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Model Reference Adaptive Control for Nonlinear Systems with Unknown Degrees†

—General Case—

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In most of the studies of model reference adaptive control, it is assumed that an upper bound on the degree of the controlled system is known. It makes the scope of application of model reference adaptive control too restrictive, since the reasonable upper bound on the degree cannot be specified a priori in many practical cases.
In the present paper, we propose a design method of model reference adaptive control systems for nonlinear systems with unknown degrees. The present adaptive controller is composed of high gain feedbacks of hierarchical structures derived from backstepping techniques, and the degree of it is independent of the degree of the controlled system. It is shown that the resulting control system is uniformly bounded, and that the tracking error converges to an arbitrarily small residual region. Finally, several simulation studies also show the effectiveness of the proposed method.

Key Words: model reference adaptive control, nonlinear system, degree, backstepping

1. Introduction

In most of the studies of model reference adaptive control, it is assumed that an upper bound on the degree of the controlled system is known[1)]. However, that assumption may become too restrictive for many practical processes, since reasonable upper bounds on the degrees of those processes cannot be specified a priori. Hence, the study of model reference adaptive control for processes with unknown degrees has been of great importance from both theoretical and practical point of view.

Recently, several attempts have been made to introduce sliding mode control or high gain feedback control techniques into adaptive control schemes, and to construct robust adaptive systems[2)]. Utilizing those techniques, we proposed design methods of model reference adaptive control for processes with unknown degrees[3), 4)], robotic manipulators with unknown nonlinear elements[5), 6)], and distributed parameter systems of infinite degrees[7), 8)]. However, those methods are effective for processes with relative degree 1 only. When the relative degree is greater than 1, the derivatives of the output are also needed.

In order to solve those problems, simple adaptive control (SAC)[9)] and VS-MRAC[10)] schemes were proposed. In SAC schems[9)], parallel feedforward compensators (PFC) are introduced in the control loop so as to make augmented systems (composed of controlled process and PFC) almost strictly positive real (ASPR), when relative degrees of controlled processes are greater than 1. However those approaches did not directly overcome the obstacle of relative degrees for original processes. VS-MRAC schems[10)] were proposed for the processes with arbitrary relative degrees using only input and output measurements. But, in that method, equivalent controls of sliding surface are needed for implementation, which are not really available, and then it was suggested that the equivalent controls are replaced by average ones for practical implementation. Therefore, global stability could not be assured, and convergence properties were not fully investigated in that approach. On the contrary, we proposed a design method of model reference adaptive control for nonlinear systems with unknown degrees and uncertain relative degrees 1 or 2[11)] by extending universal controllers of Morse[12)]. But it was not clear whether those strategies could be applied to general relative degree cases.

In the present paper, we partially extend the previous results[3)∼8), 11)] to the general relative degree case, that is, the relative degree of the process is known, but greater than 1 or 2. From only input-output measurement, the stable model reference adaptive control systems can be constructed for nonlinear systems with unknown degrees and with known relative degrees by utilizing high gain feedbacks of hierarchical structures derived from backstepping techniques[13)]. It is shown that the degree of the present adaptive controller is independent of the degree of the controlled process, and that the tracking error

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converges to a small residual region whose amplitude can be prescribed arbitrarily.

2. Problem Statement

We consider a single-input single-output nonlinear system described as follows :

$$\frac{d}{dt}x(t) = Ax(t) + bu(t) + Gf(y(t), t) \quad (1)$$

$$y(t) = c^T x(t) \quad (2)$$

where $x(t) \in \mathbf{R}^n$, $A \in \mathbf{R}^{n\times n}$, $b, c \in \mathbf{R}^n$; $G \in \mathbf{R}^{n\times m}$; $f(y(t), t)$ ($\in \mathbf{R}^m$) is an unknown nonlinear term or disturbance. For that controlled system (1), (2), only the input $u(t)$ and the output $y(t)$ are assumed to be measurable, but the state $x(t)$ and A, b, c, G, $f(y(t), t)$ are unknown. Also, the degrees n and m are unknown. The following assumptions are introduced.

Assumption 1.
1) The zeros of $c^T(sI - A)^{-1}b$ lie in $\mathbf{C}^-$.
2) The relative degree of $c^T(sI - A)^{-1}b$, denoted as n^*, is known a priori. And the sign of the high frequency gain b_0 of $c^T(sI - A)^{-1}b$ is also known. In the following context, it is assumed that $b_0 > 0$ without loss of generality.
3) For uniformly bounded y_M, and $e \equiv y - y_M$, $f(y, t) = f(y_M + e, t)$ is evaluated in the following:

$$\|f(y(t), t)\| \leq f_0 + f_1 \cdot F(e(t)) \qquad (F(0) = 0) \quad (3)$$

$$F(e(t))^2 \leq f_2 \cdot e(t) \cdot \phi(e(t)) \qquad (\phi(0) = 0) \quad (4)$$

$$e(t) \equiv y(t) - y_M(t) \quad (5)$$

where f_i ($0 \leq f_0, f_1 < \infty$, $0 < f_2 < \infty$) are unknown parameters, which are determined by $f(y, t)$ and y_M; $F(e)$ and $\phi(e)$ are known functions; $\phi(e)$ is n^*-times differentiable with respect to e.

A scalar function $y_M(t)$ is defined as a reference signal. For that reference signal, it is assumed that

Assumption 2. $y_M(t)$, $\dot{y}_M(t)$, $\cdots$, $y_M^{(n^*)}(t)$ are uniformly bounded.

Then the control problem of this paper can be stated as follows : Given a nonlinear system (1), (2) with unknown parameters, unknown degrees n and m, an unknown state $x(t)$, and a known relative degree n^*, and given a known reference signal $y_M(t)$, determine a suitable controller such that the tracking error $e(t)$ converges to a small residual region.

Remark 1. A typical example of the nonlinear element in Assumption 1-3 is given by

$$\|f(y, t)\| \leq M \cdot |y|^m \quad (0 < M < \infty)$$

for which, it follows that $F(e) = |e|^m$ and $\phi(e) = e^{2m-1}$.

3. System Representation

In the present section, we derive an input-output representation of the controlled system (1), (2) by utilizing the zero dynamics of $c^T(sI - A)^{-1}b$. The following lemma is obtained.

Lemma 1. On Assumption 1-1 and 1-2, the controlled system is represented as follows :

$$\frac{d}{dt}y(t) = \theta y(t) + b_0 u_{fn^*-1}(t) + \mathcal{L}(y(t)) + \mathcal{L}(f(y, t)) + \epsilon(t) \quad (6)$$

$$\frac{d}{dt}u_{fn^*-i}(t) = -\lambda_i u_{fn^*-i}(t) + u_{fn^*-i-1}(t), \quad (7)$$
$$(1 \leq i \leq n^* - 1) \quad (u_{f0}(t) \equiv u(t))$$

where θ, b_0 are unknown system parameters; λ_i are arbitrary positive constants (known); $\epsilon(t)$ is an exponentially decaying term; $\mathcal{L}(v(t))$ is defined by

$$\mathcal{L}(v(t)) \equiv G_0(s)v(t) \qquad (G_0(s) \in \mathcal{RH}^\infty) \quad (8)$$

The proof is easily obtained by utilizing Assumption 1-1 and 1-2, and by extending Lemma 1 in 11).

4. Nonlinear adaptive control

4.1 Preliminary

The next lemma is also needed for stability analysis of the adaptive systems.

Lemma 2. The following inequality holds.

$$\int_0^t v(\tau)\{\mathcal{L}(y(\tau)) + \mathcal{L}(f(y, \tau)) + \epsilon(\tau)\}d\tau \leq \frac{1}{2C}\int_0^t v(\tau)^2 d\tau + CL_0 t + CL_1\int_0^t e(\tau)^2 d\tau + CL_2\int_0^t F(e(\tau))^2 d\tau + CL_3\int_0^t \epsilon(\tau)^2 d\tau \quad (9)$$

where $L_0 \sim L_3 > 0$ are unknown system parameters determined by $\mathcal{L}(y(t))$, $\mathcal{L}(f(y(t), t)$ and $y_M(t)$; $C(> 0)$ is an arbitrary constant (known).

The proof is derived from Assumption 1-3, and Lemma 3 in 11).

4.2 Construction Method I

The design procedure consists of n^* steps derived from backstepping techniques[13].

