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Determining 3-D Shape from the Distribution of Normal Vector on a Curved Surface†

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This paper describes the shape representation of 3-D objects which utilizes the normal vector obtained from 2-D images. It is proposed to use the 2-D plane which is obtained by projecting the Gaussian sphere perspectively as the characteristic space for representing 3-D objects, and the shape of the surface of any 3-D object corresponds to the figure on this plane. The characteristic figures on the plane are easily extracted using techniques for image processing. Moreover, the method, by which both principal curvature and the tangential line vector are used, is proposed for representing the object which cannot be satisfactorily represented only in the distribution of the normal vector. It is also shown that the characteristic space proposed earlier is also applicable to the method, because the change of the vector is shown as the figure in this characteristic space.

Key Words:
computer vision, intrinsic characteristic, Gaussian sphere, surface orientation

1. Introduction

When we measure and recognize the 3-D shape of an object using a computer vision system, information about the relationship between the shape of the object and the characteristics should be extracted. Obviously, the amount of information decreases in a process of projecting the 3-D scene into a 2-D image. Therefore, it is a significant problem in computer vision to recover the lacked information. To resolve the recovery problem, we usually use the information of constraints about either 3-D objects or multiple images.

When there is an object having curved surfaces in a scene, intrinsic characteristics, which represent the curved object, can be extracted through camera images and used in order to measure and recognize the object. A method to use such information about normal vectors of curved surfaces has been proposed by B.K.P. Horn as a study on shape from shading problem [1)]. Woodham has proposed the photometric stereo method to extract normal vectors of the curved surface from grey images [4)]. After that, the method using a normal vector space has become one of popular methods [1)~3)].

Generally, the Gaussian sphere is used as a normal vector space in the field of differential geometry. However, some other kinds of quantified normal vector spaces are employed for computer processing. For the representation of a scene, a gradient space is used since the image processing is easier than other methods [4)]. Extended Gaussian Image (EGI) proposed by Ikeuchi is a digitized Gaussian sphere and keeps good symmetry [5)]. Using this feature, EGI is adequate to apply a model matching technique. These two characteristic spaces are intrinsically equivalent normal vector spaces each other. However, in each method, there is a peculiar method to representing curved objects and it is based on the following characteristics; a dual relationship between line drawings and figures in the gradient space [6)], and the definition of the moment of inertia with each patches in EGI.

In this paper, we explain the technique to represent the shape of curved objects on a characteristic space made by the orthographic projection of a Gaussian sphere. At first, we compare this characteristic space with other normal vector spaces such as a gradient space and EGI. Then, we propose a new representation method of 3-D objects. At last, we expand this normal vector space into a tangential vector space and show a representation method of curved objects using this characteristic space.

2. Representation of Characteristics using GPM

Each start point of normal vectors is moved to the center of an unit sphere in Gaussian projection. The direction of the normal vector is represented by the location on a surface of the sphere. So, all of the normal vectors calculated in each pixel of an input image are projected on the sphere. On the other hand, it is impossible to perform the inverse projection since information of the location of each pixel is lost in the process of Gaussian projection.

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However, such projection gives a new characteristic to us.

Using GPM (Gaussian sphere Projection Map) method proposed in this paper, a Gaussian sphere is projected onto an orthogonal plane to the view direction and conventional image processing techniques can be applied to the projected image (**Fig.1**). A set of normal vectors projected from an image appears only in a hemisphere of a Gaussian sphere. Therefore, in this projection process, there is no loss of data. Moreover, information about the location of normal vectors is ignored so that global image processing can be performed on the image. These characteristic is useful for a bottom-up approach in image processing.

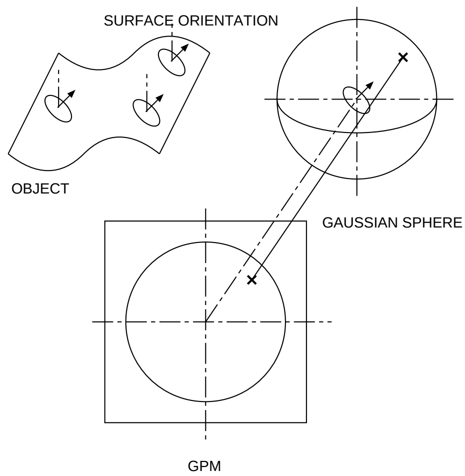


Fig. 1 GPM(Gaussian sphere Projection Map)

3. Distribution of normal vectors on GPM

3.1 Pattern of normal vectors of curved surface

At first, we will explain how primitive surfaces (plane, cylindrical surface, conical surface and sphere) are represented on GPM. When a curved surface is described with parameters u and v, a normal vector of the curved surface $\mathbf{e}$ can be formed as follows.

$$\mathbf{e} = \frac{\mathbf{P}_u \times \mathbf{P}_v}{|\mathbf{P}_u \times \mathbf{P}_v|} \quad (1)$$

Where, $\mathbf{P}_u, \mathbf{P}_v$ are tangential vectors along u, v curves on the surface respectively. In the projection from the Gaussian sphere into the GPM, the element of the normal vector corresponding to the direction of a view vector is set as zero.

