Trans. of the Society of Instrument and Control Engineers
Vol.E-3, No.1, 19 / 24 (2004)

An Optimal Design of Sampled-Data Systems with Communication Constraints†

Hisaya FUJIOKA* and Kensaku ITO**

This paper proposes a design method for NCSs (networked control systems), where plant and controller are linked through a serial communication network. The network has limitted capacity and control inputs and measured outputs are updated/sampled partially at each step. Assuming that the controller-plant communication is periodic, the design problem is formulated as one for sampled-data feedback systems with periodic discrete-time components. A necessary and sufficient condition for existence of discrete-time periodic controller is given in terms of LMIs, and a controller construction algorithm is derived. The proposed controller (if exists) is stabilizing and sub-optimizes the $\mathbf{L}_2$-induced norm of the resultant NCS.

Key Words: networked control systems, sampled-data control, $\mathbf{H}_\infty$ control

1. Introduction

Control systems constructed through a serial communication network is called NCSs (networked control systems) [11]. In comparison to conventional control system connected in a point-to-point manner, NCSs are superior from the following viewpoints: low cost, high reliability, less wiring, easy maintenance and wiring, etc. Hence the use of NCSs is now widely spreading as an implementation method in the field of automobiles, production plants, and airplanes.

Since the communication in NCSs is in a serial manner, the number of sensors and actuators those can access to the controller at a time is limited [7]. The communication constraints do not exist in a point-to-point communication, and is a particular difficulty in the design of NCSs.

Under such communication constraints, it would be natural to switch sensors/actuators that can access controller periodically [7]. A stabilization problem under the periodic switching is considered in References 7), 14), 15) and a necessary and sufficient condition for the stabilization problem and a construction algorithm of a stabilizing controller are provided in References 14), 15). There is, however, no discussion on the performance of the whole system.

The purpose of this paper is to propose a design method of a controller which optimizes the closed-loop performance, in addition to the internal stability, of the NCSs with communication constraints. The closed-loop performance will be evaluated by $\mathbf{L}_2$-induced norm, where we will treat NCSs as a special class of sampled-data feedback control systems. We will derive a necessary and sufficient condition for the existence of the sub-optimizing controller for a given performance level and a synthesis procedure to construct the sub-optimizing controller.

This paper is organized as follows: Section 2 describes the communication constraints in NCSs and formulate it as a sampled-data systems design problem. Section 3 provides a necessary and sufficient condition in terms of LMIs for the synthesis problem of NCSs with communication constraints. A controller synthesis algorithm is also given. Some numerical examples are given in Section 4. Section 5 contains some concluding remarks.

Notation: For a given matrix $A \in \mathbb{R}^{n\times m}$, (i) A' denotes its transpose, (ii) $X \in \mathbb{R}^{(n-r)\times n}$ satisfying $XA = 0$, $XX' > 0$ is denoted by $A^\perp$ where $r := \operatorname{rank} A$. If $A \in \mathbb{R}^{np\times nm}$ has the following structure:

$$A = \begin{bmatrix} A_{11} & 0 & \dots & 0 \\ A_{21} & A_{22} & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix}, \quad A_{ij} \in \mathbb{R}^{p\times m},$$

we write $A \in \mathsf{BLT}(p, m, n)$. For a given positive integer ν, the blocking [9] of a discrete-time signal x for period ν is denoted by $\mathcal{B}_\nu x$, namely,

$$(\mathcal{B}_\nu x)[k] := \begin{bmatrix} x[\nu k] \\ x[\nu k+1] \\ \vdots \\ x[\nu(k+1)-1] \end{bmatrix}.$$

For given systems G and H with appropriate sizes and underlying time domain, the feedback connection of G and H is denoted by $G \star H$ supposing the well-posedness implic-

* Graduate School of Infomatics, Kyoto University, Kyoto
** Kimitsu Works, Nippon Steel Corporation, Kimitsu, Chiba