<u>**Step 1)**</u> Define $z_1(t)$ by

$$z_1(t) \equiv e(t) = y(t) - y_M(t) \quad (10)$$

and take the time derivative of it by using Lemma 1.

$$\dot{z}_1(t) = \theta y(t) - \dot{y}_M(t) + b_0 u_{fn^*-1}(t) + w(t) \quad (11)$$

$$w(t) \equiv \mathcal{L}(y(t)) + \mathcal{L}(f(y(t), t)) + \epsilon(t) \quad (12)$$

If $n^* = 1$, then $u_{fn^*-1}(t) = u(t)$. In that case, the design procedure is completed by setting $u(t) \equiv \alpha_1(t)$, and $z_2(t) \equiv 0$ in the following context. Otherwise, when $n^* \geq 2$, introduce new variables $z_2(t)$ and $\alpha_1(t)$ such that

$$z_2(t) \equiv u_{fn^*-1}(t) - \alpha_1(t) \quad (13)$$

$$\alpha_1(t) = -\hat{k}_{11}(t)z_1(t) - \hat{k}_{12}(t)\phi(z_1(t)) - \hat{k}_{13}(t)y(t) + \hat{k}_{14}(t)\dot{y}_M(t) \quad (14)$$

where $\hat{k}_{1j}(t)\,(1 \leq j \leq 4)$ are tuning parameters defined by

$$\begin{aligned}
\dot{\hat{k}}_{11}(t) &= g_{11}N[\,\|z(t)\|\,]z_1(t)^2 \\
\dot{\hat{k}}_{12}(t) &= g_{12}N[\,\|z(t)\|\,]\phi(z_1(t))z_1(t) \\
\dot{\hat{k}}_{13}(t) &= g_{13}N[\,\|z(t)\|\,]y(t)z_1(t) \\
\dot{\hat{k}}_{14}(t) &= -g_{14}N[\,\|z(t)\|\,]\dot{y}_M(t)z_1(t) \quad (15)
\end{aligned}$$

$(g_{11} \sim g_{14} > 0)$

$$N[\,\|z(t)\|\,] \equiv \begin{cases} 1 & \text{if } \|z(t)\| \geq \epsilon^* \quad \text{(Case I)} \\ 0 & \text{if } \|z(t)\| < \epsilon^* \quad \text{(Case II)} \end{cases} \quad (16)$$

$$\epsilon^* > 0 \quad (17)$$

$$z(t) \equiv [z_1(t),\, z_2(t),\, \cdots,\, z_{n^*}(t)]^T \quad (18)$$

$z_i(t)\,(3 \leq i \leq n^*)$ are signals to be determined later, and $\epsilon^*(> 0)$ is a design parameter which prescribes the magnitude of the output $e(t)(= z_1(t))$. For stability analysis, we define a positive function $V_1(t)$ by

$$V_1(t) \equiv \frac{1}{2}z_1(t)^2 + \frac{b_0}{2}\sum_{j=1}^{4}\{k_{1j} - \hat{k}_{1j}(t)\}^2/g_{1j} \quad (19)$$

$$k_{13} \equiv \theta/b_0, \quad k_{14} \equiv 1/b_0 \quad (20)$$

where k_{11} and k_{12} are positive constants to be determined later. We take the time derivative of $V_1(t)$ along its trajectory when $\|z(t)\| \geq \epsilon^*$ (Case I).

$$\dot{V}_1(t) = b_0z_1(t)z_2(t) + z_1(t)w(t) - b_0k_{11}z_1(t)^2 - b_0k_{12}z_1(t)\phi(z_1(t)) \quad (21)$$

Here, we integrate $\dot{V}_1(\tau)$ over an interval $t_0 \leq \tau \leq t$ ($\{t_0 \leq \tau \leq t : \|z(\tau)\| \geq \epsilon^*\} \equiv$ Case I), then obtain the following inequality by utilizing Lemma 2.

$$\begin{aligned}
&V_1(t) - V_1(t_0) \\
&\leq (\frac{1}{2C} + CL_1 - b_0k_{11})\int_{t_0}^{t} z_1(\tau)^2 d\tau \\
&+(CL_2 - \frac{b_0k_{12}}{f_2})\int_{t_0}^{t} F(z_1(\tau))^2 d\tau + CL_0(t - t_0) \\
&+b_0\int_{t_0}^{t} z_1(\tau)z_2(\tau)d\tau + CL_3\int_{t_0}^{t} \epsilon(\tau)^2 d\tau \quad (22)
\end{aligned}$$

Step 2) Take the time derivative of $z_2(t)$.

$$\dot{z}_2(t) = -\lambda_1 u_{fn^*-1}(t) + u_{fn^*-2}(t) - \dot{\alpha}_1(t) \quad (23)$$

$$\dot{\alpha}_1(t) = \beta_1(t) + \gamma_1(t)\{\theta y(t) + b_0 u_{fn^*-1}(t) + w(t)\} \quad (24)$$

$$\beta_1(t) \equiv \frac{\partial\alpha_1}{\partial\hat{K}_1}\dot{\hat{K}}_1(t) + \frac{\partial\alpha_1}{\partial Y_M^{(1)}}\dot{Y}_M^{(1)}(t) - \frac{\partial\alpha_1}{\partial z_1}\dot{y}_M(t) \quad (25)$$

$$\gamma_1(t) \equiv \frac{\partial\alpha_1}{\partial z_1}, \quad (26)$$

$$\hat{K}_1(t) \equiv [\hat{k}_{11}(t),\, \hat{k}_{12}(t),\, \hat{k}_{13}(t),\, \hat{k}_{14}(t)]^T \quad (27)$$

$$Y_M^{(1)}(t) \equiv [y_M(t),\, \dot{y}_M(t)]^T. \quad (28)$$

If $n^* = 2$, then $u_{fn^*-2}(t) = u(t)$. In that case, the design procedure is completed by setting $u(t) \equiv \alpha_2(t)$, and $z_3(t) \equiv 0$ in the following context. Otherwise, when $n^* \geq 3$, following variables $z_3(t)$ and $\alpha_2(t)$ are introduced.

$$z_3(t) \equiv u_{fn^*-2}(t) - \alpha_2(t) \quad (29)$$

$$\begin{aligned}
\alpha_2(t) = {} & \lambda_1 u_{fn^*-1}(t) + \tilde{\beta}_1(t) + \hat{\theta}^{(2)}(t)\gamma_1(t)y(t) \\
&+\hat{b}_0^{(2)}\{\gamma_1(t)u_{fn^*-1}(t) - z_1(t)\} \\
&-\hat{k}_{21}(t)\gamma_1(t)^2z_2(t) - \hat{k}_{22}(t)z_2(t) \quad (30)
\end{aligned}$$

where $\hat{\theta}^{(2)}(t)$, $\hat{b}_0^{(2)}(t)$, $\hat{k}_{21}(t)$, $\hat{k}_{22}(t)$ are tuning parameters defined by

$$\begin{aligned}
\dot{\hat{\theta}}^{(2)}(t) &= -g_{21}N[\,\|z(t)\|\,]\gamma_1(t)y(t)z_2(t) \\
\dot{\hat{b}}_0^{(2)}(t) &= -g_{22}N[\,\|z(t)\|\,]\{\gamma_1(t)u_{fn^*-1}(t) - z_1(t)\}z_2(t) \\
\dot{\hat{k}}_{21}(t) &= g_{23}N[\,\|z(t)\|\,]\gamma_1(t)^2z_2(t)^2 \\
\dot{\hat{k}}_{22}(t) &= g_{24}N[\,\|z(t)\|\,]z_2(t)^2 \quad (31)
\end{aligned}$$

$\tilde{\beta}_1(t)$ is a variable composed of the same elements as those in $\beta_1(t)$, an analytic function of its components, and satisfies the following statement.

$$z_2(t)\{\tilde{\beta}_1(t) - \beta_1(t)\} \leq 0 \quad \text{when } \|z(t)\| \geq \epsilon^* \quad (32)$$

For stability analysis, a positive function $V_2(t)$ is introduced.

