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Robust Receding Horizon Control for Hybrid Systems based on Constrained Positively Invariant Sets

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This paper presents a robust receding horizon control algorithm for a class of hybrid systems by exploiting the equivalence between piecewise linear systems and mixed logical dynamical systems. The control algorithm consists of two control modes which are a state feedback mode and a receding horizon control mode. In the receding horizon control mode, the constrained positively invariant sets are used for a terminal constraint. This control algorithm guarantees that the state converges to a union of constrained positively invariant sets with no constraint violation. An illustrative example is presented to show that the control algorithm satisfies the stability and constraints.

Key Words: receding horizon control, model predictive control, robust control, hybrid systems, invariant set

1. Introduction

In recent years, receding horizon control (model predictive control) has attracted the attention of researchers in not only the field of process control, but also the field of robotics and aerospace. Furthermore, robust control problems have been studied for receding horizon control [1]. It is well known that, in the practical applications, physical bounds on the state and control input are present, so the control law is required to guarantee that the closed-loop system fulfills these constraints. The receding horizon control strategy optimizes an open-loop control sequence at each time, to minimize an objective function subject to some state and input constraints. For these problems terminal constraints play important role in stabilization problems [2]. In 3) feedback min-max model predictive control for linear time invariant discrete-time systems is presented. This control algorithm is effective for the bounded disturbance case. On the other hand, hybrid systems arise in a large number of application areas, and are attracting increasing attention in both academic theory-oriented circles as well as in industry [4,5]. Bemporad et al. have proposed a new class of hybrid systems called Mixed Logical Dynamical (MLD) systems [6]. It is capable to model a broad class of systems: linear hybrid dynamical systems, hybrid automata, some class of discrete event systems, linear systems with constraints, etc. Further in 9) equivalences among some classes of hybrid systems are established. Bemporad et al. have proposed receding horizon control algorithm for mixed logical dynamical systems named Mixed Integer Predictive Control (MIPC) law [6] and have proved that it stabilizes MLD systems to the equilibrium state or the desired reference trajectory. However when disturbances or model mismatch are present, closed-loop performance can be poor with likely violations of the constraints and no convergence can be guaranteed. Feedback min-max model predictive control for hybrid systems is considered, however it is not easy to extend the algorithm in 3). Since the system formulation is restricted to linear time invariant discrete-time systems, the control can not deal with hybrid systems directly and a method to construct the end set constraint is not given clearly. Furthermore, robust receding horizon control for hybrid systems has been researched and some formulations which address these issue have been proposed. However it is not clear how to construct the terminal constraints.

In this paper, we take into account the effects of unknown bounded disturbances and propose a robust receding horizon control algorithm for piecewise linear systems. It is based on feedback min-max model predictive control [3], but in the control algorithm we employ the equivalence of piecewise linear systems and MLD systems and propose the end set constraint that consists of constrained positively invariant sets [10,14]. Switching control using constrained positively invariant set is also reported in 13). This control law guarantees convergence to the set and satisfying the constraints for unknown bounded disturbance. A simple example is shown to illustrate the effects of the control law.

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The reminder of this paper is organized as follows. Section 2 describes the problem formulation and constrained positively invariant sets. In section 3 we propose a robust receding horizon control procedure for piecewise linear systems that solves a min-max type optimization problem. Section 4 shows an illustrative example of the simulation result when the control procedure is applied. Section 5 describes a conclusion of our research.

2. System Representation

2.1 Piecewise Linear Systems with Disturbance

In this paper, piecewise linear systems with disturbance are described by the following equation (1),

$$x(t+1)=A^i x(t)+B^i u(t)+B_w w(t) \quad x(t)\in\mathcal{X}_i \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^m$ is the input, $\mathcal{X}_i$ is the partition of the state set that satisfies $\mathcal{X}_i\cap\mathcal{X}_j=\emptyset$ and $\forall i\neq j$, $\cup_{i=1}^s\mathcal{X}_i=\mathbb{X}$ and we assume that (A^i,B^i) is controllable. The vector $w(t)\in\mathbb{W}\subset\mathbb{R}^l$ is an unknown bounded disturbance and the set $\mathbb{W}$ is convex and contains the origin. In addition the system is subject to constraints on either or both the states and the control inputs i.e. $x(t)\in\mathbb{X}$, $u(t)\in\mathbb{U}$, $\forall t\in\mathbb{N}$. We assume $\mathbb{X}$ and $\mathbb{U}$ are convex polyhedral. Consider the output to be constrained

$$y_c(t)=Cx(t)+Du(t)+D_w w(t). \tag{2}$$

By an appropriate choice of matrix C, D and a set $\mathbb{Y}$, all constraint defined in this paper can be summarized by

$$y_c(t)\in\mathbb{Y}. \tag{3}$$

Assume that the set $\mathbb{Y}$ is convex and contains the origin.