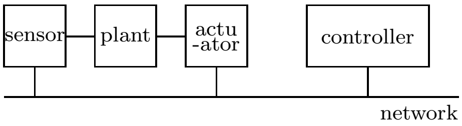

Fig. 1 Networked control system

itly. For a given finite-dimensional discrete-time system G_d with a state-space realization:

$$\begin{bmatrix} x[k+1] \\ y[k] \end{bmatrix} = \begin{bmatrix} A[k] & B[k] \\ C[k] & D[k] \end{bmatrix} \begin{bmatrix} x[k] \\ u[k] \end{bmatrix},$$

we use the following packed notation:

$$G_d \overset{\text{ssr}}{=} \left[\begin{array}{c|c} A[k] & B[k] \\ \hline C[k] & D[k] \end{array}\right]. \tag{1}$$

We also use the same notation for continuous-time systems. $\mathbf{L}_2$- and ℓ_2-induced norms are denoted by $\|G_c\|$ and $\|G_d\|$ for an $\mathbf{L}_2$-stable continuous-time system G_c and an ℓ_2-stable discrete-time system, respectively.

2. Control System Synthesis with Communication Constraints

2.1 Communication Constraints in NCSs

An example of NCSs is depicted in Fig. 1. The plant is connected to the communication network through the sensor(s) and the actuator(s). We assume that each sensors, each actuators, and the controller are independent network nodes.

Sensors and actuators are connected to controller in a point-to-point manner in conventional control systems, and hence the controller can access all sensors and actuators simultaneously. There, however, exists communication constraints in NCSs, e. g., the controller can access one of the sensor and the actuator at a time if the plant is SISO and the network is of the bus structure.

Under such communication constraints, it would be natural to switch sensors/actuators that can access controller periodically [7]. We also deal with the communication constraints by the periodic input/output switching in this paper.

2.2 Control Systems Synthesis

In this subsection, we formulate a control systems synthesis problem with communication constraints as that for sampled-data feedback systems with periodic discrete-time elements (Fig. 2).

In Fig. 2, G_c is a continuous-time FDLTI (finite-dimensional linear time-invariant) system:

$$G_c \overset{\text{ssr}}{:=} \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & 0 & 0 \end{array}\right],$$

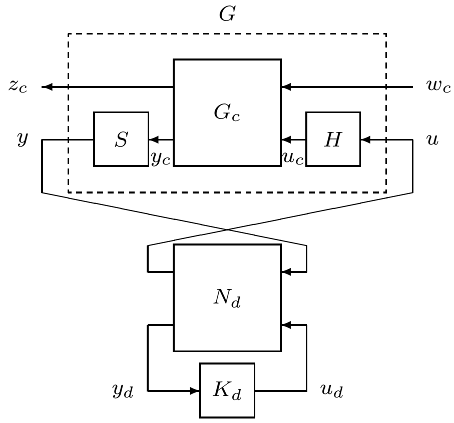

Fig. 2 Sampled-data system with periodic discrete-time components

where $D_{c21} = 0$ and $D_{c22} = 0$ are assumed and the assumption is a necessary condition for the $\mathbf{L}_2$-stability of the closed loop system. S and H are an ideal sampler and a zero-order hold for sampling period h respectively:

$$S : y_c \mapsto y : \; y[k] = y_c(kh),$$

$$H : u \mapsto u_c : \; u_c(kh+\theta) = u[k], \quad \theta \in [0, h).$$

N_d is a periodic discrete-time system with period ν representing the communication channel switching periodically:

$$N_d \overset{\text{ssr}}{:=} \left[\begin{array}{c|cc} A_N[k] & B_{N1}[k] & B_{N2}[k] \\ \hline C_{N1}[k] & 0 & D_{N12}[k] \\ C_{N2}[k] & D_{N21}[k] & 0 \end{array}\right], \tag{2}$$

$$\begin{bmatrix} A_N[k] & B_{N1}[k] & B_{N2}[k] \\ C_{N1}[k] & 0 & D_{N12}[k] \\ C_{N2}[k] & D_{N21}[k] & 0 \end{bmatrix} = \begin{bmatrix} A_N[k+\nu] & B_{N1}[k+\nu] & B_{N2}[k+\nu] \\ C_{N1}[k+\nu] & 0 & D_{N12}[k+\nu] \\ C_{N2}[k+\nu] & D_{N21}[k+\nu] & 0 \end{bmatrix}.$$

K_d is a discrete-time controller to be designed.