$$V_2(t) = V_1(t) + \frac{1}{2}z_2(t)^2 + \frac{1}{2}\sum_{j=1}^{4}\{\psi_{2j} - \hat{\psi}_{2j}(t)\}^2/g_{2j} \quad (33)$$

$$\begin{aligned}
&\psi_{21} = \theta, \quad \psi_{22} = b_0, \quad \psi_{23} = k_{21}, \quad \psi_{24} = k_{22}, \\
&\hat{\psi}_{21}(t) = \hat{\theta}^{(2)}(t), \quad \hat{\psi}_{22}(t) = \hat{b}_0^{(2)}(t), \\
&\hat{\psi}_{23}(t) = \hat{k}_{21}(t), \quad \hat{\psi}_{24}(t) = \hat{k}_{22}(t) \quad (34)
\end{aligned}$$

where k_{21}, k_{22} are positive constants to be determined later. Considering (23)~(32), then $V_2(\tau)$ in Case I ($t_0 \leq \tau \leq t : \|z(\tau)\| \geq \epsilon^*$), is evaluated as follows:

$$\begin{aligned}
&V_2(t) - V_2(t_0) \\
&\leq (\frac{1}{2C} + 2CL_1 - b_0k_{11})\int_{t_0}^{t} z_1(\tau)^2 d\tau \\
&+(2CL_2 - \frac{b_0k_{12}}{f_2})\int_{t_0}^{t} F(z_1(\tau))^2 d\tau \\
&+2CL_0(t - t_0) + (\frac{1}{2C} - k_{21})\int_{t_0}^{t} \gamma_1(\tau)^2 z_2(\tau)^2 d\tau
\end{aligned}$$

$$-k_{22}\int_{t_0}^{t} z_2(\tau)^2 d\tau + \int_{t_0}^{t} z_2(\tau)z_3(\tau)d\tau + 2CL_3\int_{t_0}^{t}\epsilon(\tau)^2 d\tau \quad (35)$$

Step i) $(3 \le i \le n^*-1)$ Define $z_i(t)$ and take the time derivative of it.

$$z_i(t) \equiv u_{fn^*-i+1}(t) - \alpha_{i-1}(t), \quad (36)$$

$$\dot{z}_i(t) = -\lambda_{i-1}u_{fn^*-i+1}(t) + u_{fn^*-i}(t) - \dot{\alpha}_{i-1}(t) \quad (37)$$

$$\dot{\alpha}_{i-1}(t) = \beta_{i-1}(t) + \gamma_{i-1}(t)\{\theta y(t) + b_0 u_{fn^*-1}(t) + w(t)\} \quad (38)$$

$$\beta_{i-1}(t) \equiv \frac{\partial\alpha_{i-1}}{\partial u_f^{(n^*-i+2)}}\dot{u}_f^{(n^*-i+2)}(t) + \frac{\partial\alpha_{i-1}}{\partial\hat{K}_{i-1}}\dot{\hat{K}}_{i-1}(t) + \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial z_j}\{-\lambda_{j-1}u_{fn^*-j+1}(t) + u_{fn^*-j}(t) - \beta_{j-1}(t)\} + \frac{\partial\alpha_{i-1}}{\partial Y_M^{(i-1)}}\dot{Y}_M^{(i-1)}(t) - \frac{\partial\alpha_{i-1}}{\partial z_1}\dot{y}_M(t) \quad (39)$$

$$\gamma_{i-1}(t) \equiv \frac{\partial\alpha_{i-1}}{\partial z_1} - \sum_{j=2}^{i-1}\frac{\alpha_{i-1}}{\partial z_j}\gamma_{j-1}(t) \quad (40)$$

$$\hat{K}_{i-1}(t) \equiv [\hat{K}_{i-2}(t)^T, \hat{\theta}^{(i-1)}(t), \hat{b}_0^{(i-1)}(t), \hat{k}_{i-1,1}(t), \hat{k}_{i-1,2}(t)]^T \quad (41)$$

$$u_f^{(n^*-i+2)}(t) \equiv [u_{fn^*-1}(t), \cdots, u_{fn^*-i+2}(t)]^T \quad (42)$$

$$\left(u_f^{(n^*-1)}(t) \equiv u_{fn^*-1}(t)\right) \quad (43)$$

$$Y_M^{(i-1)}(t) \equiv [y_M(t), \dot{y}_M(t), \cdots, y_M^{(i-1)}(t)]^T \quad (44)$$

For $z_i(t)$, new variables $z_{i+1}(t), \alpha_i(t)$ are introduced in the following:

$$z_{i+1}(t) \equiv u_{fn^*-i}(t) - \alpha_i(t) \quad (45)$$

$$\alpha_i(t) = \lambda_{i-1}u_{fn^*-i+1}(t) + \tilde{\beta}_{i-1}(t) + \hat{\theta}^{(i)}(t)\gamma_{i-1}(t)y(t) + \hat{b}_0^{(i)}(t)\gamma_{i-1}(t)u_{fn^*-1}(t) - \hat{k}_{i1}(t)\gamma_{i-1}(t)^2 z_i(t) - \hat{k}_{i2}(t)z_i(t) - z_{i-1}(t) \quad (46)$$

$$\begin{aligned}\dot{\hat{\theta}}^{(i)}(t) &= -g_{i1}N[\|z(t)\|]\gamma_{i-1}(t)y(t)z_i(t)\\ \dot{\hat{b}}_0^{(i)}(t) &= -g_{i2}N[\|z(t)\|]\gamma_{i-1}(t)u_{fn^*-1}(t)z_i(t)\\ \dot{\hat{k}}_{i1}(t) &= g_{i3}N[\|z(t)\|]\gamma_{i-1}(t)^2 z_i(t)^2\\ \dot{\hat{k}}_{i2}(t) &= g_{14}N[\|z(t)\|]z_i(t)^2\end{aligned} \quad (47)$$

$\tilde{\beta}_{i-1}(t)$ is a variable composed of the same elements as those in $\beta_{i-1}(t)$, an analytic function of its components, and satisfies the following statement.

$$z_i(t)\{\tilde{\beta}_{i-1}(t) - \beta_{i-1}(t)\} \le 0 \quad \text{when } \|z(t)\| \ge \epsilon^* \quad (48)$$

For stability analysis, a positive function $V_i(t)$ is introduced.

$$V_i(t) = V_{i-1}(t) + \frac{1}{2}z_i(t)^2 + \frac{1}{2}\sum_{j=1}^{4}\{\psi_{ij} - \hat{\psi}_{ij}(t)\}/g_{ij} \quad (49)$$

$$\begin{aligned}&\psi_{i1} = \theta,\ \psi_{i2} = b_0,\ \psi_{i3} = k_{i1},\ \psi_{i4} = k_{i2},\\ &\hat{\psi}_{i1}(t) = \hat{\theta}^{(i)}(t),\ \hat{\psi}_{i2}(t) = \hat{b}_0^{(i)}(t),\\ &\hat{\psi}_{i3}(t) = \hat{k}_{i1}(t),\ \hat{\psi}_{i4}(t) = \hat{k}_{i2}(t)\end{aligned} \quad (50)$$

where k_{i1}, k_{i2} are positive constants to be determined later. Considering (36)∼(48), then $V_i(\tau)$ in Case I ($t_0 \le \tau \le t : \|z(\tau)\| \ge \epsilon^*$), is evaluated in the following:

$$\begin{aligned}V_i(t) - V_i(t_0) &\le \left(\frac{1}{2C} + iCL_1 - b_0k_{11}\right)\int_{t_0}^{t} z_1(\tau)^2 d\tau\\ &+\left(iCL_2 - \frac{b_0k_{12}}{f_2}\right)\int_{t_0}^{t} F(z_1(\tau))^2 d\tau\\ &+iCL_0(t-t_0)\\ &+\sum_{j=2}^{i}\left(\frac{1}{2C} - k_{j1}\right)\int_{t_0}^{t}\gamma_{j-1}(t)^2 z_j(\tau)^2 d\tau\\ &-\sum_{j=2}^{i}k_{j2}\int_{t_0}^{t} z_j(\tau)^2 d\tau\\ &+\int_{t_0}^{t} z_i(\tau)z_{i+1}(\tau)d\tau + iCL_3\int_{t_0}^{t}\epsilon(\tau)^2 d\tau\end{aligned} \quad (51)$$

Step n^*) Define $z_{n^*}(t)$ and take the time derivative of it.

