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A Reference Governor in a Piecewise State Affine Function: Its Implementation and Experimental Validation

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This paper proposes a reference management technique for closed-loop systems with state and control constraints. The management rule is represented in the form of a piecewise state affine function obtained from an explicit solution to a multi-parametric quadratic programming problem, which explicitly considers not only constraint fulfillment but also tracking performance. Therefore, a successive and optimal management of a given constant reference online can be achieved. However, as our approach theoretically has no robustness for modeling errors and noises, we perform its experimental validation using a real position servomechanism and show the results of the effectiveness and practicability. Additionally, from the aspect of its implementation we also illustrate the experimental responses in the case where each sampling time of the local closed loop system and our reference governor is different, and state an issue regarding the reference governor in the future.

Key Words: reference governor, multi-parametric quadratic programming, piecewise affine function, experimental validation, implementation

1. Introduction

Constraints are inherent characteristics in almost all practical control systems. They appear most commonly as actuator bounds on control variables, but physical limits on state variables are also ubiquitous. It is known that violations of such constraints drastically degrade system performances and in the worst case lead to instability [5)].

In recent years, colorful control approaches to systems with input and/or state-related constraints have been well studied. Above all, reference governor control schemes have received considerable attention [3),6),8),14),16),18)]. The most important and distinctive role of reference governors is to modify a reference signal supplied to a closed-loop system so as to enforce fulfillment of the constraints. Another property is that the problem of obtaining a good local control design for each specification can be decoupled from the problem of meeting constraints that typically become an issue when there is a large change in reference signals.

The reference governor approaches proposed in 3), 6), 18) are general in the sense that they allow an arbitrary time-varying external reference input w. Basically, the reference governors on-line select the largest possible time-varying scalar gain $\alpha(t) \in [0,\ 1]$ such that $r(t+1) = (1-\alpha(t))r(t)+\alpha(t)w(t+1)$, so as not to violate the specified constraints, where signals r managed by a reference governor will be actual inputs to the constrained system. However, selecting the largest $\alpha(t)$ does not directly optimize the closed-loop tracking performance. In addition, the generality of these approaches (such that they allow arbitrary time-varying external reference inputs) leads to conservatism in the constraint satisfaction. By these motivations, the reference management techniques under an assumption of constant references, which consider tracking performance explicitly, are reported in 14), 16). With the techniques, we have to finish managing all of the original references in advance according to the known initial state, and simply input the managed references to the constrained system in concurrence with the starting control. The reason to do so off-line is that it requires too much computation to solve the optimization problem for each interval time of samplings in especially mechanical systems. Furthermore, the approaches are weak against noises and modeling errors.

To achieve an on-line type reference governor, we have to reduce the computation cost. Since the technique for reducing the cost in the model predictive control, proposed in 2), is the most useful, this paper applies a part of it to the reference governor construction. Here, it can not guarantee the feasibility of the corresponding optimization problem appearing in the model predictive control, but our approach can formulate the reference management optimization problem that is always feasible. Therefore, the point of guaranteeing the feasibility is different from the approach in 2).

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On the other hand, they have almost never reported the experimental demonstration of the reference governor. Although only 10) has reported a case study with a real plant, it is difficult to understand an unfamiliar plant and to apply their approach to any constrained systems because their reference governor requires specialized Lyapunov functions, strongly depending on the plant. The authors think that performing experimental validations is very important in order to determine whether the reference governor is one of the practical and effectual control schemes for constrained systems.

Therefore, this paper proposes an on-line reference management technique for general discrete-time systems with state and/or control constraints, which is represented by a piecewise state affine function. Consequently, the authors expect the technique to be more robust for noises and unmodeled dynamics and to decrease far more on-line computation cost than previous work [16]. We show a procedure to assure the feasibility of the reference management optimization problem. The effectiveness of our control approach is illustrated by simulations. Moreover, since influences of modeling errors and noises necessarily appear in the practical control, we perform experimental validations.

The structure of this paper is as follows. In Section **2** it is shown that a discrete-time system with the constraint condition is formulated. In Section **3**, how to manage the reference signal in the reference governor for the constrained system is reduced to a kind of an optimization problem. In Section **4**, from the optimization obtained in Section **3**, the reference governor in a piecewise state affine is derived, and then, Section **5** illustrates some experimental results to show the effectiveness and practicability of the proposed method. Finally, we conclude this paper in Section **6**.