Example 1. Consider the case when the following three conditions hold: (i) the plant is SISO, (ii) the network capacity is 1, and (iii) y_d and u_d are updated by tern. In the case $\nu = 2$ and y_d and u are given by

$$y_d[k] = \begin{cases} y[k] & k = 0, 2, 4, \ldots, \\ y[k-1] & k = 1, 3, 5, \ldots \end{cases}$$

$$u[k] = \begin{cases} u_d[k-1] & k = 0, 2, 4, \ldots, \\ u_d[k] & k = 1, 3, 5, \ldots \end{cases}.$$

Consequently this case can be represented in the framework of Fig. 2 by setting

$$N_d \overset{\text{ssr}}{=} \begin{cases} \left[\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] & k = 0,\, 2,\, 4,\, \ldots, \\ \left[\begin{array}{c|cc} 0 & 0 & 1 \\ \hline 0 & 0 & 1 \\ 1 & 0 & 0 \end{array}\right] & k = 1,\, 3,\, 5,\, \ldots \end{cases}.$$

Remark 1. The number of sensors/actuators those the controller K_d can access at time k is given by

$$\text{rank}\begin{bmatrix} 0 & D_{N12}[k] \\ D_{N21}[k] & 0 \end{bmatrix}.$$

In this paper, the following control system synthesis problem is considered(1):

Problem 1. For a given G and N_d, find a controller K_d satisfying

(i) $G \star N_d \star K_d$ is internally stable, and

(ii) $\|G \star N_d \star K_d\| < 1$, where

$$G := \begin{bmatrix} I & 0 \\ 0 & S \end{bmatrix} Gc \begin{bmatrix} I & 0 \\ 0 & H \end{bmatrix}. \tag{3}$$

Remark 2. A design procedure of K_d satisfying the specification (i) is found in [14), 15)].

2.3 Reductions of Specifications

The following lemma is standard in the sampled-data control theory [1)]:

Lemma 1. Suppose that $G \star N_d \star K_d$ is internally stable.

$$\|G \star N_d \star K_d\| \geq \|\mathbf{D}_{11}\|$$

where

$$(\mathbf{D}_{11}w)(\theta) := C_{c1}\int_0^{\theta} \mathrm{e}^{A_c(\theta-\tau)} B_{c1} w(\tau)\,\mathrm{d}\tau + D_{c11} w(\theta).$$

This is a direct consequence of the fact that $\mathbf{D}_{11}$ is the restriction of $G \star N_d \star K_d$ on $[0,\, h]$.

Hence the following assumption is a necessary condition to satisfy the specification above:

Assumption 1. For a given G in (3), the following holds:

$$\|\mathbf{D}_{11}\| < 1. \tag{4}$$

Under Assumption 1, we get the following lemma:

Lemma 2. For given G satisfying Assumption 1, N_d, and K_d, define G_∞ by an $\mathbf{H}_\infty$ norm bound preserving discrete-time system for G given in [1)]. The following statements are equivalent:

(i) $G \star N_d \star K_d$ is internally stable and $\|G \star N_d \star K_d\| < 1$.

(ii) $G_d \star K_d$ is internally stable and $\|G_d \star K_d\| < 1$, where

$$G_d := G_\infty \star N_d. \tag{5}$$

(iii) $\tilde{G}_d \star \tilde{K}_d$ is internally stable and $\left\|\tilde{G}_d \star \tilde{K}_d\right\| < 1$, where

$$\tilde{G}_d := \begin{bmatrix} \mathcal{B}_\nu & 0 \\ 0 & \mathcal{B}_\nu \end{bmatrix} G_d \begin{bmatrix} \mathcal{B}_\nu^{-1} & 0 \\ 0 & \mathcal{B}_\nu^{-1} \end{bmatrix}, \tag{6}$$

$$\tilde{K}_d := \mathcal{B}_\nu K_d \mathcal{B}_\nu^{-1}. \tag{7}$$

Proof: The equivalence between (i) and (ii) is a straightforward extension of [1)], while the equivalent between (ii) and (iii) is trivial since the blocking is isometric (Property 1 in Appendix B). ■

Remark 3. G_d is ν-periodic, and hence no conservatism is introduced if we restrict K_d to ν-periodic systems [2)].