$$z_{n^*}(t) \equiv u_{f1}(t) - \alpha_{n^*-1}(t) \quad (52)$$

$$\dot{z}_{n^*}(t) = -\lambda_{n^*-1}u_{f1}(t) + u(t) - \dot{\alpha}_{n^*-1}(t) \quad (53)$$

$$\dot{\alpha}_{n^*-1}(t) = \beta_{n^*-1}(t) + \gamma_{n^*-1}(t)\{\theta y(t) + b_0 u_{fn^*-1}(t) + w(t)\} \quad (54)$$

$$\beta_{n^*-1}(t) \equiv \frac{\partial\alpha_{n^*-1}}{\partial u_f^{(2)}}\dot{u}_f^{(2)}(t) + \frac{\partial\alpha_{n^*-1}}{\partial\hat{K}_{n^*-1}}\dot{\hat{K}}_{n^*-1}(t) + \sum_{j=2}^{n^*-1}\frac{\partial\alpha_{n^*-1}}{\partial z_j}\{-\lambda_{j-1}u_{fn^*-j+1}(t) + u_{fn^*-j}(t) - \beta_{j-1}(t)\} + \frac{\partial\alpha_{n^*-1}}{\partial Y_M^{(n^*-1)}}\dot{Y}_M^{(n^*-1)}(t) - \frac{\partial\alpha_{n^*-1}}{\partial z_1}\dot{y}_M(t) \quad (55)$$

$$\gamma_{n^*-1}(t) \equiv \frac{\partial\alpha_{n^*-1}}{\partial z_1} - \sum_{j=2}^{n^*-1}\frac{\alpha_{n^*-1}}{\partial z_j}\gamma_{j-1}(t) \quad (56)$$

$$\hat{K}_{n^*-1}(t) \equiv [\hat{K}_{n^*-2}(t)^T, \hat{\theta}^{(n^*-1)}(t), \hat{b}_0^{(n^*-1)}(t), \hat{k}_{n^*-1,1}(t), \hat{k}_{n^*-1,2}(t)]^T \quad (57)$$

$$u_f^{(2)}(t) \equiv [u_f^{(3)}(t)^T, u_{f2}(t)]^T \quad (58)$$

$$Y_M^{(n^*-1)}(t) \equiv [y_M(t),\ \dot{y}_M(t),\ \cdots,\ y_M^{(n^*-1)}(t)]^T \quad (59)$$

Note that the actual control input $u(t)$ appears in $\dot{z}_{n^*}(t)$. $u(t)$ is determined in the following.

$$\begin{aligned} u(t) &= \lambda_{n^*-1}u_{f1}(t) + \tilde{\beta}_{n^*-1}(t) \\ &\quad -z_{n^*-1}(t) + \hat{\theta}^{(n^*)}(t)\gamma_{n^*-1}(t)y(t) \\ &\quad +\hat{b}_0^{(n^*)}(t)\gamma_{n^*-1}(t)u_{fn^*-1}(t) \\ &\quad -\hat{k}_{n^*1}(t)\gamma_{n^*-1}(t)^2 z_{n^*}(t) - \hat{k}_{n^*2}(t)z_{n^*}(t) \end{aligned} \quad (60)$$

where $\hat{\theta}^{(n^*)}(t)$, $\hat{b}_0^{(n^*)}(t)$, $\hat{k}_{n^*1}(t)$, $\hat{k}_{n^*2}(t)$ are defined by

$$\begin{aligned} \dot{\hat{\theta}}^{(n^*)}(t) &= -g_{n^*1}N[\,\|z(t)\|\,]\gamma_{n^*-1}(t)y(t)z_{n^*}(t) \\ \dot{\hat{b}}_0^{(n^*)}(t) &= -g_{n^*2}N[\,\|z(t)\|\,]\gamma_{n^*-1}(t)u_{fn^*-1}(t)z_{n^*}(t) \\ \dot{\hat{k}}_{n^*1}(t) &= g_{n^*3}N[\,\|z(t)\|\,]\gamma_{n^*-1}(t)^2 z_{n^*}(t)^2 \\ \dot{\hat{k}}_{n^*2}(t) &= g_{n^*4}N[\,\|z(t)\|\,]z_{n^*}(t)^2 \end{aligned} \quad (61)$$

$\tilde{\beta}_{n^*-1}(t)$ is constructed in the same way as $\tilde{\beta}_{i-1}(t)$ ((32),(48)). For stability analysis, a positive function $V_{n^*}(t)$ is introduced.

$$\begin{aligned} V_{n^*}(t) &= V_{n^*-1}(t) + \frac{1}{2}z_{n^*}(t)^2 \\ &\quad +\frac{1}{2}\sum_{j=1}^{4}\{\psi_{n^*j} - \hat{\psi}_{n^*j}(t)\}/g_{n^*j} \end{aligned} \quad (62)$$

where ψ_{n^*j}, $\hat{\psi}_{n^*j}(t)$ $(1 \le j \le 4)$ are defined by (50) $(i = n^*)$, and $\psi_{n^*3} = k_{n^*1}$, $\psi_{n^*4} = k_{n^*2}$ are positive constants to be determined later. Considering (52)~(61), then $V_{n^*}(\tau)$ in Case I $(t_0 \le \tau \le t : \|z(\tau)\| \ge \epsilon^*)$, is evaluated as follows:

$$\begin{aligned} &V_{n^*}(t) - V_{n^*}(t_0) \\ &\le (\frac{1}{2C} + n^*CL_1 - b_0k_{11})\int_{t_0}^{t} z_1(\tau)^2 d\tau \\ &\quad +(n^*CL_2 - \frac{b_0k_{12}}{f_2})\int_{t_0}^{t} F(z_1(\tau))^2 d\tau \\ &\quad +n^*CL_0(t-t_0) \\ &\quad +\sum_{j=2}^{n^*}(\frac{1}{2C} - k_{j1})\int_{t_0}^{t}\gamma_{j-1}(t)^2 z_j(\tau)^2 d\tau \\ &\quad -\sum_{j=2}^{n^*}k_{j2}\int_{t_0}^{t} z_j(\tau)^2 d\tau \\ &\quad +n^*CL_3\int_{t_0}^{t}\epsilon(\tau)^2 d\tau \end{aligned} \quad (63)$$

Step n^*+1) Stability Analysis Note that $\epsilon(t) \in \mathcal{L}^2$, and let $V(t)$ be defined by

$$V(t) \equiv V_{n^*}(t) + n^*CL_3\int_{t}^{\infty}\epsilon(\tau)^2 d\tau \quad (64)$$

Here we set the parameter k_{ij} $(1 \le i \le n^*,\ j = 1,2)$ such that

$$\begin{aligned} k_{11} &> (\frac{1}{2C} + n^*CL_1 + \tilde{k}_{11})/b_0 \\ k_{12} &> n^*CL_2f_2/b_0 \\ \tilde{k}_{11} &> n^*CL_0/\epsilon^{*2} \\ k_{j1} &> \frac{1}{2C} \quad k_{j2} > n^*CL_0/\epsilon^{*2} \quad (2 \le j \le n^*) \end{aligned} \quad (65)$$

where C is an any positive constant. Then, $V(\tau)$ in Case I $(t_0 \le \tau \le t : \|z(\tau)\| \ge \epsilon^*)$, is evaluated in the following:

$$\begin{aligned} &V(t) - V(t_0) \\ &\le -\tilde{k}_{11}\int_{t_0}^{t} z_1(\tau)^2 d\tau - \sum_{i=2}^{n^*}k_{i2}\int_{t_0}^{t} z_i(\tau)^2 d\tau \\ &\quad +n^*CL_0(t-t_0) \\ &\le -(n^*CL_0/\epsilon^{*2})\int_{t_0}^{t}\{\|z(\tau)\|^2 - \epsilon^{*2}\}d\tau \le 0 \end{aligned} \quad (66)$$

On the other hand, when $\|z(t)\| < \epsilon^*$ (Case II), tuning parameters $\hat{k}_{ij}(t)$, $\hat{\theta}^{(i)}(t)$, $\hat{b}_0^{(i)}(t)$ are constant. Therefore, it is shown that $V(t) \in \mathcal{L}^\infty$, and that $z_i(t) \in \mathcal{L}^\infty$, $\hat{K}_{n^*}(t) \equiv [\hat{K}_{n^*-1}(t)^T, \hat{\theta}^{(n^*)}(t), \hat{b}_0^{(n^*)}(t),\ \hat{k}_{n^*1}(t), \hat{k}_{n^*2}(t)]^T \in \mathcal{L}^\infty$, $F(z_1(t))$, $f(z_1(t),t) \in \mathcal{L}^\infty$. Note that $\alpha_1(t) \in \mathcal{L}^\infty$ and $u_{fn^*-1}(t) = z_2(t)+\alpha_1(t)$, then it follows that $u_{fn^*-1}(t) \in \mathcal{L}^\infty$. Repeating the similar procedure, it is also shown that $\alpha_2(t) \sim \alpha_{n^*-1}(t) \in \mathcal{L}^\infty$, $u_{fn^*-2}(t) \sim u_{f1}(t) \in \mathcal{L}^\infty$, and that $u(t) \in \mathcal{L}^\infty$. Hence, it is proved that the resulting control system is uniformly bounded. Next, we analyze the convergence property of $z(t)$. First, from above facts, it follows that $\dot{z}_i(t) \in \mathcal{L}^\infty$ $(1 \le i \le n^*)$. Since $\dot{\hat{k}}_{11}(t) \ge 0$, $\hat{k}_{11}(t) \in \mathcal{L}^\infty$, and $\dot{\hat{k}}_{i2}(t) \ge 0$, $\hat{k}_{i2}(t) \in \mathcal{L}^\infty$ $(2 \le i \le n^*)$, we say that $\hat{k}_{11}(\infty)$, $\hat{k}_{i2}(\infty)$ $(2 \le i \le n^*)$ exist. Hence, it is shown that $\|z(t)\|(N[\,\|z(t)\|\,])^{1/2} \in \mathcal{L}^2$. Considering that $\dot{z}_i(t) \in \mathcal{L}^\infty$, we see that

$$\lim_{t\to\infty}\|z(t)\|^2 N[\,\|z(t)\|\,] = 0 \quad (67)$$

and derive the following statement.

$$z(t) \to S(\epsilon^*) \equiv \{z \equiv (z_1,\ \cdots,\ z_{n^*}) : \|z\| < \epsilon^*\} \quad (68)$$

This concludes Construction Method I. The next theorem is one of the main theorems of this paper.