2.2 The Mixed Logical Dynamical Form of Piecewise Linear Systems

Here the mixed logical dynamical form [6], which is equivalent to piecewise linear systems is introduced. Consider the following general piecewise linear system

$$x(t+1)=A^i x(t)+B^i u(t) \quad \text{for } x(t)\in\mathcal{X}_i \tag{4}$$

$$y(t)=C_i x(t)+D_i u(t) \tag{5}$$

where $x(t)\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^m$ is the input, $\mathcal{X}_i$ is a partition of the state set that satisfies $\mathcal{X}_i\cap\mathcal{X}_j=\emptyset$ and $\forall i\neq j$, $\cup_{i=1}^s\mathcal{X}_i=\mathbb{X}$ and we assume that (A^i,B^i) is controllable. Piecewise affine systems are described by the state space equations

$$\begin{array}{l} x(t+1)=A^i x(t)+B^i u(t)+f_i \\ y(t)=C_i x(t)+g_i \end{array}, \text{for} \begin{bmatrix} x(t)\\ u(t)\end{bmatrix}\in\mathcal{X}_i \tag{6}$$

Since f_i, g_i can be thought as generated by integrators with no input, the piecewise linear system and the piecewise affine system are equivalent. The piecewise affine system (6) can be transformed into mixed logical dynamical systems formulation [6]. And it also is reported that Mixed Logical Dynamical systems are formally equivalent to piecewise affine systems [8]. The key idea of mixed logical dynamical systems is technique that can transform into propositional logic into mixed integer inequalities, i.e. inequalities involving both continuous and 0-1 variables.

The general mixed logical dynamical form [6] is:

$$x(t+1)=Ax(t)+B_1u(t)+B_2\delta(t)+B_3z(t) \tag{7a}$$

$$E_2\delta(t)+E_3z(t)\le E_1u(t)+E_4x(t)+E_5 \tag{7b}$$

where

$$x=\begin{bmatrix} x_c\\ x_l\end{bmatrix}, x_c\in\mathbb{R}^{n_c}, x_l\in\{0,1\}^{n_l}, n:=n_c+n_l$$

$$u=\begin{bmatrix} u_c\\ u_l\end{bmatrix}, u_c\in\mathbb{R}^{m_c}, u_l\in\{0,1\}^{m_l}, m:=m_c+m_l$$

is the state of the system, whose components are distinguished between continuous x_c and 0-1 x_l, is the command input. $\delta\in\{0,1\}^{r_l}$ and $z\in\mathbb{R}^{r_c}$ represent respectively auxiliary logical and continuous variables.

The piecewise linear system (1) can be transformed mixed logical dynamical system formulation. The mixed logical dynamical systems form is

$$x(t+1)=Ax(t)+B_1u(t)+B_2\delta(t)+B_3z(t)+B_pw(t) \tag{8a}$$

$$E_2\delta(t)+E_3z(t)\le E_1u(t)+E_4x(t)+E_5 \tag{8b}$$

Assume that system (8) is completely well-posed [6], which in words means that for all x, u, w within a bounded set the variables δ, z are uniquely determined, i.e. there exist functions F, G such that, at each time t, $\delta(t)=F(x(t),u(t),w(t))$, $z(t)=G(x(t),u(t),w(t))$.

A robust receding horizon control algorithm for the hybrid systems by exploiting the equivalence between piecewise linear systems and mixed logical dynamical systems. A description language HYSDEL (HYbrid Systems DEscription Language) [12] is proposed. In this paper, we assume that the equivalent representations is obtained by using HYSDEL [12].

3. Robust Receding Horizon Control Algorithm

3.1 Control Laws

Since the state can not be steered to the origin due to existing disturbance $w(t)$, the control objective is to drive the state of the system to the set that is constructed by

invariant sets. In this paper, we propose 2 modes for the control law by using two system representations (1), (8).
mode-1: The control law is set as $u = K_i x$ for the system (1).
mode-2: Here $l \in \mathcal{L}$ denotes indexes of possible disturbance. At time t, predictions for possible disturbance are represented by $\{w^l(j|t)\}$, and $\{u^l(j|t)\}$ denotes the input sequence for a disturbance realization l. $\{z^l(j|t)\}$, $\{\delta^l(j|t)\}$ are similarly defined respectively.