Remark 4. Suppose that K_d is ν-periodic. Both $\tilde{G}_d$ and $\tilde{K}_d$ are time-invariant (Property 2 in Appendix B). Note also that the 'D'-matrix of $\tilde{K}_d$ must have a certain structure (Property 3 in Appendix B).

3. Main Results: LMI-Based Design of NCSs

In this section, a solution to the design problem formulated in the previous section will be given.

Let a state-space form of $\tilde{G}_d$ is given by

$$\tilde{G}_d \overset{\text{ssr}}{=}: \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]. \tag{8}$$

Let also sizes of A and D_{11} be denoted by $n \times n$ and $\pi \times \mu$ respectively. The following theorems give a solution to Problem 1:

Theorem 1. For given G satisfying Assumption 1 and N_d, Problem 1 has a solution if and only if there exist $X = X' \in \mathbb{R}^{n\times n}$, $Y = Y' \in \mathbb{R}^{n\times n}$, and $Z \in \mathsf{BLT}(m,\, p,\, \nu)$ satisfying (9)—(12):

$$\check{B}^{\perp}\left(\check{A}\check{X}_\mu\check{A}' - \check{X}_\pi\right)\left(\check{B}^{\perp}\right)' < 0, \tag{9}$$

$$\left(\check{C}'\right)^{\perp}\left(\check{A}'\check{Y}_\pi\check{A} - \check{Y}_\mu\right)\left(\left(\check{C}'\right)^{\perp}\right)' < 0, \tag{10}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \tag{11}$$

$$\Delta := \begin{bmatrix} \check{X}_\pi & -(\check{A} + \check{B}Z\check{C}) \\ -(\check{A} + \check{B}Z\check{C})' & \check{Y}_\mu \end{bmatrix} > 0. \tag{12}$$

where

$$\check{A} := \begin{bmatrix} A & B_1 \\ C_1 & D_{11} \end{bmatrix}, \quad \check{B} := \begin{bmatrix} B_2 \\ D_{12} \end{bmatrix}, \quad \check{C} := \begin{bmatrix} C_2 & D_{21} \end{bmatrix},$$

(1) Other difficulities in the design of NCSs such that the random time-delay are ignored in this paper.

$$\check{X}_q := \begin{bmatrix} X & 0 \\ 0 & I_q \end{bmatrix}, \quad \check{Y}_q := \begin{bmatrix} Y & 0 \\ 0 & I_q \end{bmatrix}.$$

Theorem 2. Suppose that Problem 1 has a solution. We can construct a solution K_d by the following procedure:

Step 1: Determine $X = X' \in \mathbb{R}^{n\times n}$, $Y = Y' \in \mathbb{R}^{n\times n}$, and $Z \in \mathsf{BLT}(m, p, \nu)$ by solving (9)—(12).

Step 2: Determine $\bar{K}_d$ by

$$\bar{K}_d \overset{\text{ssr}}{:=} \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & Z \end{array}\right] \tag{13}$$