Theorem 1. Consider a controlled system (1), (2) with Construction Method I. Suppose that Assumption 1 and Assumption 2 can be met. Then it follows that all the signals in the resulting adaptive control system are uniformly bounded, and that the state variables $z_1(t) \sim z_{n^*}(t)$(where $z_1(t) \equiv e(t)$: output error) converge to the residual region defined by $S(\epsilon^*)$ (68), where ϵ^* is an arbitrary positive constant.

Remark 2. The relations of each signals are depicted in **Fig. 1**. First, $z_1(t) \equiv e(t)$ is defined, then $\alpha_1(t)$ is determined from $z_1(t)$, and $z_2(t)$ is derived from $u_{fn^*-1}(t)$ and $\alpha_1(t)$. Similarly, $\alpha_2(t)$, $z_3(t)$, $\cdots$, $\alpha_{n^*-1}(t)$, $z_{n^*}(t)$ are obtained in sequence, and finally, the actual control input $u(t)$ is determined. The design parameter ϵ^* is a ar-

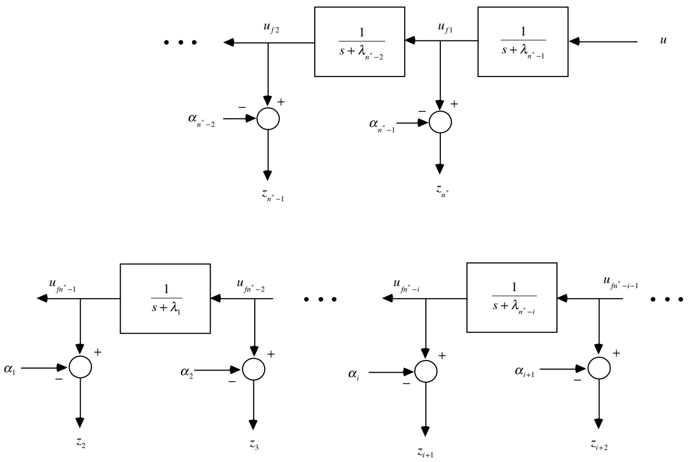

Fig. 1 Design procedure of adaptive controller

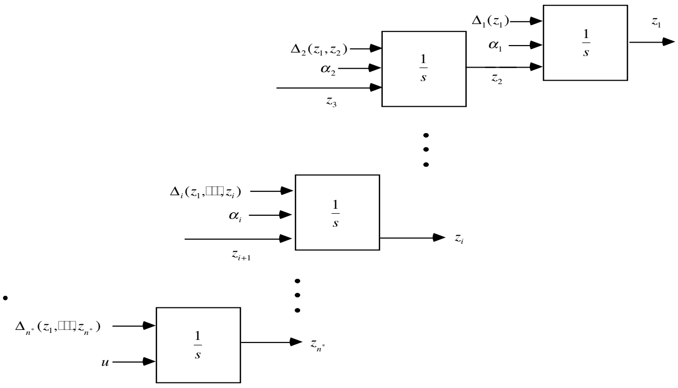

Fig. 2 Configuration of overall system

bitrary positive constant, but it cannot be equal to zero, that is, the magnitude of n^*-dimensional state vector $z(t)$ where $e(t) = z_1(t)$ is included, can be made arbitrarily small, but cannot be made 0. For smaller output errors, ϵ^* should be set smaller, but it may give rise to large control inputs, since necessary control parameters (65) become large. Hence, ϵ^* should be chosen properly considering the magnitudes of both output errors and control inputs.

Remark 3. By setting the positive constant C properly in (65), either k_{j1} or k_{j2} $(2 \leq j \leq n^*)$ can be made arbitrary positive constants. Hence, either $\hat{k}_{j1}(t)$ or $\hat{k}_{j2}(t)$ $(2 \leq j \leq n^*)$ may be any positive constants. Tuning parameters $\hat{\theta}^{(i)}(t)$ and $\hat{b}_0^{(i)}(t)$ are current estimates of θ and b_0 at step i) $(2 \leq i \leq n^*)$. Then, $2(n^*-1)$ different estimates are needed for two system parameters θ and b_0. Thus, the minimal number of tuning parameters in the present Method I is $3n^*+1$. In the next section, it will be shown that this number can be made less.

Remark 4. One way to obtain $\tilde{\beta}_i(t)$ from $\beta_i(t)$ is to replace $N[||z(t)||]$ in $\beta_{i-1}(t)$ with 1. Then, the resulting $\tilde{\beta}_i(t)$ is analytic, and satisfy $\tilde{\beta}_{i-1}(t) = \beta_{i-1}(t)$ in Case I.

Remark 5. $y^{(n^*-1)}(t) \sim \dot{y}(t)$ are included in $u_{f1}(t) \sim u_{fn^*-1}(t)$. This is an extension of indirect configuration of time derivative of output signals in 12). In each steps, high-gain feedback techniques are applied for relative-degree 1 systems by utilizing virtual input $\alpha_i(t)$. Then, overall systems are constructed by high-gain feedback control schemes of hierarchical structure depicted in **Fig. 2**.

4.3 Construction Method II

Contrary to the previous Construction Method I, in the present Construction Method II, the necessary current estimates of θ and b_0 are $\hat{\theta}(t)$ and $\hat{b}_0(t)$ only, instead of $2(n^*-1)$ estimates $\hat{\theta}^{(i)}(t)$ and $\hat{b}_0^{(i)}(t)$ $(2 \leq i \leq n^*)$. The design procedure is composed of n^* steps, too, but several auxiliary signals are added.

Step 1) Same as Construction Method I.

Step 2) Take the time derivative of $z_2(t)$. If $n^* = 2$, then the procedure is completed. On the other hand, when $n^* \geq 3$, define $z_3(t)$ by (29), and determine $\alpha_2(t)$ in the following:

$$\begin{aligned} \alpha_2(t) &= \lambda_1 u_{fn^*-1}(t) + \tilde{\beta}_1(t) + \hat{\theta}(t)\gamma_1(t)y(t) \\ &\quad + \hat{b}_0\{\gamma_1(t)u_{fn^*-1}(t) - z_1(t)\} \\ &\quad - \hat{k}_{21}(t)\gamma_1(t)^2 z_2(t) - \hat{k}_{22}(t)z_2(t) \end{aligned} \quad (69)$$

$$\begin{aligned} \dot{\hat{k}}_{21}(t) &= g_{21}N[\,||z(t)||\,]\gamma_1(t)^2 z_2(t)^2 \\ \dot{\hat{k}}_{22}(t) &= g_{22}N[\,||z(t)||\,]z_2(t)^2 \end{aligned} \quad (70)$$