2. Constrained System

We consider a closed-loop system Σ, illustrated in **Fig. 1**, that consists of a plant Σ_p and a controller Σ_c. The system Σ is written in discrete time below.

$$\Sigma : \quad \begin{aligned} x(t+1) &= Ax(t) + Bw(t), && \text{(1a)} \\ z_0(t) &= C_0 x(t) + D_0 w(t), && \text{(1b)} \\ z_1(t) &= C_1 x(t), && \text{(1c)} \end{aligned}$$

where $x = [x_p' \ \ x_c']' \in \Re^n$ $(n = n_p + n_c)$ is a state of Σ, and $x_p \in \Re^{n_p}$ and $x_c \in \Re^{n_c}$ are respectively states of the plant Σ_p and the controller Σ_c. An initial state of Σ is given by $x(0) = x_0 \in \Re^n$. $w \in \Re^{p_1}$ is an external reference

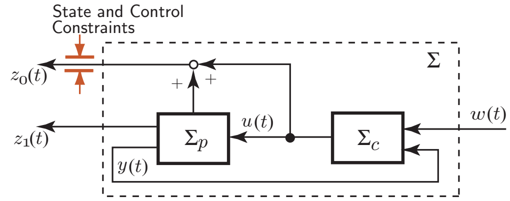


Fig. 1 Closed-loop systems with state and control constraints

input and $z_1 \in \Re^{p_1}$ is a controlled output. Additionally, $z_0 \in \Re^{p_0}$ is constrained signals on state and/or control, and in Σ there exists the following constraint condition:

$$z_0(t) \in \mathcal{Z} \quad \forall\, t \in \mathbb{Z}^+, \tag{2}$$

where $\mathbb{Z}^+$ denotes a set of non-negative integers.

Remark 1. A polytope set $\mathcal{Z}$ is described by $\mathcal{Z} = \{z_0 \in \Re^{p_0} \,|\, M_Z z_0 \le m_Z\}$ using $M_Z \in \Re^{s_z \times p_0}$ and $m_Z \in \Re^{s_z}$. Note that these inequalities in the above equations imply component-wise.

Remark 2. Our interest is focused on an additional reference management technique, which is applied to the primary designed closed-loop system Σ in **Fig. 2**. We assume that the controller Σ_c has already been designed by using abundant results of linear control theories, and in the absence of specified constraints the controller Σ_c provides the desired tracking performance.

Remark 3. Since Σ_c provides the desired performance in the absence of constraints, we can consider the primary purpose of a reference governor as altering the reference signal during the *shortest* possible horizon in order to fulfill the specified constraints. On the other hand, the reference governor is a device that produces an actual reference input to the primal control system Σ. Therefore, it also has an ability to improve the closed-loop tracking performance.

This paper deals with a control problem of tracking z_1 to w and fulfilling the constraint condition (2). Our method is for constraint satisfaction under not the arbitrary reference signal [6),18)] but the constant [14),16)]. Additionally, we at least assume that the specified state and control constraints are satisfied at an equilibrium state (corresponding to the external constant reference $\bar{w}$).

Assumption 1. $\bar{w} \in \Re^{p_1}$ $\forall t \in \mathbb{Z}^+$ satisfies

$$\begin{aligned} &\bar{w} \in \operatorname{int} W \\ &W = \{\bar{w} \in \Re^{p_1} \,|\, M_Z(D_0 + C_0(I-A)^{-1}B)\bar{w} \le m_Z\}, \end{aligned}$$

where int$S$ is an interior of the set S.

The constrained system and the problem setting described above are the same as that of 16); refer to it for details. This paper has the following assumption in order to construct the reference governor that manages the reference on-line under every sampling of the state.

Assumption 2. A full state x of Σ can be measured at a current time t.