where

$$\begin{aligned} A_K &:= -R^{-1}\Big(YB_2\Theta_C + \Theta_B' C_2 X + Y(A - B_2 Z C_2)X \\ &\quad + \begin{bmatrix} -\mathcal{I}_\pi' & \mathcal{I}_\pi' \check{Y}_\pi \check{A} + \Theta_B' \check{C} \end{bmatrix} \Delta^{-1} \\ &\quad \times \begin{bmatrix} \check{A}\check{X}_\mu \mathcal{I}_\mu + \check{B}\Theta_C \\ -\mathcal{I}_\mu \end{bmatrix}\Big)(Q')^{-1}, \\ B_K &:= R^{-1}(-YB_2 Z + \Theta_B'), \\ C_K &:= (-ZC_2X + \Theta_C)\left(Q'\right)^{-1}, \\ I - XY &=: QR', \\ \Theta_B &:= \begin{bmatrix} I_{\nu p} & 0 \end{bmatrix} \begin{bmatrix} 0_{\nu p} & \begin{bmatrix} 0 & \check{C} \end{bmatrix} \\ \begin{bmatrix} 0 \\ \check{C}' \end{bmatrix} & -\Delta \end{bmatrix}^{\dagger} \begin{bmatrix} 0_{\nu p, n} \\ \mathcal{I}_\mu \\ -\check{A}'\check{Y}_\pi \mathcal{I}_\pi \end{bmatrix}, \\ \Theta_C &:= \begin{bmatrix} I_{\nu m} & 0 \end{bmatrix} \begin{bmatrix} 0_{\nu m} & \begin{bmatrix} \check{B}' & 0 \end{bmatrix} \\ \begin{bmatrix} \check{B} \\ 0 \end{bmatrix} & -\Delta \end{bmatrix}^{\dagger} \begin{bmatrix} 0_{\nu m, n} \\ -\check{A}\check{X}_\mu \mathcal{I}_\mu \\ \mathcal{I}_\pi \end{bmatrix}, \\ \mathcal{I}_q &:= \begin{bmatrix} I_n & 0_{n,q} \end{bmatrix}'. \end{aligned}$$

Step 3: Determine $\tilde{K}_d$ by

$$\begin{aligned} \tilde{K}_d &:= (-D_{22}) \star \begin{bmatrix} I \\ I \end{bmatrix} \bar{K}_d \begin{bmatrix} I & I \end{bmatrix} \qquad (14) \\ &\overset{\text{ssr}}{=} \left[\begin{array}{c|c} A_K - B_K\tilde{R}^{-1}D_{22}C_K & B_K\tilde{R}^{-1} \\ \hline (I + ZD_{22})^{-1}C_K & Z\tilde{R}^{-1} \end{array}\right], \end{aligned}$$

where $\tilde{R} := I + D_{22}Z$.

Step 4: Determine K_d by

$$K_d := \mathcal{B}_\nu^{-1}\tilde{K}_d\mathcal{B}_\nu.$$

Proof: See Appendix A.

Remark 5. A solution K_d to Problem 1 constructed in Theorem 2 is ν-periodic.

Remark 6. Problem 1 might not have a solution even if the $\mathbf{H}_\infty$ control problem for $\tilde{G}_d$ has a solution. We can find a similar situation in the multirate $\mathbf{H}_\infty$ problem (e. g. 4), 10), 12)). In fact, we can apply solutions for the multirate $\mathbf{H}_\infty$ problem to Problem 1. Theorem 1 is a specialized alternative solution providing a new LMI-based formula. We also note that LMIs in Theorem 1 contain less LMI-variables in compare to LMIs in 10) when n is large and ν is small.

4. Numerical Examples

Consider a system in Fig. 2 with

$$G_c = \begin{bmatrix} W_1 & 0 \\ 0 & I_2 \end{bmatrix} P_c + \begin{bmatrix} 0 & W_2 \\ 0 & 0 \end{bmatrix},$$

$$P_c \overset{\text{ssr}}{:=} \left[\begin{array}{cc|ccc} 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

where $p = m = 2$. W_1 and W_2 are determined by the following transfer functions:

$$\hat{W}_1(s) = \frac{s}{s+1}, \quad \hat{W}_2(s) = \frac{0.1s}{s+1}\begin{bmatrix} 1 & 1 \end{bmatrix}.$$

Let the sampling period be chosen to $h = 0.1$ and the network capacity be $\gamma = 3$. We further assume that K_d can access 2 actuators simultaneously, namely, we consider N_d having the following form:

$$N_d = \begin{bmatrix} 0 & I_2 \\ N_y & 0 \end{bmatrix}.$$

We compare the following design methods:

Method 0 $N_y = I_2$ (no communication constraint)

Method 1a $N_y = \begin{bmatrix} I & 0 \end{bmatrix}$ (no communication constraint, the second sensor is ignored.)

Method 1b $N_y = \begin{bmatrix} 0 & I \end{bmatrix}$ (no communication constraint, the first sensor is ignored.)