Adaptive laws of $\hat{\theta}(t)$ and $\hat{b}_0(t)$ are to be determined later. Define $V_2(t)$ by

$$\begin{aligned} V_2(t) &= V_1(t) + \frac{1}{2}z_2(t)^2 + \frac{1}{2}\sum_{j=1}^{2}\{k_{2j} - \hat{k}_{2j}(t)\}^2/g_{2j} \\ &\quad + \frac{1}{2}\{\theta - \hat{\theta}(t)\}^2/g_{23} + \frac{1}{2}\{b_0 - \hat{b}_0(t)\}^2/g_{24} \\ &\quad (g_{21} \sim g_{24} > 0) \end{aligned} \quad (71)$$

and determine variables $\tau_{\theta 2}(t), \tau_{b2}(t)$ in the following:

$$\tau_{\theta 2}(t) \equiv -g_{22}\gamma_1(t)y(t)z_2(t) \quad (72)$$

$$\tau_{b2}(t) \equiv -g_{23}\{\gamma_1(t)u_{fn^*-1}(t) - z_1(t)\} \quad (73)$$

Then $V_2(\tau)$ in Case I $(t_0 \leq \tau \leq t : ||z(\tau)|| \geq \epsilon^*)$, is evaluated as follows:

$$\begin{aligned} & V_2(t) - V_2(t_0) \\ &\leq (\frac{1}{2C} + 2CL_1 - b_0 k_{11})\int_{t_0}^{t} z_1(\tau)^2 d\tau \\ &\quad + (2CL_2 - \frac{b_0 k_{12}}{f_2})\int_{t_0}^{t} F(z_1(\tau))^2 d\tau \\ &\quad + 2CL_0(t - t_0) \\ &\quad + (\frac{1}{2C} - k_{21})\int_{t_0}^{t} \gamma_1(\tau)^2 z_2(\tau)^2 d\tau \end{aligned}$$

$$-k_{22}\int_{t_0}^{t} z_2(\tau)^2 d\tau$$
$$+\int_{t_0}^{t} z_2(\tau)z_3(\tau)d\tau + 2CL_3\int_{t_0}^{t}\epsilon(\tau)^2 d\tau$$
$$+\int_{t_0}^{t}\{\theta-\hat\theta(\tau)\}\{-\dot{\hat\theta}(\tau)+\tau_{\theta 2}(\tau)\}d\tau/g_{23}$$
$$+\int_{t_0}^{t}\{b_0-\hat b_0(\tau)\}\{-\dot{\hat b}_0(\tau)+\tau_{b2}(\tau)\}d\tau/g_{24} \quad (74)$$

Step 3) Take the time derivative of $z_3(t)$.

$$\dot z_3(t) = -\lambda_2 u_{fn^*-2}(t) + u_{fn^*-3}(t) - \dot\alpha_2(t) \quad (75)$$
$$\dot\alpha_2(t) = \beta_2(t) + \gamma_2(t)\{\theta y(t) + b_0 u_{fn^*-1}(t) + w(t)\} + \frac{\partial\alpha_2}{\partial\hat\theta}\dot{\hat\theta}(t) + \frac{\partial\alpha_2}{\partial\hat b_0}\dot{\hat b}_0(t) \quad (76)$$
$$\hat K_2(t) \equiv [\hat k_{11}(t), \hat k_{12}(t), \hat k_{13}(t), \hat k_{14}(t), \hat k_{21}(t), \hat k_{22}(t)]^T \quad (77)$$

$\beta_2(t)$ is defined similarly to (39), but $\hat K_2(t)$ is different from the previous (41). All other variables are defined in the same way as Method I. If $n^*=3$, then the procedure is completed. For $n^*\geq 4$, introduce $z_4(t)$ and $\alpha_3(t)$

$$\alpha_3(t) = \lambda_2 u_{fn^*-2}(t) + \tilde\beta_2(t) + \hat\theta(t)\gamma_2(t)y(t) + \hat b_0(t)\gamma_2(t)u_{fn^*-1}(t) - \hat k_{31}(t)\gamma_2(t)^2 z_3(t) - \hat k_{32}(t)z_3(t) - z_2(t) + \frac{\partial\alpha_2}{\partial\hat\theta}\tau_{\theta 3}(t) + \frac{\partial\alpha_2}{\partial\hat b_0}\tau_{b3}(t) \quad (78)$$
$$\dot{\hat k}_{31}(t) = g_{31}N[\|z(t)\|]\gamma_2(t)^2 z_3(t)^2$$
$$\dot{\hat k}_{32}(t) = g_{32}N[\|z(t)\|]z_3(t)^2 \quad (79)$$

Also, $\tilde\beta_2(t)$ is the same as the previous one. For stability analysis, a positive function $V_3(t)$ is introduced.

$$V_3(t) = V_2(t) + \frac{1}{2}z_3(t)^2 + \frac{1}{2}\sum_{j=1}^{2}\{k_{3j}-\hat k_{3j}(t)\}^2/g_{3j} \quad (80)$$

By setting $\tau_{\theta 3}(t), \tau_{b3}(t)$ in the following:

$$\tau_{\theta 3}(t) \equiv \tau_{\theta 2}(t) - g_{22}\gamma_2(t)y(t)z_3(t) \quad (81)$$
$$\tau_{b3}(t) \equiv \tau_{b2}(t) - g_{23}\gamma_2(t)u_{fn^*-1}(t)z_3(t) \quad (82)$$

then $V_3(\tau)$ in Case I ($t_0 \leq \tau \leq t : \|z(\tau)\| \geq \epsilon^*$), is evaluated as follows:

$$V_3(t) - V_3(t_0)$$
$$\leq (\frac{1}{2C} + 3CL_1 - b_0 k_{11})\int_{t_0}^{t} z_1(\tau)^2 d\tau$$
$$+(3CL_2 - \frac{b_0 k_{12}}{f_2})\int_{t_0}^{t} F(z_1(\tau))^2 d\tau$$
$$+3CL_0(t-t_0)$$
$$+\sum_{j=2}^{3}(\frac{1}{2C} - k_{j1})\int_{t_0}^{t}\gamma_{j-1}(t)^2 z_j(\tau)^2 d\tau$$
$$-\sum_{j=2}^{3}k_{j2}\int_{t_0}^{t} z_j(\tau)^2 d\tau$$
$$+\int_{t_0}^{t} z_3(\tau)z_4(\tau)d\tau + 3CL_3\int_{t_0}^{t}\epsilon(\tau)^2 d\tau$$
$$+\int_{t_0}^{t}\frac{\partial\alpha_2}{\partial\hat\theta}\{\tau_{\theta 3}(\tau) - \dot{\hat\theta}(\tau)\}z_3(\tau)d\tau$$
$$+\int_{t_0}^{t}\frac{\partial\alpha_2}{\partial\hat b_0}\{\tau_{b3}(\tau) - \dot{\hat b}_0(\tau)\}z_3(\tau)d\tau$$
$$+\int_{t_0}^{t}\{\theta-\hat\theta(\tau)\}\{-\dot{\hat\theta}(\tau)+\tau_{\theta 3}(\tau)\}d\tau/g_{23}$$
$$+\int_{t_0}^{t}\{b_0-\hat b_0(\tau)\}\{-\dot{\hat b}_0(\tau)+\tau_{b3}(\tau)\}d\tau/g_{24} \quad (83)$$

Step i) ($4 \leq i \leq n^*-1$) Define $z_i(t)$ similarly, and take the time derivative of it.

$$\dot z_i(t) = -\lambda_{i-1}u_{fn^*-i+1}(t) + u_{fn^*-i}(t) - \dot\alpha_{i-1}(t) \quad (84)$$
$$\dot\alpha_{i-1}(t) = \beta_{i-1}(t) + \gamma_{i-1}(t)\{\theta y(t) + b_0 u_{fn^*-1}(t) + w(t)\} + \gamma_{\theta i-1}(t)\dot{\hat\theta}(t) + \gamma_{bi-1}(t)\dot{\hat b}_0(t) \quad (85)$$
$$\hat K_{i-1}(t) \equiv [\hat K_{i-2}(t)^T, \hat k_{i-1,1}(t), \hat k_{i-1,2}(t)]^T \quad (86)$$
$$\gamma_{\theta i-1}(t) \equiv \frac{\partial\alpha_{i-1}}{\partial\hat\theta} - \sum_{j=3}^{i-1}\gamma_{\theta j-1}(t)\frac{\partial\alpha_{i-1}}{\partial z_j} \quad (87)$$
$$\left(\gamma_{\theta 2}(t) \equiv \frac{\partial\alpha_2}{\partial\hat\theta}\right) \quad (88)$$
$$\gamma_{bi-1}(t) \equiv \frac{\partial\alpha_{i-1}}{\partial\hat b_0} - \sum_{j=3}^{i-1}\gamma_{bj-1}(t)\frac{\partial\alpha_{i-1}}{\partial z_j} \quad (89)$$
$$\left(\gamma_{b2}(t) \equiv \frac{\partial\alpha_2}{\partial\hat b_0}\right) \quad (90)$$

$\beta_{i-1}(t)$ is similar to (39), but the definition of $\hat K_{i-1}(t)$ is different from (41). All other signals are defined in the same way as Method I. Introduce variables $z_{i+1}(t)$ and $\alpha_i(t)$

$$\alpha_i(t) = \lambda_{i-1}u_{fn^*-i+1}(t) + \tilde\beta_{i-1}(t) + \hat\theta(t)\gamma_{i-1}(t)y(t) + \hat b_0(t)\gamma_{i-1}(t)u_{fn^*-1}(t) - \hat k_{i1}(t)\gamma_{i-1}(t)^2 z_i(t) - \hat k_{i2}(t)z_i(t) - z_{i-1}(t) + \gamma_{\theta i-1}(t)\tau_{\theta i}(t) + \gamma_{bi-1}(t)\tau_{bi}(t) + \tilde\alpha_i(t) \quad (91)$$
$$\dot{\hat k}_{i1}(t) = g_{i1}N[\|z(t)\|]\gamma_{i-1}(t)^2 z_i(t)^2$$
$$\dot{\hat k}_{i2}(t) = g_{i2}N[\|z(t)\|]z_i(t)^2 \quad (92)$$