$x^l(j+1|t)$ is defined as follows

$$x^l(k+1|t) = Ax^l(k|t)+B_1u^l(k|t) +B_2\delta^l(k|t)+B_3z^l(k|t)+B_pw^l(k|t) \quad (9a)$$

$$E_2\delta^l(k|t)+E_3z^l(k|t) \leq E_1u^l(k|t)+E_4x^l(k|t)+E_5 \quad l \in \mathcal{L} \quad (9b)$$

At current time t, let $x(t)$ be the current state. Consider the following optimal control problem (10),

$$\min_{\{U_0^{N-1}\}} \max_{l \in \mathcal{L}} J(U_0^{N-1}, x(t)) := \sum_{k=0}^{N-1} L(x^l(k|t), u^l(k|t)) \quad (10)$$

$$\text{subject to} \begin{cases} x^l(N|t) \in \mathcal{P} \\ x^l(k+1|t) = Ax^l(k|t) + B_1u^l(t) \\ \quad +B_2\delta^l(k|t) + B_3z^l(k|t) + B_pw^l(k|t) \\ E_2\delta^l(k|t) + E_3z^l(k|t) \\ \quad \leq E_1u^l(t) + E_4x^l(k|t) + E_5 \\ y_c(t) \in \mathbb{Y} \end{cases}$$

where N is predictive horizon and $Q_1 = Q_1' > 0$, $Q_2 = Q_2' > 0$ are weighting matrices respectively. For the sake of simplicity we define $x(k|t) := x(t+k, x(t), u_0^{k-1})$ and $\delta(k|t)$, $z(k|t)$ are defined similarly. $x(N|t)^l \in \mathcal{P}$ denotes an end set constraint. It is assumed that L is a convex function and

$$L = \begin{cases} 0 & x \in \mathcal{P} \\ L(x,u) \geq \alpha(d(x,\mathcal{P})) & x \notin \mathcal{P} \end{cases} \quad (11)$$

where $\alpha(\cdot)$ is a K-function and $d(x,\mathcal{P})$ denotes the distance between x and the set $\mathcal{P}$.

At each time step, solve the optimization problem (10). Assume that the optimal solution $\mathcal{U}$ exists. Let the optimal control sequence be

$$U_0^{N-1}(t) := \{\bar{u}(0|t), \ldots, \bar{u}(N-1|t)\}.$$

Then the actual control applied at time t is the first element of this sequence i.e. $u(t) = \bar{u}(0|t)$. Because of the linearity of the system (9) and convexity of the constraints and cost, we consider only the extreme disturbance realization. We show the optimal problem considering only extreme disturbance realization leads to a stabilizing control law that satisfies the constraints for all disturbance realizations that lie inside the convex hull of the realizations considered in the optimization. Here, assume that $\mathbb{W} \subset \mathbb{R}^n$ is polyhedron. The disturbances that take values of vertices of the convex set $\mathbb{W}$ are considered.

Then let $\mathcal{L}_v$ denote the set of indexes l, such that possible realizations of the disturbance take the values on the vertices of $\mathbb{W}$. Then the optimization problem (10) are replaced by the following problem (12), which can be solved by finite optimizations.

$$\min_{\{U_0^{N-1}\}} \max_{l \in \mathcal{L}_v} J(U_0^{N-1}, x(t)) = \sum_{k=0}^{N-1} L(x^l(k|t), u^l(k|t)) \quad (12)$$

$$\text{subject to} \begin{cases} x^l(N|t) \in \mathcal{P} \\ x^l(k+1|t) = Ax^l(k|t) + B_1u^l(t) \\ \quad +B_2\delta^l(k|t) + B_3z^l(k|t) + B_pw^l(k|t) \\ E_2\delta^l(k|t) + E_3z^l(k|t) \\ \quad \leq E_1u^l(t) + E_4x^l(k|t) + E_5 \\ y_c(t) \in \mathbb{Y} \end{cases}$$

Predictive variables can be represented as follows.

$$x(k|t) = A^kx(t) + \sum_{i=0}^{k-1} A^i[B_1u(k-1-i|t) + B_2\delta(k-1-i|t) +B_3z(k-1-i|t)+B_pw(k-1-i|t)] \quad (13)$$

By plugging (13) into (12), and by defining the vectors

$$\Omega^l := \begin{bmatrix} u^l(0) \\ \vdots \\ u^l(T-1) \end{bmatrix}, \Delta^l := \begin{bmatrix} \delta^l(0|t) \\ \vdots \\ \delta^l(T-1|t) \end{bmatrix},$$

$$\Xi^l := \begin{bmatrix} z^l(0|t) \\ \vdots \\ z^l(T-1|t) \end{bmatrix}, \quad \Gamma^l := \begin{bmatrix} w^l(0|t) \\ \vdots \\ w^l(T-1|t) \end{bmatrix}, \forall l \in \mathcal{L}_v \quad (14)$$

we obtain the following formulation of the optimization problem that is equivalent to (12). Assume that the set $\mathcal{P}$ is polytope.

$$\begin{cases} \min_{\Omega,\Delta,\Xi} \max_{l \in \mathcal{L}_v} \begin{bmatrix} \Omega^l \\ \Delta^l \\ \Xi^l \\ \Gamma^l \end{bmatrix}' S_1 \begin{bmatrix} \Omega^l \\ \Delta^l \\ \Xi^l \\ \Gamma^l \end{bmatrix} + 2(S_2 + x'(t)S_3) \begin{bmatrix} \Omega^l \\ \Delta^l \\ \Xi^l \\ \Gamma^l \end{bmatrix} \\ \text{subject to} \quad F_1 \begin{bmatrix} \Omega^l \\ \Delta^l \\ \Xi^l \\ \Gamma^l \end{bmatrix} \leq F_2 + F_3x(t) \end{cases} \quad (15)$$

where $S_i, F_i, i = 1,2,3$ are suitably defined.