Here, some notations are defined. Regarding the output vector z_0, the sequence from time 0 to $k-1$ is denoted by $\hat{z}_0^k \in \Re^{p_0 k}$. Similarly, we define $\hat{z}_1^k$ and $\hat{w}^k$. For example, a vector $\hat{z}_1^k$ is written in

$$\hat{z}_1^k = \begin{bmatrix} z_1(0) \\ z_1(1) \\ \vdots \\ z_1(k-1) \end{bmatrix}, \quad \hat{w}^k = \begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(k-1) \end{bmatrix}.$$

With $x(0)=x_0$, the output $\hat{z}_0^k$ and $\hat{z}_1^k$ can be written in $\hat{z}_0^k = Q_{01}^k x_0 + Q_{02}^k \hat{w}^k$ and $\hat{z}_1^k = Q_{11}^k x_0 + Q_{12}^k \hat{w}^k$ by a linearity of Σ, where the coefficient matrices $Q_{01}^k \in \Re^{p_0 k\times n}$, $Q_{11}^k \in \Re^{p_1 k\times n}$, $Q_{02}^k \in \Re^{p_0 k\times p_1 k}$, $Q_{12}^k \in \Re^{p_1 k\times p_1 k}$ are below,

$$Q_{01}^k = \begin{bmatrix} C_0 \\ C_0 A \\ C_0 A^2 \\ \vdots \\ C_0 A^{k-1} \end{bmatrix}, \quad Q_{11}^k = \begin{bmatrix} C_1 \\ C_1 A \\ C_1 A^2 \\ \vdots \\ C_1 A^{k-1} \end{bmatrix},$$

$$Q_{02}^k = \begin{bmatrix} D_0 & & & & \\ C_0 B & D_0 & & \mathbf{0} & \\ C_0 AB & C_0 B & D_0 & & \\ \vdots & \vdots & \vdots & \ddots & \\ C_0 A^{k-2} B & C_0 A^{k-3} B & \dots & C_0 B & D_0 \end{bmatrix},$$

$$Q_{12}^k = \begin{bmatrix} \mathbf{0} & & & & \\ C_1 B & \mathbf{0} & & \mathbf{0} & \\ C_1 AB & C_1 B & \mathbf{0} & & \\ \vdots & \vdots & \vdots & \ddots & \\ C_1 A^{k-2} B & C_1 A^{k-3} B & \dots & C_1 B & \mathbf{0} \end{bmatrix}.$$

Especially, when the reference is constant, $w(t)=\bar{w}\ \forall t \in \mathbb{Z}^+$, denote the sequence vector as $\hat{\bar{w}}^k$. The managed signal by the reference governor is denoted as $r \in \Re^{p_1}$ and the first component of r is as $\hat{r}^k(1)$, i.e., $\hat{r}^{T^*}(1) = r(0)$.

3. Constraint Fulfillment

This section shows the condition to achieve constraint fulfillment for an infinite horizon. For the goal, we use a maximal output admissible set [7),17)] as follows.

Definition. (Maximal Output Admissible Set) Let $z_0(t; x_0, \bar{w})$ denote the output (1c) of Σ for the initial condition $x(0)=x_0$ and constant reference input $\bar{w} \in W$. Define the $\bar{w}$ dependent maximal output admissible set

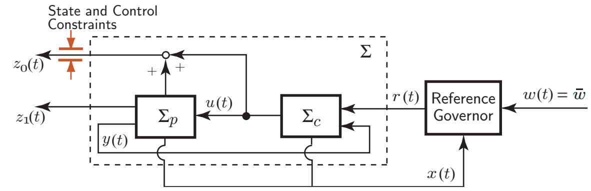


Fig. 2 Closed-loop systems equipped with reference governor

by [7),17)]

$$O_\infty(\bar{w}) = \{\, x_0 \in \Re^n \mid z_0(t; x_0, \bar{w}) \in \mathcal{Z} \ \ \forall t \in \mathbb{Z}^+ \}.$$

Remark 4. Linear programming-based computational procedures of $O_\infty(\bar{w})$ have been proposed in 7) and 17). The maximal output admissible set $O_\infty(\bar{w})$ is a convex polyhedral set and is realized in the form of

$$O_\infty(\bar{w}) = \{\, x \in \Re^n \mid Mx \le m \},$$

where $M \in \Re^{s\times n}$ and $m \in \Re^s$ are the matrices and vectors to describe the linear constraints.