$\tilde\alpha_i(t)$ is an auxiliary signal to be determined later. A positive function $V_i(t)$ is introduced.

$$V_i(t) = V_{i-1}(t) + \frac{1}{2}z_i(t)^2$$

$$+\frac{1}{2}\sum_{j=1}^{2}\{k_{ij}-\hat{k}_{ij}(t)\}^2/g_{ij} \tag{93}$$

Then, setting $\tau_{\theta i}(t)$, $\tau_{bi}(t)$ by

$$\begin{aligned}\tau_{\theta i}(t) &\equiv \tau_{\theta i-1}(t)-g_{23}\gamma_{i-1}(t)y(t)z_i(t)\\ \tau_{bi}(t) &\equiv \tau_{bi-1}(t)-g_{24}\gamma_{i-1}(t)u_{fn^*-1}(t)z_i(t)\end{aligned} \tag{94}$$

$V_i(\tau)$ in Case I ($t_0 \le \tau \le t : \|z(\tau)\| \ge \epsilon^*$), is evaluated as follows:

$$\begin{aligned}&V_i(t)-V_i(t_0)\\ &\le(\frac{1}{2C}+iCL_1-b_0k_{11})\int_{t_0}^{t}z_1(\tau)^2d\tau\\ &+(iCL_2-\frac{b_0k_{12}}{f_2})\int_{t_0}^{t}F(z_1(\tau))^2d\tau\\ &+iCL_0(t-t_0)\\ &+\sum_{j=2}^{i}(\frac{1}{2C}-k_{j1})\int_{t_0}^{t}\gamma_{j-1}(\tau)^2z_j(\tau)^2d\tau\\ &-\sum_{j=2}^{i}k_{j2}\int_{t_0}^{t}z_j(\tau)^2d\tau\\ &+\int_{t_0}^{t}z_i(\tau)z_{i+1}(\tau)d\tau+iCL_3\int_{t_0}^{t}\epsilon(\tau)^2d\tau\\ &+\sum_{j=3}^{i}\int_{t_0}^{t}\gamma_{\theta j-1}(\tau)\{\tau_{\theta j}(\tau)-\dot{\hat{\theta}}(\tau)\}z_j(\tau)d\tau\\ &+\sum_{j=3}^{i}\int_{t_0}^{t}\gamma_{bj-1}(\tau)\{\tau_{bj}(\tau)-\dot{\hat{b}}_0(\tau)\}z_j(\tau)d\tau\\ &+\int_{t_0}^{t}\{\theta-\hat{\theta}(\tau)\}\{-\dot{\hat{\theta}}(\tau)+\tau_{\theta i}(\tau)\}d\tau/g_{23}\\ &+\int_{t_0}^{t}\{b_0-\hat{b}_0(\tau)\}\{-\dot{\hat{b}}_0(\tau)+\tau_{bi}(\tau)\}d\tau/g_{24}\\ &+\sum_{j=4}^{i}\int_{t_0}^{t}\tilde{\alpha}_j(\tau)z_j(\tau)d\tau\end{aligned} \tag{95}$$

Step n*) Define $z_{n^*}(t)$ similarly and take the time derivative of it.

$$\dot{z}_{n^*}(t)=-\lambda_{n^*-1}u_{f1}(t)+u(t)-\dot{\alpha}_{n^*-1}(t) \tag{96}$$

$$\begin{aligned}\dot{\alpha}_{n^*-1}(t)=&\ \beta_{n^*-1}(t)+\gamma_{n^*-1}(t)\{\theta y(t)\\ &+b_0u_{fn^*-1}(t)+w(t)\}+\gamma_{\theta n^*-1}(t)\dot{\hat{\theta}}(t)\\ &+\gamma_{bn^*-1}(t)\dot{\hat{b}}_0(t)\end{aligned} \tag{97}$$

$$\hat{K}_{n^*-1}(t)\equiv[\hat{K}_{n^*-2}(t)^T,\hat{k}_{n^*-1,1}(t),\hat{k}_{n^*-1,2}(t)]^T \tag{98}$$

$$\gamma_{\theta n^*-1}(t)\equiv\frac{\partial\alpha_{n^*-1}}{\partial\hat{\theta}}-\sum_{j=3}^{n^*-1}\gamma_{\theta j-1}(t)\frac{\partial\alpha_{n^*-1}}{\partial z_j} \tag{99}$$

$$\gamma_{bn^*-1}(t)\equiv\frac{\partial\alpha_{n^*-1}}{\partial\hat{b}_0}-\sum_{j=3}^{n^*-1}\gamma_{bj-1}(t)\frac{\partial\alpha_{n^*-1}}{\partial z_j} \tag{100}$$

$\beta_{n^*-1}(t)$ is defined similar to (55), but $\hat{K}_{n^*-1}(t)$ is different from (57). The definitions of other signals are the same as the previous case. The control input $u(t)$ is determined such that

$$\begin{aligned}u(t)=&\ \lambda_{n^*-1}u_{f1}(t)+\tilde{\beta}_{n^*-1}(t)+\hat{\theta}(t)\gamma_{n^*-1}(t)y(t)\\ &+\hat{b}_0(t)\gamma_{n^*-1}(t)u_{fn^*-1}(t)\\ &-\hat{k}_{n^*1}(t)\gamma_{n^*-1}(t)^2z_{n^*}(t)\\ &-\hat{k}_{n^*2}(t)z_{n^*}(t)-z_{n^*-1}(t)\\ &+\gamma_{\theta n^*-1}(t)\tau_{\theta n^*}(t)+\gamma_{bn^*-1}(t)\tau_{bn^*}(t)\\ &+\tilde{\alpha}_{n^*}(t)\end{aligned} \tag{101}$$

$$\begin{aligned}\dot{\hat{k}}_{n^*1}(t)&=g_{n^*1}N[\|z(t)\|]\gamma_{n^*-1}(t)^2z_{n^*}(t)^2\\ \dot{\hat{k}}_{n^*2}(t)&=g_{n^*2}N[\|z(t)\|]z_{n^*}(t)^2\end{aligned} \tag{102}$$

$\tilde{\alpha}_{n^*}(t)$ is an auxiliary signal to be determined later. For stability analysis, a positive function $V_{n^*}(t)$ is introduced.

$$\begin{aligned}V_{n^*}(t)=&\ V_{n^*-1}(t)+\frac{1}{2}z_{n^*}(t)^2\\ &+\frac{1}{2}\sum_{j=1}^{2}\{k_{n^*j}-\hat{k}_{n^*j}(t)\}^2/g_{n^*j}\end{aligned} \tag{103}$$

We determine $\tau_{\theta n^*}(t)$, $\tau_{bn^*}(t)$ and the adaptive law of $\hat{\theta}(t)$ and $\hat{b}(t)$ in the following:

$$\tau_{\theta n^*}(t)=\tau_{\theta n^*-1}(t)-g_{22}\gamma_{n^*-1}(t)y(t)z_{n^*}(t) \tag{104}$$

$$\tau_{bn^*}(t)=\tau_{bn^*-1}(t)-g_{23}\gamma_{n^*-1}(t)u_{fn^*-1}(t)z_{n^*}(t) \tag{105}$$

$$\dot{\hat{\theta}}(t)=N[\|z(t)\|]\tau_{\theta n^*}(t) \tag{106}$$

$$\dot{\hat{b}}_0(t)=N[\|z(t)\|]\tau_{bn^*}(t) \tag{107}$$

Then $V_{n^*}(\tau)$ in Case I ($t_0 \le \tau \le t : \|z(\tau)\| \ge \epsilon^*$), is evaluated by

$$\begin{aligned}&V_{n^*}(t)-V_{n^*}(t_0)\\ &\le(\frac{1}{2C}+n^*CL_1-b_0k_{11})\int_{t_0}^{t}z_1(\tau)^2d\tau\\ &+(n^*CL_2f_1^2-\frac{b_0k_{12}}{f_2})\int_{t_0}^{t}F(z_1(\tau))^2d\tau\\ &+n^*CL_0(t-t_0)\\ &+\sum_{j=2}^{n^*}(\frac{1}{2C}-k_{j1})\int_{t_0}^{t}\gamma_{j-1}(t)^2z_j(\tau)^2d\tau\\ &-\sum_{j=2}^{n^*}k_{j2}\int_{t_0}^{t}z_j(\tau)^2d\tau+n^*CL_3\int_{t_0}^{t}\epsilon(\tau)^2d\tau\\ &+\sum_{j=3}^{n^*-1}\int_{t_0}^{t}\gamma_{\theta j-1}(\tau)\{\tau_{\theta j}(\tau)-\tau_{\theta n^*}(\tau)\}z_j(\tau)d\tau\\ &+\sum_{j=3}^{n^*-1}\int_{t_0}^{t}\gamma_{bj-1}(\tau)\{\tau_{bj}(\tau)-\tau_{bn^*}(\tau)\}z_j(\tau)d\tau\end{aligned}$$

$$+\sum_{j=4}^{n^*}\int_{t_0}^{t}\tilde{\alpha}_j(\tau)z_j(\tau)d\tau \quad (108)$$