3.2 End Set Constraint

In this section, we explain how to construct the end set $\mathcal{P}$. Consider the control input $u = K_ix$ for the system (1) then the system can be rewritten as

$$x(t+1) = (A^i + B^i K_i)x(t) + B_w w(t) \quad (16)$$

$$y_c(t) = (C + DK_i)x(t) + D_w w(t) \quad (17)$$

Maximal constrained positively invariant set for the system (16) in the set i is defined as $\mathcal{O}_{\infty i}$ (see Appendix Appendix A). Next we define a set that is used for an end set constraint of receding horizon control as

$$\mathcal{P} := \cap_{i=1}^{s} \mathcal{O}_{\infty i}. \quad (18)$$

Maximal constrained positively invariant set $\mathcal{O}_{\infty i}$ can be obtained by the recursive process proposed in 10), 14). Notice that we have to design so that the set $\mathcal{P}$ is not empty (1).

3.3 Control Algorithm

We propose a robust receding horizon control algorithm for the piecewise linear systems (1) with disturbance $w(t) \in \mathbb{W}$ as follows.

Algorithm 1: Data: $x(t)$

Algorithm: IF $x(t) \in \mathcal{P}$ THEN (mode-1) $u(t) = K_i x(t)$.
ELSE (mode-2) Solve the optimal control problem (10) for the system (8), and set $u(t)$ to the first element of the optimal input sequence. □

Theorem 1. Assume that the optimal control problem (10) has feasible solutions. The control for the system (1) given by Algorithm 1 satisfies the constraints (3) and drives the state $x(t)$ to the set $\cup_{i=1}^{s} \mathcal{O}_{\infty i}$.

Proof: At time t, state $x(t)$, let $\{w^l(j|t)\}$, $l \in \mathcal{L}$ denote the optimal control sequences that respond to disturbance realizations $\{\bar{u}^l(j|t)\}$. Let $\{x^l(j|t)\}$ and $J^l(\mathcal{U}_t^{l*}, x(t))$ denote the state trajectories and the costs for the case of a disturbance realization l. The optimal cost is defined as

$$\bar{V}(t) = \max_{l \in \mathcal{L}} J^l(t) \quad (19)$$

At time t, the first element of the optimal sequence is applied, and disturbance takes a certain value $w(t)$. The set of indexes such that $w^l(t|t) = w(t)$, $\forall l \in \mathcal{L}_1$ and $w^l(t|t) \neq w(t)$, $\forall l \notin \mathcal{L}_1$ as $\mathcal{L}_1$ is defined. At time $t+1$, the state $x(t+1)$ has moved along a trajectory that coincides with the predicted state trajectories indexed by $l \in \mathcal{L}_1$. Consider the optimal sequence defined as $\mathcal{U}_1 = \{\bar{u}^l(k+1|t),\ \bar{u}^l(k+2|t),\ \cdots,\ \bar{u}^l(k+N-1|t),\ K_i x(k+N|t)^l\}$, $\forall l \in \mathcal{L}_1$. Then we obtain the following inequality.

$$V^l(t+1) \le J^l(\mathcal{U}_1, x(t+1)) \quad (20)$$

Using the relations $\delta^l(k+1|t) = \delta^l(k|t+1)$ and $z^l(k+1|t) = z^l(k|t+1)$, $\forall l \in \mathcal{L}$ the inequality (20) is represented as

$$V^l(t) - L(x(t), u(t)) + L(x^l(N|t), u^l(N|t)). \quad (21)$$

From the definition of L an end set constraint $x^l_{k+N|t} \in \mathcal{P}$ provides $L(x^l(t+N|t), u^l(t+N|t)) = 0$, and consequently

$$V^l(t+1) \le V^l(t) - L(x(t), u(t)). \quad (22)$$

Let $\bar{V}(t+1)$ denote the optimal cost at time $t+1$, then we obtain

$$\bar{V}(t+1) \le \max_{l \in \mathcal{L}_1} V^l(t) - L(x(t), u^l(t)). \quad (23)$$