Method 2 Determine N_y by

$$y_d[k] = \begin{cases} \begin{bmatrix} y_1[k] \\ y_2[k-1] \end{bmatrix}; & (k : \text{even}), \\ \begin{bmatrix} y_1[k-1] \\ y_2[k] \end{bmatrix}; & (k : \text{odd}) \end{cases}$$

and solve Problem 1 to determine K_d.

Method 3 Determine N_y by

$$y_d[k] = \begin{cases} \begin{bmatrix} y_1[k] + y_2[k] \\ y_1[k-1] + y_2[k-1] \end{bmatrix}; & (k : \text{even}), \\ \begin{bmatrix} y_1[k-1] + y_2[k-1] \\ y_1[k] + y_2[k] \end{bmatrix}; & (k : \text{odd}) \end{cases}$$

and solve Problem 1 to determine K_d.

Method 4 Determine N_y by

$$y_d[k] = \begin{cases} \begin{bmatrix} y_1[k] + 1.5y_2[k] \\ y_1[k-1] + y_2[k-1] \end{bmatrix}; & (k : \text{even}), \\ \begin{bmatrix} y_1[k-1] + 1.5y_2[k-1] \\ y_1[k] + y_2[k] \end{bmatrix}; & (k : \text{odd}) \end{cases}$$

Table 1 Performance of resultant system

	$\mathbf{L}_2$-induced norm
Method 0	0.07
Method 1a	unstable
Method 1b	0.7
Method 2	0.18
Method 3	0.13
Method 4	0.09

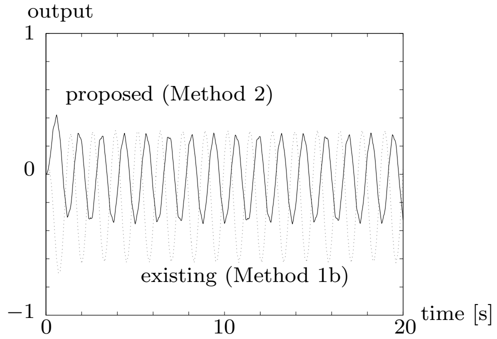

Fig. 3 Sinusoid response

and solve Problem 1 to determine K_d.

The achieved values of the $\mathbf{L}_2$-induced norm by applying the design method are summarized in Table 1. Note that the value for Method 0 will be the limits of performance of the NCSs with communication constraints. We can observe that the resultant performance of Method 2 is closer to the limit of the performance, in comparison to those by Methods 1a and 1b. Time responses of resultant systems of Methods 1b and 2 for sinusoid disturbance at 5 [rad/sec] are depicted in Fig. 3.

We also note that we can further optimizes the performance by properly choosing the sensors and/or actuators. In this example, more than 30% performance improvement has been achieved by Method 4, where we compare between Cases 2 and 4.

5. Concluding Remarks

In this paper, a design problem for NCSs has been considered. The network has limitted capacity and control inputs and measured outputs are updated/sampled partially at each step. Assuming that the controller-plant communication is periodic, the design problem is formulated as one for sampled-data feedback systems with periodic discrete-time components. A necessary and sufficient condition for existence of discrete-time periodic controller has been given in terms of LMIs, and a controller construction algorithm has been derived. The proposed controller (if exists) stabilizes and sub-optimizes the $\mathbf{L}_2$-induced norm of the resultant NCSs.

Appendix A. Proof of Theorems 1 and 2

Denote the 'D'-matrix of a state-space system P by $\mathsf{D}(P)$. Invoking Lemma 2 and Property 3, Problem 1 has a solution if and only if there exists a stabilizing controller $\tilde{K}_d$ satisfying

(i) $\left\|\tilde{G}_d \star \tilde{K}_d\right\| < 1$.

(ii) $\mathsf{D}(\tilde{K}_d) \in \mathsf{BLT}(p,\, m,\, \nu)$.

supposing Assumption 1. Noting that the 'D_{22}'-matrix of N_d is zero, and D_{22} in (8) satisfies

$$D_{22} \in \mathsf{BLT}(p,\, m,\, \nu)$$

and the diagonal blocks of D_{22} are all zero (See Property 2), we have the following lemma:

Lemma 3. Suppose Assumption 1 holds. Problem 1 has a solution if and only if there exists a stabilizing controller $\bar{K}_d$ satisfying the following specifications:

(i) $\left\|\bar{G}_d \star \bar{K}_d\right\| < 1$.