It is easy to understand the distribution maps of normal vectors of the both a plane and a sphere on the GPM. So we will describe the condition of distribution of normal vectors of both cylindrical surfaces and conical surfaces. Let us think a regular cone; of which axis is parallel to y axis and the angle between of which generating line and axis is τ. Next this cone is rotated by an angle θ around x axis and then rotated by an angle ψ around z axis. Distributed pattern of the normal vectors of this cone can be represented with the following formula.

$$\left(\frac{f\cos\psi + g\sin\psi}{\cos\tau}\right)^2 + \left(\frac{f\sin\psi - g\cos\psi - \cos\theta\sin\tau}{\sin\theta\cos\tau}\right)^2 = 1 \quad (2)$$

Consequently, an ellipse appears on the GPM. The long span and the short span of the ellipse are $\cos\tau$ and $\sin\theta\cos\tau$ respectively. The gradient of the ellipse is equal to ψ, and the distance between the origin of the GPM and the center of the ellipse is given as $\cos\theta\sin\tau$. Where, f, g and h mean the axes of the Gaussian sphere corresponding to x, y and z axes of a camera coordinate system. In this case, only a visible hemisphere of the Gaussian sphere is projected into the GPM. Therefore, when we assume that a view vector is $(0, 0, -1)$, a half of the ellipse appears for the condition $h \leq 0$. Concerning with a cylindrical surface, it is enough that we consider only the simple formula set as $\tau = 0$.

Table 1 shows the comparison among some normal vector spaces. A difference among these normal vector space is how normal vectors of a cone including a cylinder appears in the space. Basically, the way to represent a conical surface in a normal vector space is equal to determining three parameters of a distributed pattern of normal vectors of the cone on a Gaussian sphere; i.e. the position and pose of the plane crossing the Gaussian sphere. Therefore, the three spaces shown in Table 1 are equivalent each other. However, the parameter spaces, which are planar, are more useful than a Gaussian sphere, because 2-D array description is easy to be used for a computer. Moreover, an infinite plane such as a gradient space is not adequate when global processing is applied to the parameter space. Namely, GPM represented as a finite plane is useful. In GPM, both of a cylinder and a cone are simply represented by an ellipse. So, many methods to extrac-

Table 1 Comparison among normal vector spaces

	plane	cylindrical	conical	spherical
Gaussian sphere (sphere)	point	planer curve	planer curve	distributed points
Gradient space (infinite plane)	point	straight lines	hyperbolic line	distributed points
GPM (unit circle)	point	ellipse	ellipse	distributed points

t the ellipse proposed by many researchers can be also applied to GPM. In other words, a certain surface which consists of multiple developable surfaces can be extracted on GPM.

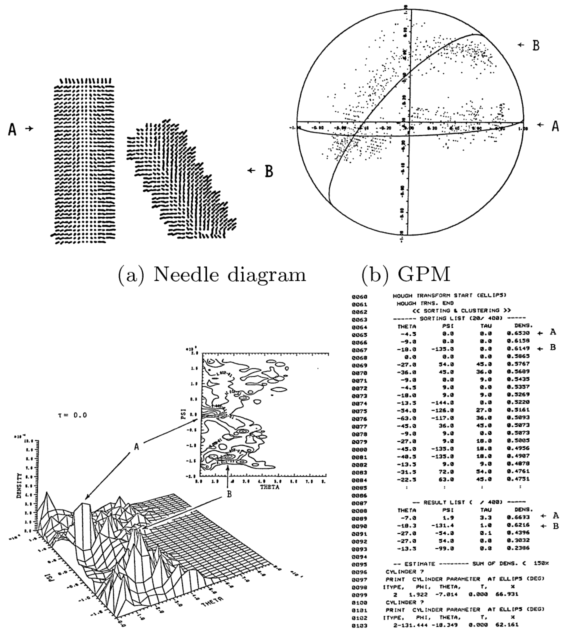
3. 2 Method to extract the characteristics of distributed normal vectors

In our experiment, normal vectors were extracted by the photometric stereo method with three gray images $128 \times 128 \times 8bit$, which were obtained under different light conditions. As we described in the previous section, it is equivalent to find local peak points on GPM to extract planes. In order to extract cylindrical and conical surfaces, we employed the Hough transform method to extract multiple and discontinuous ellipses.