Since $\max_{l \in \mathcal{L}_1} V^l(t) \le \max_{l \in \mathcal{L}} V^l(t) = \bar{V}(t)$, we obtain

$$\bar{V}(t+1) \le \bar{V}(t) - L(x(t), u(t)). \quad (24)$$

The cost is monotonically nonincreasing. As it is bounded below by zero, it must consequently converge to a constant value, so that $\bar{V}(t) - \bar{V}(t+1) \to 0$ as $t \to \infty$. From (24), we have $L(x(t), u(t)) \le \bar{V}(t) - \bar{V}(t+1)$ and it follows that $L(x(t), u(t)) \to 0$ as $t \to \infty$. In view of Assumption 1, we conclude that $d(x(t), \mathcal{P}) \to 0$, as $t \to \infty$. Hence the state converges to $\mathcal{P}$. Further when the state in the set $\mathcal{P}$, the control law changes to $u = K_i x$. Consequently the control algorithm satisfies the constraints and drives the state in $\cup_{i=1}^{s} \mathcal{O}_i$. ∎

Theorem 1 is guarantees that the state of the system can be steered to the set $\cup_{i=1}^{s} \mathcal{O}_{\infty i}$ with no constraint violation. The set $\mathcal{O}_{\infty i}$ depends on the design of feedback gain K_i, $\mathbb{X}$ and $\mathbb{U}$. If we consider PWA system of a single region, the control algorithm corresponds to the algorithm for a linear system. Thus Theorem 1 includes the theorem 1 in 3) as a special case. It is difficult, however, to obtain the optimal solution of the optimal control problem (10) since $l \in \mathcal{L}$ has infinite canditates. Algorithm 1 is extended to the following.

Algorithm 2: Data: $x(t)$

Algorithm: IF $x(t) \in \mathcal{P}$ THEN (mode-1) $u(t) = K_i x(t)$.
ELSE (mode-2) Solve the optimal control problem (12) for the system (8), and set $u(t)$ to the first element of the optimal input sequence. □

Theorem 2. Assume that the optimal control problem (12) has feasible solutions. The control for the system (1) given by Algorithm 2 satisfies the constraints (3) and drives the state $x(t)$ to the set $\cup_{i=1}^{s} \mathcal{O}_{\infty i}$.

Proof: From the assumptions the constraints (3) are satisfied. Since L is convex, we can proof in a similar way of the proof of Theorem 1. ∎

Theorem 2 is also guarantees that the state of the system can be steered to the set $\cup_{i=1}^{s} \mathcal{O}_{\infty i}$ with no constraint

(1) One possible way to satisfy the requirement is setting the gain K_i value so that $(A^i + B^i K_i) = 0$. It makes the state $x(t+1) = B_w w(t)$, $w \in \mathbb{W}$ then it is possible to construct the CPI sets. Since the set $\mathbb{W}$ includes the origin, the end set $\mathcal{P}$ is obtained by design the gains.

violation. The set $\mathcal{O}_{\infty i}$ is depend on the design of feedback gain K_i, $\mathbb{X}$ and $\mathbb{U}$. Theorem 2 is also extension of the theorem 2 in 3).

In Algorithm 2 the control mode 1 is mode for keeping the state in the set $\mathcal{P}$ and mode 2 is mode for steering the state to the set $\mathcal{P}$. According to the each gain K_i, the constraint sets $\mathbb{X}$ and $\mathbb{U}$, the maximal constrained positively invariant set $\mathcal{O}_{\infty i}$ is designed. Then we can construct the end set as $\cup_{i=1}^{s}\mathcal{O}_{\infty i}$. The $\mathcal{P}$ can be obtained by the way (footnote 1). The feasibility of Algorithm 2 depends on the set $\mathcal{P}$ and the feasibility of the optimization problem in mode-2. If the optimization problem in mode-2 is not feasible, it is necessary to make the horizon longer, the end set $\mathcal{P}$ larger or to tune weighting matrices. However, the computation of the algorithm is demanding since mode 2 solves the min-max optimization problem each time steps. The optimization problem is solved as mixed integer quadratic programming (MIQP). For MIQP some computational algorithms are proposed and available e.g. branch and bound method.

Then the proposed procedure is summarized as follows.

(1) Set the feedback gain K_i for each region i.

(2) Calculate the maximal constrained positively invariant set $\mathcal{O}_{\infty i}$ for each region i.

(3) Let $\cap_{i=1}^{s}\mathcal{O}_{\infty i}$ be the end set constraint $\mathcal{P}$.

(4) Set horizon N and weight matrices Q_s, $s = 1, 2$.

(5) Algorithm .