(ii) $\mathsf{D}(\bar{K}_d) \in \mathsf{BLT}(m,\, p,\, \nu)$.

where

$$\bar{G}_d := \tilde{G}_d - \begin{bmatrix} 0 & 0 \\ 0 & D_{22} \end{bmatrix}.$$

More over if a solution exists, $\tilde{K}_d$ is given by (15).

Proof: Note that $I - \mathsf{D}(\bar{K}_d)D_{22}$ is invertible and

$$(I - \mathsf{D}(\bar{K}_d)D_{22})^{-1} \in \mathsf{BLT}(m,\, m,\, \nu).$$

Lemma 3 directly follows.

Hence we will consider the synthesis problem of a stabilizing controller $\bar{K}_d$ satisfying the specifications (i), (ii) in Lemma 3 in the sequel. It is well-known that there exists a stabilizing controller satisfying (i) if and only if there exists $X = X' \in \mathbb{R}^{n\times n}$ and $Y = Y' \in \mathbb{R}^{n\times n}$ satisfying (9) – (11) (See e.g.,[6] and references therein). The following lemma[6] provides an $\mathbf{H}_\infty$ controller synthesis procedure:

Lemma 4. For given $\bar{G}_d$, a stabilizing controller $\bar{K}_d$ satisfying (i) of Lemma 3 is obtained (if exists) by the following steps:

Step 1: Find $X = X' \in \mathbb{R}^{n\times n}$ and $Y = Y' \in \mathbb{R}^{n\times n}$ satisfying (9) – (11).

Step 2: Find $Z \in \mathbb{R}^{\nu m\times \nu p}$ satisfying (12).

Step 3: Determine $\bar{K}_d$ by (13).

Noting that Steps 2 and 3 are independent, and $\mathsf{D}(\bar{K}_d) = Z$, $\bar{K}_d$ satisfies (ii) of Lemma 3 if we put an additional constraint $Z \in \mathsf{BLT}(m,\, p,\, \nu)$ in Step 2 of Lemma 4. Conversely, if such Z does not exist, there is not $\bar{K}_d$ satisfying (ii) of Lemma 3. This completes the proof.

Appendix B. Blocking Related Properties

In this appendix, some properties related to the blocking technique are introduced without proofs. Proofs are found in literature.

Property 1. For a given discrete-time signal $x \in \ell_2$,

$$\|x\|_2 = \|\mathcal{B}_\nu x\|_2.$$

Property 2. For a given ν-periodic discrete-time system P_d:

$$P_d \overset{\text{ssr}}{:=} \left[\begin{array}{c|c} A[k] & B[k] \\ \hline C[k] & D[k] \end{array}\right],$$

$$\begin{bmatrix} A[k+\nu] & B[k+\nu] \\ C[k+\nu] & D[k+\nu] \end{bmatrix} = \begin{bmatrix} A[k] & B[k] \\ C[k] & D[k] \end{bmatrix}, \; k = 0,\, 1,\, 2,\, \ldots,$$

$\mathcal{B}_\nu P_d \mathcal{B}_\nu^{-1}$ is time-invariant:

$$\mathcal{B}_\nu P_d \mathcal{B}_\nu^{-1} \overset{\text{ssr}}{=} \left[\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & \tilde{D} \end{array}\right],$$

$$\tilde{A} := \prod_{i=0}^{\nu-1} A[i],$$

$$\tilde{B} := \begin{bmatrix} \prod_{i=0}^{\nu-2} A[i]B[0] & \cdots & A[\nu-2]B[\nu-2] & B[\nu-1] \end{bmatrix},$$

$$\tilde{C} := \begin{bmatrix} C[0] \\ C[1]A[0] \\ \vdots \\ C[\nu-1]\prod_{i=0}^{\nu-2} A[i] \end{bmatrix},$$

$$\tilde{D} := \begin{bmatrix} D[0] & 0 & \cdots & 0 \\ C[1]B[0] & D[1] & & \vdots \\ \vdots & & \ddots & 0 \\ C[\nu-1]\prod_{i=1}^{\nu-2} A[i]B[0] & \cdots & \cdots & D[\nu-1] \end{bmatrix}.$$