In the Hough transform method, there is one serious problem concerning the computational cost. This computational cost is reduced assuming that an ellipse consists of two or more points. Moreover, GPM is divided into a square tessellation (a 2-D array) in order to store the number of pixels corresponding to the direction of the normal vector in each cell. Accumulating this value in the Hough transform, the computational costs is radically reduced.

The accuracy of the Hough transform method is related to the fineness of the parameter space. On the other hand, we should note that parameters describing an ellipse θ, ψ, τ are not symmetrical. Namely, we cannot change these parameters uniformly in order to accumulate them in the N-dimensional array. To reduce the influence from this problem, θ and ψ, which are related to the pose of the cone, are changed in proportion to a solid angle.

Fig.2 shows the experimental result that two objects in a scene are classified into two cylinders which have different poses by the use of the Hough transform method. Fig.2(c) is a bird view of a density contour map of $\theta - \psi$ plane at $\tau = 0$. Fig.2(d) shows the process of clustering of data in an accumulating array of the Hough transform method. Two arrows in each figures show the corresponding results of calculation. The number of ellipses to be extracted is related to the sum of areas of surfaces. Therefore, the process to extract the ellipses is terminated when the sum of the number of normal vectors of the extracted ellipses becomes bigger than the area of surfaces in the scene.



(a) Needle diagram (b) GPM

(c) Parameter space ($\tau = 0.0$) (d) Process of clustering

Fig. 2 Detection of ellipses

4. Density distribution on GPM

4. 1 Size of area of surfaces and density map on GPM

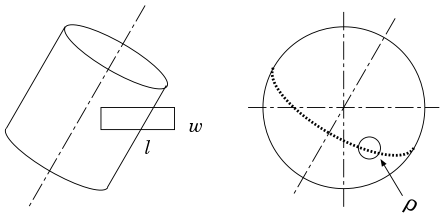
The difference among a gradient space, EGI and GPM is concerned with the way to quantify the characteristic space. In this section, we will explain the relationship between the size of areas of surfaces and the density map on GPM.

At first, let's think something about a sphere. Normal vectors of the surface of the sphere are uniformly distributed. However, the size of the view area producing the normal vector is reduced in proportion to the magnitude of an angle between the normal vector and the view vector. Therefore, an unit sphere is produced as if the sphere in the scene is resized into GPM and the density distribution on GPM becomes uniform. In other words, Gaussian curvature, which is a rate between the area of the curved surface and the area corresponding to the curved surface projected into a Gaussian sphere, is related to the density on GPM. Therefore, instead of finding ellipses in the GPM, we can extract developable surfaces using this characteristic.

4. 2 Extraction of cylinder using density map

For use of a density map on the GPM in order to extract curved surfaces in a scene, we have to think about the concrete method and its accuracy. In this section, we will show the method and experimental results.

The curvature k of a cylindrical surface C, which has the height h and the radius of bottom r, is $1/r$. When the cylindrical object can be observed without occlusion, we can obtain the relationship $\lambda s/h = r$. Where, s is the size of its visible area and λ is a coefficient concerning the pose of the object. If this cylinder is projected onto the GPM and the ellipse appears, we can obtain s as ρs_G. Where, s_G is the length of the contour of the ellipse and ρ is a line density of the ellipse. Then we can calculate k with h. However, there is occlusion in a scene frequently and s and h can not be obtained. So, k can not be estimated. In order to solve the influence of the occlusion, we set a small window whose width and height are w and l respectively and we assume that the size of the window is enough small than the occlusion expected in advance. Projecting the data in the small window into the GPM and then obtaining s_G and rho on the GPM, we can estimate the curvature k (**Fig.3**).



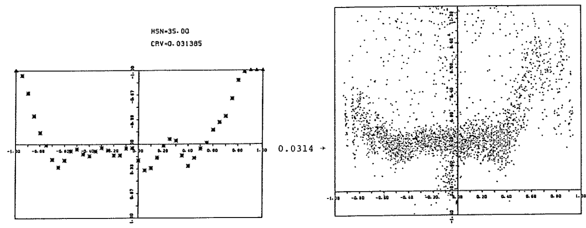
(a) Image (b) GPM

Fig. 3 Principle of the method

In our experiment, GPM was separated into multiple cells such as square tessellation and the density of each cell was calculated. To obtain the line density on GP-M approximately, the number of cells on the ellipse was used. In this case, GPM should be divided into $2n \times 2n$ at least, if a radius of a cylinder in a scene is bigger than n.