4. Illustrative Example

We consider the following simple system:

$$x(t+1) = 0.8\begin{bmatrix}\cos\alpha(t) & -\sin\alpha(t)\\ \sin\alpha(t) & \cos\alpha(t)\end{bmatrix}x(t) + \begin{bmatrix}0\\1\end{bmatrix}u(t) + \begin{bmatrix}0\\1\end{bmatrix}w(t) \tag{25}$$

$$\alpha(t) = \begin{cases} \frac{\pi}{3} & \text{if} \quad [\ 1 \quad 0\ ]x(t) \geq 0 \\ -\frac{\pi}{3} & \text{if} \quad [\ 1 \quad 0\ ]x(t) < 0. \end{cases}$$

This system has constraints $x(t) \in [-10, 10] \times [-10, 10]$, $u(t) \in [-2, 2]$. The disturbance $w(t)$ is assumed to be $w(t) \in [-0.2, 0.2]$. Hence, the output to be constrained $\mathbb{Y}$ is defined as

$$\mathbb{Y} := \left\{ x \middle| \begin{bmatrix}-1 & 0\\ 0 & -1\\ 1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}u \leq \begin{bmatrix}10\\10\\10\\10\\2\\2\end{bmatrix} \right\}. \tag{26}$$

Consider the state feedback $u(t) = K_1x(t)$ if the state of the system is in $\mathcal{X}_1$ and $u(t) = K_2x(t)$ if it is in $\mathcal{X}_2$, and calculate the constrained positively invariant sets $\mathcal{O}_{\infty 1}$ and $\mathcal{O}_{\infty 2}$ for each closed loop system. The sets $\mathcal{O}_{\infty 1}$ and $\mathcal{O}_{\infty 2}$ obtained from

$$K_1 = [-1.2 \quad -0.6],\ K_2 = [0.6 \quad -1.2] \tag{27}$$

are shown in **Fig. 1**. Then we define the set that are used for the end set constraint of receding horizon control as $\mathcal{P} = \mathcal{O}_{\infty 1} \cap \mathcal{O}_{\infty 2}$. The set $\mathcal{P}$ is represented by a gray region in Fig. 1.

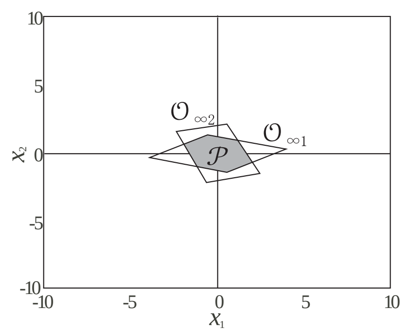

Fig. 1 Sets $\mathcal{O}_{\infty 1}$ and $\mathcal{O}_{\infty 2}$

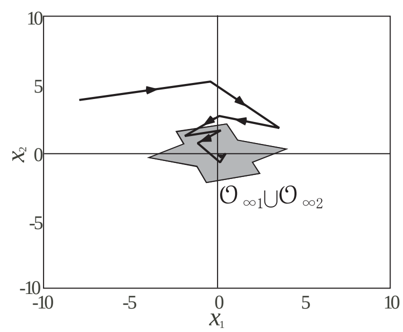

Fig. 2 State trajectory

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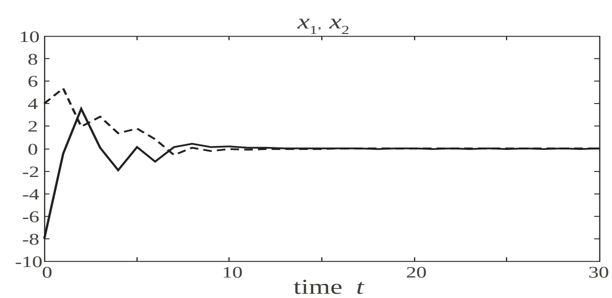

Fig. 3 State response (solid line: x_1, dashed line: x_2)

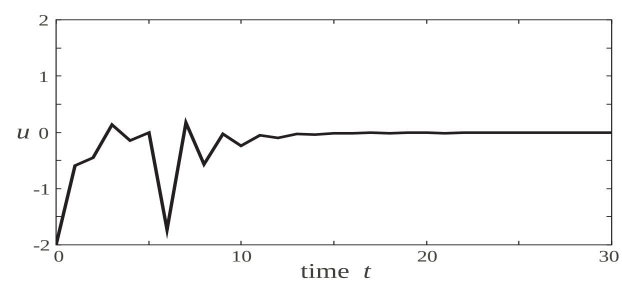

Fig. 4 Input response

Transform PWA system (25) into the corresponding

MLD system (see Appendix Appendix B). L is defined as follows,

$$L(x,u) := \begin{cases} 0 & (x \in \mathcal{P}) \\ \|x(k|t)\|_{Q_1}^2 + \|u(k|t)\|_{Q_2}^2 & (x \notin \mathcal{P}). \end{cases} \quad (28)$$

The control law given by Algorithm 2 is adapted along with the predictive horizon $N = 3$, a terminal constraint $x(N|t) \in \mathcal{P}$ and weights $Q_1 = 10I,\ Q_2 = 1$. In **Fig. 2–4**, we show the resulting trajectories obtained with $x(0) = [-8 \quad 4]'$ and $w(t) = 0.2/t$, $t \geq 1$. Fig. 3 shows the state, which has the constraint $x(t) \in [-10, 10] \times [-10, 10]$ and Fig. 4 shows the constrained control input $u(t) \in [-2, 2]$. Finally we find that in spite of the disturbance the control law given by Algorithm 1 drives the state to the set $\mathcal{O}_1 \cup \mathcal{O}_2$ allowing no constraint violations.