Consider the closed-loop systems Σ with the constant reference input $\bar{w}$. The necessary and sufficient condition for constraint fulfillment is that $x(0) \in O_\infty(\bar{w})$ holds. In that case, the reference governor does not need to alter the constant reference input $\bar{w}$, and it can be directly applied to the closed-loop system without any constraint violations. Therefore, from the property of a maximal output admissible set the following lemma is known for a general $T \in \mathbb{Z}^+$, not only $T=0$.

Lemma 1. Assumption 1 holds. For a constant reference $\bar{w}$ and a given $T \in \mathbb{Z}^+$, $z_0(t) \in \mathcal{Z}\ \forall t \ge T$ if and only if a terminal condition $x(T) \in O_\infty(\bar{w})$ is satisfied.

Proof. See 16) in detail.

Remark 5. From **Lemma 1**, we can understand the following. In order to fulfill the constraints for an infinite horizon, it is important to find an input sequence $\hat{r}^T \in \Re^{p_1 T}$ subject to both $z_0(\tau) \in \mathcal{Z}$, $\tau = 0, 1, \cdots, T-1$ and $x(T) \in O_\infty(\bar{w})$ for the given $T \in \mathbb{Z}^+$ and $x_0 \in \Re^n$. That sequence corresponds to a signal managed by the reference governor.

Remark 6. Once the state of the constrained system steps inside the maximal output admissible set, it can transit to the equilibrium point with the reference $\bar{w}$ fixed, satisfying the constraints. Therefore, if the terminal condition $x(T) \in O_\infty(\bar{w})$ at a certain time T, then after the time T the reference governor directly inputs the original reference to the closed-loop system without any change.

From **Lemma 1** and **Remarks 5** and **6**, the main role of the reference governor is to let the state move into the maximal output admissible set in T steps. This paper chooses the *shortest* step T^* subject to the terminal condition $x(T) \in O_\infty(\bar{w})$. To obtain it, we introduce the following recursively defined set $O_{-k}(\bar{w})$ [15]:

$$O_{-k}(\bar{w}) = \{x \in \Re^n \mid \exists r \in \Re^{p_1}$$
$$Ax + Br \in O_{-(k-1)}(\bar{w}),\ C_0x + D_0r \in \mathcal{Z}\}$$
$$O_{-0}(\bar{w}) = O_\infty(\bar{w})$$

The set $O_{-k}(\bar{w})$ has the following properties: i) all the states inside $O_{-k}(\bar{w})$ are reachable to the maximal output admissible set $O_\infty(\bar{w})$ at least in k steps, fulfilling the constraints, and ii) if $x(t) \in O_{-k}(\bar{w}),\ k \geq 1$ at time t, then there exist such an input $r(t)$ that $x(t+1) \in O_{-(k-1)}(\bar{w})$ and $z_0(t) \in \mathcal{Z}$ hold. Therefore, the shortest horizon T^* in managing the reference $\bar{w}$ is given by

$$T^* = \min_{k \in \mathbb{Z}^+} k \quad \text{s.t.} \quad x_0 \in O_{-k}(\bar{w}) \tag{3}$$

where we assume that there exists $T \in \mathbb{Z}^+$ such that $x_0 \in O_{-T}(\bar{w})$ holds for the initial state x_0.

As a result, for the constant reference $\bar{w} \in W$, there exists an input sequence $\hat{r}^{T^*}$ subject to $z_0(\tau) \in \mathcal{Z}, \tau = 0, 1, \cdots, T^*-1$ and $x(T^*) \in O_\infty(\bar{w})$, where the shortest horizon T^* is given by (3).

Next, a procedure to get the shortest step T^* is shown using a numerical example.

Example. Consider a closed-loop system Σ that consists of the following plant Σ_p and controller Σ_c.