Fig.4 shows the case of a cylinder. Fig.4(a) is normalized by the value of the density, which is of the case that data are distributed uniformly. Using the same input image, Fig.4(b) shows the graph of curvatures calculated by the first fundamental form of a surface [7), 8)]. In comparison of these two graphs, we were able to confirm that the method to calculate the curvature using the density map was equivalent with the method using first fundamental form.

In the method using the fundamental form of a surface, the differential value of an irradiance of pixels is needed but the value often have an error for the noises. Therefore, a normalizing method and a noise reduction method are required to reduce the influence of noises. On the other hand, in the proposed method, a main process is only to count the number of pixels observed through the square window. So its computational cost is relatively small. Moreover, the accuracy in computing the density



(a) Distribution of density (b) Distribution of curvature

Fig. 4 Result

is the same as that of the method using the fundamental form of a surface, if the size of the window is adequate. Using these good characteristics, we can use this method for a coarse-to-fine algorithm by changing the size of the window.

5. Tangential vectors on GPM

5. 1 Tangential vector space

Using the information of a distribution map of normal vectors, we can represent some curved surfaces. However, when there are multiple objects in the same scene or there are more complex objects with occlusion in the scene, we are not able to represent these objects individually. For example, we cannot distinguish the difference between multiple cylinders and a torus, and also cannot distinguish the difference between a plane having noise uniformly and a part of a sphere. This is because the continuousness among normal vectors in a local area is not represented. So in order to deal with this weak point, the change of neighboring normal vectors should be represented as a curvature. Generally, principal curvatures and principal directions can be obtained by solving the second fundamental form of curved surfaces. They have five parameters in comparing to 2-D GPM so it is not adequate to make a new characteristic space using these features for computer processing.

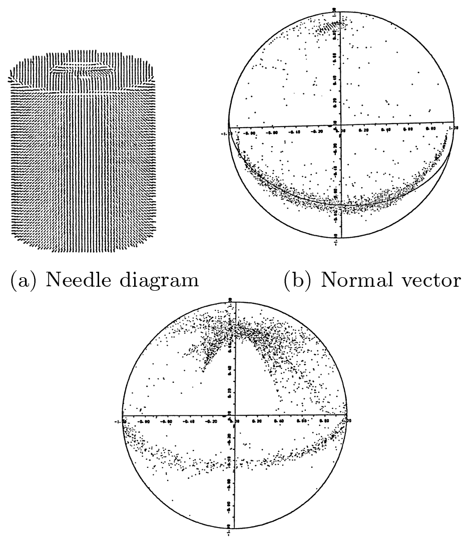
As we know, two principal directions are orthogonal each other. So we choose one of two principal directions, where the norm of the principal curvature corresponding to the chosen principal direction is smaller than another one. In this case, this chosen principal direction corresponds to the generating line of a cylinder and the principal curvature of orthogonal direction shows the radius of the cylinder. So we employ the particular tangential vector described above as a feature to represent objects. This feature means that a curved surface can be approximately represented as some cylindrical surfaces. A radius of a Gaussian sphere is proportional to the magnitude of a principal curvature in this parameter space. So, a shape of a GPM projected from such a Gaussian sphere becomes three dimensional parameter space such as a cylinder.

A figure in this GPM produced by this method is similar to the sweeping trajectory of a generalized cylinder method[9] because the figure in the GPM shows the change of a pose of a cylinder. For example, a figure in a GPM corresponding to a torus is an ellipse, which shows the rotation of the axis of a cylinder. In a case of a cone, a different type of an ellipse, which shows the rotation of the generating line of the cone, appears and the ellipse is swept along an axis indexing the norm of a principal curvature. A plane also makes an ellipse on GPM because the plane is equal to a cylinder having curvature = 0 and the direction of its axis is indeterminant. In a case of a sphere, the magnitude of the curvature is constant but the principal direction is not uniquely decided so the end points of projected vectors are distributed in a GPM. So trajectory of tangential vectors selected in our method can be used as a new characteristic value.