5. Conclusion

In this paper we have proposed robust receding horizon control algorithm for piecewise linear systems. The robust receding horizon control has been implemented by solving a min– max type optimization problem employing mixed logical dynamical form with an end set constraint that have consisted of constrained positively invariant sets. Proposed control algorithm has guaranteed convergence to the set and no constraint violations. A simple example has been shown to illustrate the effects of the procedure. However, in the proposed algorithm the computation of the algorithm is demanding since mode 2 solves the min-max optimization problem each time steps. If we control the system on-line, it is hoped that the computation is improved.

References

1) A. Bemporad and M. Morari: Robust Model Predictive Control: A Survey, in *Robustness in Identification and Control*, A. Garulli, A. Tesi and A. Vicino (Eds.), Lecture Notes in Control and Information Sciences, vol. 245, pp. 207–226, Springer-Verlag, 1999.
2) D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert: Constrained Model Predictive Contrl: Stablility and Optimality, *Automatica*, vol. 36, no. 6, pp. 790–814, 2000.
3) P. O. M. Scokaert and D. Q. Mayne: Min-Max Feedback Model Predictive Control for Constrained Linear Systems, *IEEE Trans. Automat. Contr.*, vol. 43, no. 8, pp. 1136–1142, 1998.
4) R. Izadi-Zamanabadi, P. Amann, M. Blanke, V. Cocquempot, G.L. Gissinger, E.C. Kerrigan, T. F. Lootsma, J.M. Perronne, and G. Schreier: Ship Propulsion Control and Reconfiguration, in K. J. Astrom e.a. (Eds.), *Control of Complex Systems*, Springer-Verlag, 2000.
5) E.C. Kerrigan, A. Bemporad, D. Mignone, M. Morari and J. M. Maciejowski: Multi-objective Prioritisation and Reconfiguration for the Control of Constrained Hybrid Systems, *Proc. of the 2000 American Control Conference*, pp. 1694–1698, 2000.
6) A. Bemporad and M. Morari: Control of Systems Integrating Logic, Dynamics,and Constraints, *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.
7) B. De Schutter and T. van den Boom: Model Predictive Control for Max-Puls-Linear Discrete Event Systems, *Automatica*, vol. 37, no. 7, pp. 1049–1056, 2001.
8) A. Bemporad, G. Ferrari-Trecate, and M. Morari: Observability and Controllability of Piecewise Affine and Hybrid Systems, *IEEE Trans. on Automatic Control*, vol. 45, no. 10, pp. 1864–1876, 2000.
9) W. P. M. H. Heemels, B. De Schutter, and A. Bemporad: Equivalence of Hybrid Dynamical Models, *Automatica*, vol. 37, no. 7, pp. 1085–1091, 2001.
10) K. Hirata and M. Fujita: Analysis of Conditions for nonviolation of Constraints on Linear Discrete-time Systems with Exogenous Inputs, *Proc. of the 36th IEEE Conference on Decision and Control*, pp. 1477–1478, 1997.
11) K. Hirata and M. Fujita: Analysis of Conditions for Non-Violation of Constraints on Linear Discrete-Time Systems with Exogenous Inputs (in Japanese), IEEJ Transactions on Electronics, Information and Systems, vol.118-C, no. 3, pp. 384–390, 1998.
12) F. D. Torrisi, A. Bemporad and D. Mignone: HYSDEL–A Tool for Generating Hybrid Models, Technical Report AUT00-03, Automatic Control Lab, ETH, *http : //control.ethz.ch/ hybrid/hysdel/*, 2000.
13) K. Hirata and M. Fujita: Control of Systems with State and Control Constraints via Controller Switching Strategy, *Workshop on Systems with time-domain constraints*, Eindhoven, the Netherlands, 2000.
14) I. Kolmanovsky and E. G. Gilbert: Maximal Output Admissible Sets for Discrete-Time Systems with Disturbance Inputs, *Proc. of the 1995 American Control Conference*, pp. 1995–2000, 1995.

Appendix A. Constrained Positively Invariant Sets

The constrained positively invariant set (CPI) [10), 14)] is explained in order to use it for an end set constraint of receding horizon control. The CPI set is applied to constrained control and switching control [13)]. For each closed-loop system, we define a state constraint set.

Definition. [10), 14)] State constraint set $X((C + DK_i), D_w, \mathbb{Y}, \mathbb{W})$ is defined by

$$X_i = \{x | (C + DK_i)x + D_w w \in \mathbb{Y}, \forall w \in \mathbb{W}\}. \quad (A.1)$$

Necessary and sufficient condition $y_c(t) \in \mathbb{Y}$ for possible disturbance $w(t) \in \mathbb{W}$ is $x(t) \in X_i$.