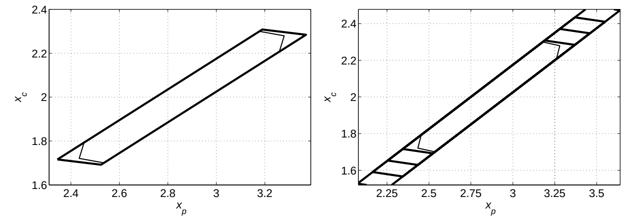
$$\Sigma_p \ : \quad \begin{bmatrix} x_p(t+1) \\ z_1(t) \end{bmatrix} = \begin{bmatrix} 1 & 0.04 \\ 0.7 & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ u(t) \end{bmatrix}$$

$$\Sigma_c \ : \quad \begin{bmatrix} x_c(t+1) \\ u(t) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.25 & -0.15 \\ 30 & 0 & -30 \end{bmatrix} \begin{bmatrix} x_c(t) \\ w(t) \\ z_1(t) \end{bmatrix}$$

where the reference is $w(t) = 2.0\ \forall t \in \mathbb{Z}^+$. There exists the constraint condition $-2.25 \leq u(t) \leq 2.25 \quad \forall t \in \mathbb{Z}^+$. Then, the corresponding maximal output admissible set $O_\infty(2)$ and the set $O_{-1}(2)$ are illustrated in **Fig. 3**(a). $O_\infty(2)$ and $O_{-1}(2)$ are respectively a polytope bounded by a thin line and by a thick line. Similarly, the set $O_{-k}(2),\ k \in \mathbb{Z}^+$ is illustrated in **Fig. 3**(b). From the figures, a part of the state space is parameterized by a step k and the inclusion $O_{-k}(2) \subset O_{-(k+1)}(2),\ k \in \mathbb{Z}^+$ holds. If the initial state of Σ is given by $x_0 = [\,2.25\ 1.6\,]'$, the shortest step T^* to be reachable to $O_\infty(2)$ is 3 because $x_0 \in O_{-k}(2),\ k \geq 3$ holds.

Here, we have the following assumption about the initial state of the constrained system.

Assumption 3. Under **Assumption 1**, for the given initial state x_0, there exists $T \in \mathbb{Z}^+$ subject to $x_0 \in O_{-T}(\bar{w})$.



(a) The maximal output admissible set $O_\infty(2)$ and the set $O_{-1}(2)$. (b) $O_{-k}(2),\ k = 0, 1, 2, \cdots$.

Fig. 3 The maximal output admissible set $O_\infty(2)$ and the sets $O_{-k}(2),\ k \in \mathbb{Z}^+$ in the state space of the closed loop system Σ.

Remark 7. When the initial state is not in $O_{-k}(\bar{w})$, i.e., $x_0 \notin \bigcup_{i=0}^{\infty} O_{-i}(\bar{w})$, then it holds that there does not exist an input $\hat{r}^{T^*}$subject to the terminal condition holds.

Thereby, we can show the following lemma.

Lemma 2. **Assumptions 1** and **3** hold. Then, for the constant reference $\bar{w} \in W$, there exists $\hat{r}^{T^*}$subject to $z_0(\tau) \in \mathcal{Z},\ \tau = 0, 1, \cdots, T^*-1$ and the terminal condition $x(T^*) \in O_\infty(\bar{w})$, where T^* is given by (3).

Proof. The proof is trivial because of the properties of the sets $O_{-k}(\bar{w})$.

Remark 8. This paper has shown how to acquire the requisite shortest horizon T^* to enforce the constraint fulfillment for the infinite horizon. Additionally, if you want to improve tracking performance, the improvement is achieved by using the longer horizons $T \geq T^*$.

4. Off-line Reference Management

From **Lemmas 1**, **2**, and **Remarks 5**, **6**, we can see that reference management for only the finite horizon $[0, T^*-1]$ makes it possible to fulfill the constraints for the infinite. This section shows how to manage the reference, explicitly taking into account the tracking performance.

4.1 Reference Management Problem

Under **Assumptions 1** and **3**, the shortest horizon T^* is available. Using it we formalize the management technique that considers both achievements of the terminal condition and the tracking performance, which is the optimization problem below,

$$\min_{\hat{r}^{T^*}} ||\hat{z}_1^{T^*} - \hat{\bar{w}}^{T^*}||_{2,P} + ||\hat{r}^{T^*} - \hat{\bar{w}}^{T^*}||_{2,Q} \tag{4a}$$

$$\text{s.t.} \quad x(T^*) \in O_\infty(\bar{w}) \tag{4b}$$

$$z_0(\tau) \in \mathcal{Z} \quad \tau = 0, \cdots, T^*-1 \tag{4c}$$

where $P = P' \succ 0$, $Q = Q' \succ 0$, and a l_2-norm of vectors is denoted by $||x||_{2,P} = x'Px$. The first term of the objective function (4a) stands for an error between the controlled output and the reference with a weight matrix P. The second is added for the signals obtained from the problem (4) not to be very vibratory, for it may be afraid of breaking the closed-loop system in practical use if such signals are input. We think that this point is an engineering-worthy explanation similar to 14).

