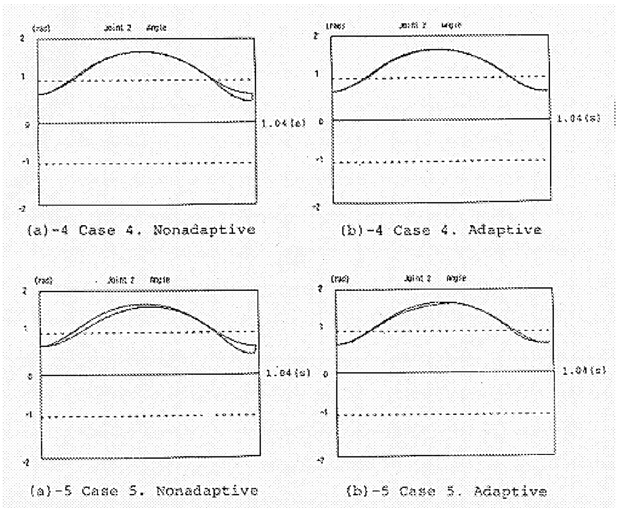

Fig. 6 Experiment results(case1 to case3)

From Fig.5(a)-1 and Fig.6(a)-1, we can say that in case of conventional scheme, due to the insufficiently nonlinear compensation effect, there are some trajectory tracking error. From Fig.5(a)-2, Fig.5(a)-3 and Fig.6(a)-2, we can see that a big load causes a big trajectory tracking error. From these results, it can be said that the conventional non-adaptive type is not powerful against the change of the dynamical property.

On the other hand, in Fig.5(b) and Fig.6(b), we can see that the trajectory tracking error becomes much smaller than results obtained by the conventional method. This implies that the proposed adaptive method is effective.

Photo 2 shows the photograph of one of the result (case 4) that the D.D.Arm wrote a straight line on the paper with a pen. In the photograph, (a) shows the result based on the nonadaptive control scheme and (b) shows the result based on the adaptive control scheme. These experimental results show the effectiveness of our method.

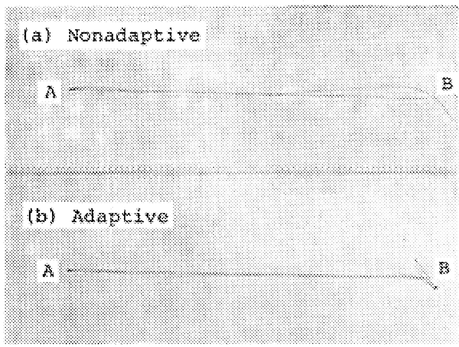

Photo. 2 Experiment result

5. Conclusion

We have proposed a design method of an adaptive control system for nonlinear mechanical systems and shown the effectiveness of the proposed method through several experiments using the D.D.Arm. Our method has the following properties. (a) Any linealizeations or approximations are used for designing the controller. (b) Therefore, the global asymptotically stable of the error between the states of the reference model and the actual states has been ensured. (c) The boundedness of the internal signals in the whole system have been ensured.

The proposed method is based on nonlinear compensation method, and all adaptive parameters in the control law has a physical meaning. Therefore, if it is needed, we can set an adaptive gain of the adaptive parameter corresponds to a parameter which is strongly unknown or will be changed big.

Furthermore, since we included all nonlinear structures of the controlled system in the control law, we can expect that our control method can be used for high speed manipulators compared with the conventional adaptive method[3)−6)].

In the future, we have to discuss the effect of using acceleration signals more precisely. And we also have to discuss the robustness of the control system against disturbances or noises.

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Appendix A.

The dynamical model of this arm can be derived as follows;

$$\begin{bmatrix} I & 0 \\ 0 & J(x) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ F(x)+u \end{bmatrix}, \qquad (A.1)$$

where,

$$x_1 = (\theta_1, \theta_2)^T, x_T = (\dot{\theta}_1, \dot{\theta}_2)^T, \qquad (A.2)$$

$$J(x) = \sum_{k=1}^{6} a_k J_k(x) \qquad (A.3)$$

$$F(x) = \sum_{k=1}^{10} a_k F_k(x) \qquad (A.4)$$

$$\begin{aligned} a &= (a_1, a_2, \cdots, a_{10}) \\ &= (J_1 + J_2, J_2^1, J_2, J_2^2, R_2^1, R_2^2, B_1, B_2, \\ &\quad f_1, f_2) \end{aligned} \qquad (A.5)$$

$$[J(x)_1 \ \vdots \ \cdots \ \vdots J_6(x)] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2\cos\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & \cos\theta_2 & 0 \end{bmatrix} \qquad (A.6)$$

$$[F_1(x) \ \vdots \ \cdots \ \vdots J_{10}(x)] = \begin{bmatrix} 0 & 0 & 0 & 0 & -2\dot{\theta}_1\dot{\theta}_2\sin\theta_2 - \dot{\theta}_2^2\sin\theta_2 & 0 & -\dot{\theta}_1 & 0 & -sgn\dot{\theta}_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dot{\theta}_2^2\sin\theta_2 & 0 & -\dot{\theta}_2 & 0 & -sgn\dot{\theta}_1 \end{bmatrix} \qquad (A.7)$$

where $J_1 = I_1 + m_1 r_1^2 + m_2 l_1^2$, $J_2 = I_2 + m_2 r_2^2$, $R_2 = m_2 r_2 l_1$, and here I_i,m_i,r_i,B_i,f_i denote the moment of inertia of the link i about the center of mass , the mass of the link i, the distance from the joint i to the center of mass of the link i, the viscous friction and Coulomb friction of the joint i, respectively.

From the above formulation, since the forms of $J_k(x)$ and $F_k(x)$ become clear, we substitute these equations to Eq.(20). Here, the parameters in Eq.(20) are tabulated in Table.A.1. The U in Eq.(20) can be written as the following.

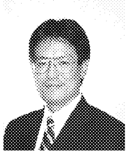
Table A.1 Parameters

J_1	0.8(kgm^2)	B_1	10.5(Nms)	f_1	1.4(Nm)
J_2	0.19(kgm^2)	B_2	1.05(Nms)	f_2	0.9(Nm)
R_2	0.1(kgm)				
Γ_1	40.0	Γ_6	0.4	λ	1.0
Γ_2	0.4	Γ_7	30.0	k_1	-64.0
Γ_3	8.0	Γ_8	6.0	k_2	-16.0
Γ_4	0.4	Γ_9	20.0	D_1	2.0
Γ_5	0.4	Γ_{10}	4.0	D_2	0.2

$$U_I = \ddot{\theta}_{di} - k_1\theta_{di} - k_2\dot{\theta}_{di} (i = 1, 2), \qquad (A.8)$$

where, θ_{di},$\dot{\theta}_{di}$, and θ_{di} denote the reference trajectory of the each joint.

Koichi Osuka (Member)



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