Definition. [10), 14)] $\mathcal{O}_i \subset \mathbb{R}^n$ contains origin in its interior. $\mathcal{O}_i$ is a CPI set, if it is a positively invariant set and is contained in $X_i((C + DK_i), D_w, \mathbb{Y}, \mathbb{W})$.
If a CPI set exists, for any initial state $x(0) \in \mathcal{O}_i$ and $w(t) \in \mathbb{W}$, then $x(t) \in \mathcal{O}_i$ for all $t \in \mathbb{Z}^+$, where $\mathbb{Z}^+$ denotes the set of nonnegative integer.

Definition. Maximal constrained positively invariant set is defined as follows

$$\mathcal{O}_{\infty i} = \{x(0) | y_c(t|x(0), w) \in \mathbb{Y}, \forall t \in \mathbb{Z}^+, \forall w \in \mathbb{W}\} \quad \text{(A. 2)}$$

Maximal CPI set $\mathcal{O}_{\infty i}$ can be obtained by recursive process proposed in 10), 14).

Appendix B. Example: MLD System Representation

In this section, PWA system (25) is transformed into MLD system representation. By defining $\delta(t)$ such that $[\delta(t)=1] \leftrightarrow [[1 \quad 0]\, x(t) \geq 0]$ that is equivalent to

$$\begin{aligned} -[1 \quad 0]\, x(t) &\leq 10\,(1-\delta(t)) \\ -[1 \quad 0]\, x(t) &\geq \varepsilon + (-10-\varepsilon)\,\delta(t) \end{aligned} \quad \text{(B. 1)}$$

where ε is small tolerance. The system (25) can be described by

$$\begin{aligned} x(t+1) = &(A_1 x(t) + B_u u(t))\,\delta(t) \\ &+ (A_2 x(t) + B_u u(t))\,(1-\delta(t)) + B_w w(t). \end{aligned} \quad \text{(B. 2)}$$

In order to transform products of $\delta(t)$, $x(t)$ and $u(t)$, auxiliary variables $z_1(t), z_2(t)$ are introduced as follows

$$\begin{cases} z_1(t) = (A_1 x(t) + B_u u(t))\,\delta(t) \\ z_2(t) = (A_2 x(t) + B_u u(t))\,(1-\delta(t)). \end{cases} \quad \text{(B. 3)}$$

The auxiliary variables $z_1(t), z_2(t)$ are equivalent to

$$\begin{cases} z_1(t) \leq M\delta(t) \\ z_1(t) \geq m\delta(t) \\ z_1(t) \leq A_1 x(t) + B_u u(t) - m\,(1-\delta(t)) \\ z_1(t) \geq A_1 x(t) + B_u u(t) - M\,(1-\delta(t)) \end{cases}$$
$$\begin{cases} z_2(t) \leq M\,(1-\delta(t)) \\ z_2(t) \geq m\,(1-\delta(t)) \\ z_2(t) \leq A_2 x(t) + B_u u(t) - m\delta(t) \\ z_2(t) \geq A_2 x(t) + B_u u(t) - M\delta(t) \end{cases} \quad \text{(B. 4)}$$

where

$$M = \begin{bmatrix} 4\,(1+\sqrt{3}) \\ 4\,(1+\sqrt{3})+1 \end{bmatrix}, m = -\begin{bmatrix} 4\,(1+\sqrt{3}) \\ 4\,(1+\sqrt{3})+1 \end{bmatrix} = -M.$$

Then we obtain

$$\begin{aligned} x(t+1) &= z_1(t) + z_2(t) + B_w w(t) \\ &= \begin{bmatrix} I & I \end{bmatrix} z(t) + B_w w(t). \end{aligned} \quad \text{(B. 5)}$$

Therefore, the PWA system (25) can be rewritten as

$$x(t+1) = \begin{bmatrix} I & I \end{bmatrix} z(t) + B_w w(t) \quad \text{(B. 6)}$$

$$\begin{bmatrix} 10 \\ -10-\epsilon \\ -M \\ -M \\ M \\ M \\ M \\ M \\ -M \\ -M \end{bmatrix} \delta(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I & 0 \\ -I & 0 \\ 0 & I \\ 0 & -I \\ I & 0 \\ -I & 0 \\ 0 & I \\ 0 & -I \end{bmatrix} z(t) \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ B_u \\ -B_u \\ B_u \\ -B_u \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & \\ 0 & \\ 0 & \\ 0 & \\ A_1 & \\ -A_1 & \\ A_2 & \\ -A_2 & \end{bmatrix} x(t) + \begin{bmatrix} 10 \\ -\varepsilon \\ 0 \\ 0 \\ M \\ M \\ M \\ M \\ 0 \\ 0 \end{bmatrix}.$$

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