5.2 Expansion of GPM for tangential vector space

In the expanded GPM, which has a new axis to index the magnitude of the curvature, we can apply the conventional image processing such as peak points extraction methods and ellipse extraction methods. **Fig.5-7** show experimental results of a cylinder, a cube and a torus. These figures were obtained by the projection of the expanded GPM along the axis of the curvature so we note that these data have the different magnitude of the curvature $-\alpha \sim +\alpha$. The cylinder was extracted as a peak point in the GPM (**Fig.5**). The cube having three visible planes has three ellipses in the GPM (**Fig.6**). **Fig.7**(a) shows the needle diagram of a torus. This torus can be considered to be made by rotating a low height cylinder around the view direction. Therefore, ellipses appear in II and IV quadrants of the GPM (Fig.7(c)) because each cylinder appears as a peak point in the GPM. In this figure, we can also find other ellipses in I and III quadrants. These were made by some cones which were in contact with either the hole of the torus or the edge of the outside of the torus.

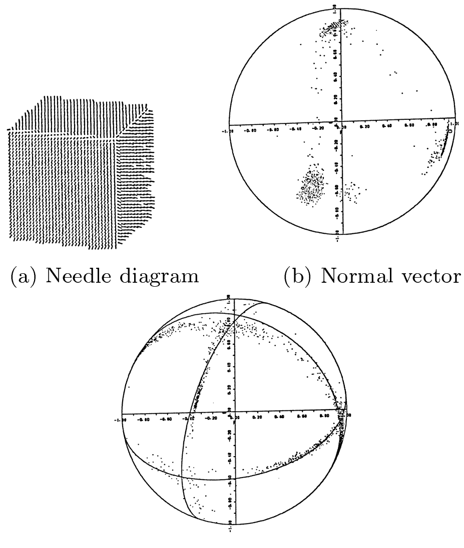
In this extended GPM method, curved objects are represented as a set of some cylinders. So both a plane and a sphere, which have been individually represented up to this time, can be represented as one kind of curved surfaces in the same characteristic space. It seems that this representation of a plane is redundant because many parameters of an ellipse are needed. However, we can note that a tangential vector of a cylinder corresponds to a crossing point of multiple ellipses, which are produced from multiple planes. Namely, this method is equal to the polygon approximation of the cylinder. The polygon approximation of 3-D objects is related to the size of the pixel of an image plane because the size is used as the measure of approximation. But information about the size of objects to be searched in the scene is not usually given in advance. So, we think that our approach using this tangential vector space is useful to estimate a curved surface having multiple partial curved surfaces by a top-down approach.



(a) Needle diagram (b) Normal vector

(c) Tangential vector

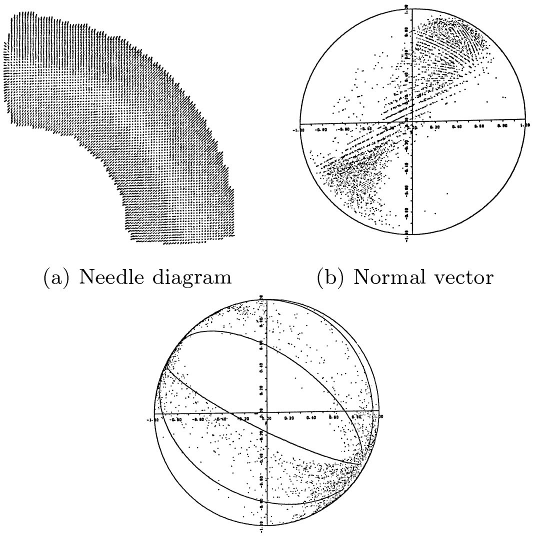
Fig. 5 CYLINDER



(a) Needle diagram (b) Normal vector

(c) Tangential vector

Fig. 6 CUBE



(a) Needle diagram (b) Normal vector

(c) Tangential vector

Fig. 7 RING(1/4)

6. Conclusion

When intrinsic characteristics of 3-D objects are given with 3-D vectors, we proposed a new method to project a Gaussian sphere involving these vectors into a plane and to estimate these characteristics as features of 2-D figures in the projected plan. These 2-D figures show the continuous change of vectors corresponding to 3-D characteristics of curved objects and can be extracted by conventional image processing methods.

A generalized cone method is one of a typical volumetric representation method for curved 3-D curved objects. On the other hand, our GPM method is a surface representation method. So our method is not equal to the former one in a strict sense. However, if the models of objects having a sweeping rule of the generalized cone method can be employed and the sweeping rule can be represented as a 2-D figure in the GPM, it is easy to find the matching between the models and objects in the scene.

A problem to be solved is how to divide a certain curved object into some basic surfaces. If an adjacency between neighboring surfaces can not be estimated clearly, complex objects might not be found sufficiently. Moreover, there is no obvious way to deal with the occlusion among objects in our method since our method is a bottom-up approach. In our future work, we are going to try to use this proposed method with a top-down approach in order to choose the intrinsic characteristics of 3-D objects adequately.

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