Property 3. For a given νm-input νp-output FDLTI discrete-time system $\tilde{P}_d$, $P_d := \mathcal{B}_\nu^{-1}\tilde{P}_d\mathcal{B}_\nu$ is ν-periodic. Moreover P_d is causal if and only if

$$\mathsf{D}(\tilde{P}_d) \in \mathsf{BLT}(p,\, m,\, \nu),$$

where $\mathsf{D}(\cdot)$ denotes the 'D'-matrix in the state-space representation.

References

1) B. A. Bamieh and J. B. Pearson: "A general framework for linear periodic systems with applications to $\mathcal{H}^\infty$ sampled-data control," IEEE Trans. Automat. Contr., vol. 37, no. 4, pp. 418–435, 1992.
2) H. Chapellat, M. Dahleh and S. Bhattacharyya, "Optimal disturbance rejection for periodic systems," Tech. Rep. 89-019, Texas A and M University, 1989.
3) T. Chen and A. Francis: Optimal Sampled-Data Control Systems, Springer, 1995.
4) T. Chen and L. Qiu, "$\mathcal{H}_\infty$ design of general multirate sampled-data control systems," Automatica, vol. 30, no. 7, pp. 1139–1152, 1994.
5) P. Gahinet and P. Apkarian: "A linear matrix inequality approach to H_∞ control," Int. J. Robust Nonlinear Control, vol. 4, pp. 421–448 (1994).
6) P. Gahinet: "Explicit controller formulas for LMI-based $\mathcal{H}_\infty$ synthesis," Automatica, vol. 32, no. 7, pp. 1007–1014 (1996).
7) D. Hristu: "Stabilization of LTI systems with communication constraints," in Proc. of 35th Amer. Contr. Conf., pp. 2342–2346 (2000)
8) T. Iwasaki and R. Skelton: "All controllers for the general H_∞ control rpoblem: LMI existence conditions and state-space formulas," Automatica, vol. 30, pp. 1307–1317 (1994).
9) J. P. Keller and B. D. O. Anderson: "A new approach to the discretization of continuous-time controllers," IEEE Trans. Automat. Contr., vol. 37, no. 2, pp. 214–223 (1992).
10) S. Lall and G. Dullerud, "An LMI solution to the robust synthesis problem for multi-rate sampled-data systems," Automatica, vol. 37, pp. 1909–1922, 2001.
11) Special Section on Networks and Control, IEEE Control Systems Magazine, vol. 21 (2001).
12) P. G. Voulgaris and B. Bamieh, "Optimal $\mathcal{H}^\infty$ and $\mathcal{H}^2$ control of hybrid multirate systems," Systems & Control Letters, vol. 20, pp. 249–261, 1993.
13) G. C. Walsh and H. Ye: "Scheduling of networked control systems," IEEE Control Systems Magazine, vol. 21, pp. 57–65, (2001).
14) S. Yamamoto et al: "Stabilization of communication systems with periodic constraints by observer-based controllers" in Proc. of SICE 1st Annual Conf. on Control Systems, pp. 9–12, 2001 (in Japanese).
15) S. Yamamoto et al: "Stabilization of monodromy in control system designs for communication systems with periodic constraints" in Proc. of 30th SICE Symp. Control Theory, pp. 273–278, 2001 (in Japanese).

Hisaya Fujioka (Member)

He received the B. E., M. E., and Ph. D. degrees from Tokyo Institute of Technology, Tokyo, Japan, in 1990, 1992, and 1995, respectively. He is currently an Associate Professor at Graduate School of Informatics, Kyoto University. His current research interests include sampled-data control, robust control, networked control systems, and computer-aided control systems design.

Kensaku Ito (Member)

He received the bachelor and the master degrees from Kyoto University, Kyoto, Japan, in 2000 and 2002, respectively. Since April, 2002, he is at Kimitsu Works, Nippon Steel Corporation, Kimitsu, Chiba.