Step $\mathbf{n^*+1}$) Stability Analysis In this final step, we determine $\tilde{\alpha}_j(t)$ $(4 \le j \le n^*)$ in order to assure the stability of the overall system. Define $\Delta_{n^*}(t)$ and evaluate it as follows:

$$\begin{aligned}\Delta_{n^*}(t) \equiv & \sum_{j=3}^{n^*-1}\gamma_{\theta j-1}(t)\{\tau_{\theta j}(t)-\tau_{\theta n^*}(t)\}z_j(t)\\ &+\sum_{j=3}^{n^*-1}\gamma_{bj-1}(t)\{\tau_{bj}(t)-\tau_{bn^*}(t)\}z_j(t)\\ &+\sum_{j=4}^{n^*}\tilde{\alpha}_j(t)z_j(t)\\ =&\sum_{j=3}^{n^*-1}\sum_{k=j+1}^{n^*}\gamma_{\theta j-1}(t)g_{23}\gamma_{k-1}(t)y(t)z_k(t)z_j(t)\\ &+\sum_{j=3}^{n^*-1}\sum_{k=j+1}^{n^*}\gamma_{bj-1}(t)g_{24}\gamma_{k-1}(t)u_{fn^*-1}(t)z_k(t)z_j(t)\\ &+\sum_{j=4}^{n^*}\tilde{\alpha}_j(t)z_j(t)\\ =&\sum_{k=4}^{n^*}\sum_{j=3}^{k-1}\gamma_{\theta j-1}(t)z_j(t)g_{23}\gamma_{k-1}(t)y(t)z_k(t)\\ &+\sum_{k=4}^{n^*}\sum_{j=3}^{k-1}\gamma_{bj-1}(t)z_j(t)g_{24}\gamma_{k-1}(t)u_{fn^*-1}(t)z_k(t)\\ &+\sum_{j=4}^{n^*}\tilde{\alpha}_j(t)z_j(t) \quad (109)\end{aligned}$$

Here, we determine $\tilde{\alpha}_j(t)$ $(4 \le j \le n^*)$ by

$$\begin{aligned}\tilde{\alpha}_j(t) =& -\sum_{k=3}^{j-1}\gamma_{\theta k-1}(t)z_k(t)g_{23}\gamma_{j-1}(t)y(t)\\ &-\sum_{k=3}^{j-1}\gamma_{bk-1}(t)z_k(t)g_{24}\gamma_{j-1}(t)u_{fn^*-1}(t)\end{aligned} \quad (110)$$

then we see that $\Delta_{n^*}(t) \equiv 0$, and that $V_{n^*}(\tau)$ in Case I $(t_0 \le \tau \le t : \|z(\tau)\| \ge \epsilon^*)$, can be evaluated in the same way as Construction Method I. Note that new components are not added in each step by introducing $\tilde{\alpha}_j(t)$. Thus, the same conclusion is derived.

Theorem 2. Consider a controlled system (1), (2) with Construction Method II. Suppose that Assumption 1 and 2 can be met. Then the same conclusion as Construction Method I is derived.

Remark 6. In Construction Method II, the minimal number of tuning parameters is n^*+5 for $n^* \ge 2$, and 4 for $n^*=1$. The adaptive laws in Method I and II are not different from each other essentially. However, it is though that the good transient property is often attained in Method II because of the few tuning parameters, although there is no large difference between two response curves in the numerical examples of the next section.

5. Numerical Example

Numerical simulation studies are performed to show the effectiveness of the proposed methods. Let us consider the following system as a controlled system ($n=52$: unknown, $m=1$: unknown, $n^*=3$: known).

$$\frac{d}{dt}x(t) = Ax(t)+bu(t)+Gf(y(t),t) \quad (111)$$

$$y(t) = c^T(t), \quad (112)$$

$$A=\begin{bmatrix} -a_1 & 0 & \cdots & 0 & 0 & 0\\ 0 & -a_2 & \ddots & \vdots & \vdots & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots & \vdots\\ 0 & \cdots & 0 & -a_{50} & 0 & 0\\ c_1 & c_2 & \cdots & c_{50} & -a_{51} & 0\\ 0 & \cdots & \cdots & 0 & 1 & -a_{52}\end{bmatrix}$$

$$b=[\underbrace{1\,1\,\cdots\,1}_{49}\;0\,0\,0]^T$$

$$c=[\underbrace{0\,0\,\cdots\,0}_{51}\;1]^T$$

$$G=[\underbrace{0\,0\,\cdots\,0}_{49}\;1\,0\,0]^T$$

$$f(y(t),t)=0.5\sin(y(t))$$

$$a_i = 0.1\cdot(i-1) \qquad c_i = 1/i \qquad (1 \le i \le 50),$$

$$a_{51}=a_{52}=-1 \quad (113)$$

Design parameters and a reference signal are in the following:

$$\lambda_1=\lambda_2=1 \qquad g_{ij}=10 \qquad \epsilon^*=10^{-6}$$

$$\phi(e)=e$$

$$\left(\frac{d}{dt}+1\right)^3 y_M(t)=\sin t$$

$$(y_M(0)=\dot{y}_M(0)=\ddot{y}_M(0)=0) \quad (114)$$

$\tilde{\beta}_i(t)$ $(i=1,2)$ are determined based on the way in Remark 4. **Fig. 3** show the results where Construction Method I is utilized. **Fig. 4** show the results where Construction Method II is utilized.

6. Conclusion

In the present paper, we proposed design methods of model reference adaptive control systems for nonlinear systems with unknown degrees and with known relative degrees. The key point of these methods are to utilize high gain feedbacks of hierarchical structures derived from

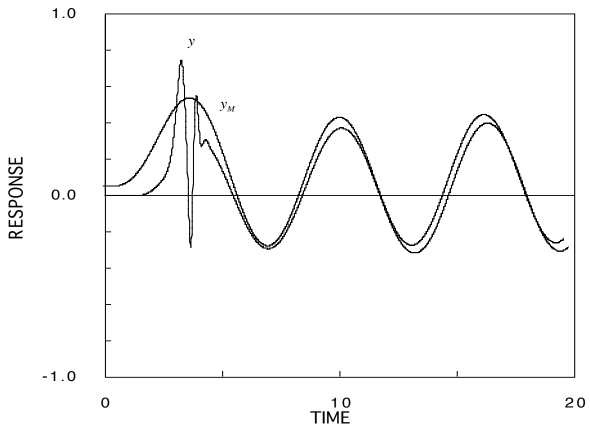

Fig. 3 Simulation result (Method I, $g_{ij} = 10$)

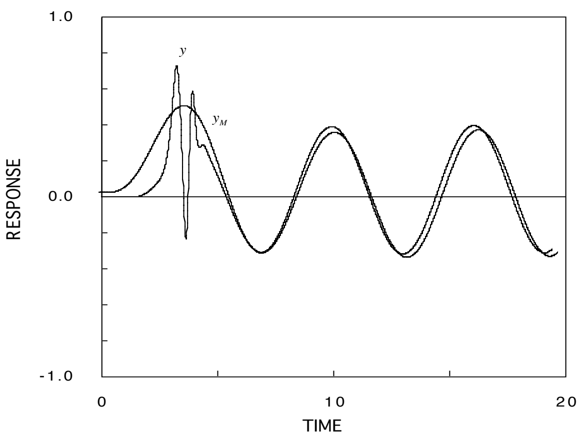

Fig. 4 Simulation result (Method II, $g_{ij} = 10$)

backstepping techniques[13], when the relative degree is greater than 1. Time derivatives of output error signals and strictly positive real error dynamics are obtained in those hierarchical structures. We showed that the resulting control system is uniformly bounded and that the tracking error converges to a residual region whose amplitude can be made arbitrarily small by the design parameter $\epsilon^*(> 0)$.

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