Remark 9. From **Lemma 2**, under **Assumptions 1** and **3** there exist some input sequences $\hat{r}^{T^*}$ subject to (4b) and (4c). Therefore, the optimization problem (4) is always feasible. Moreover, since the objective function (4a) is a convex of $\hat{r}^{T^*}$, the solution is globally optimal.

Remark 10. Since this paper considers the tracking control problem under existence of the constraints, it is sufficient to evaluate only the first term of the objective (4a). However, in that case the coefficient matrix H becomes positive-semidefinite and we can not get its inverse matrix. To avoid this, we have to set the second term.

The reference governor realized by the optimization problem (4) is an off-line type, which inputs the optimal solution as a managed signal into the closed-loop system Σ. In other words, it is not preferable for practical controls of systems under the existence of the modeling errors and noises. Then, the next section considers the reference governor that manages the reference based on the measured current state.

4.2 Management Based on Measured State

Under **Assumptions 2** and **3**, the current state $x(t)$, measured at time t, can be considered as an initial state x_0 in the reference management optimization problem (4). From the consideration, the reference governor sequentially solves the optimization problem using the measured state and inputs the managed signals $r(t) = \hat{r}^{T^*}(1)$ into the closed-loop system.

To clear such a state feedback structure of the reference governor, first, the reference management problem (4) is rearranged by a new variable $z = \hat{r}^{T^*} + H^{-1}f(x_0)$ to acquire formulation that has the initial state x_0 on a rhs of inequality constraints, and then, the initial state in the formulation is replaced with the measured one $x(t)$. Finally, the following quadratic programming problem is obtained:

$$V_z(x(t)) = \min_z z'Hz \quad \text{s.t.} \quad Gz \le N + Sx(t) \tag{5}$$

with H, $f(x(t))$, N, S, and $V_z(x(t))$:

$$\begin{aligned} H &= Q_{12}'PQ_{12} + Q \qquad (6)\\ f(x(t)) &= Q_{12}'PQ_{11}x(t) - (Q_{12}'P + Q)\hat{w}\\ N &= W - GH^{-1}(Q_{12}'P + Q)\hat{w}\\ S &= E + GH^{-1}Q_{12}'PQ_{11}\\ V_z(x(t)) &= V(x(t)) - (Q_{11}x(t) - \hat{w})'P(Q_{11}x(t) - \hat{w})\\ &\quad - \hat{w}'Q\hat{w} + f(x(t))'H^{-1}f(x(t)) \end{aligned}$$

where $V(x(t))$ denotes the objective function (4a), $E \in \Re^{q\times n}$, $W \in \Re^q$ are coefficients of the inequality $G\hat{r}^{T^*} \le W + Ex_0$, obtained by (4b) and (4c). q is a number of the inequalities. Here, we omit subscriptions $k\,(= T^*)$.

Remark. On the second term of the objective (5), from (6), $Q \succ 0$ and $P \succ 0$ imply that $H = H' \succ 0$. Therefore, an optimizer is uniquely defined according to a certain state $x(t)$.

Remark 12. Since the optimization problem (5) is equivalent to the problem (4), the optimization (5) is solvable as well from **Lemma 2** when $x(t) = x_0$ holds under **Assumptions 1** and **3**. Furthermore, the optimizer $z^*(x_0)$ provides $\hat{r}^{T^*} = z^*(x_0) - H^{-1}f(x_0)$ and is also equivalent to the global solution to the problem (4).

Under **Assumption 2**, the optimization (5) is the reference governor that reflects the current state in managing the reference. However, the optimization is hard to solve on-line in a kind of mechanical system with short sampling times because of computation burden and so on. Then, in section **5.**, we will introduce a technique about computation burden reduction and apply it to the reference governor construction.

Remark 13. The state $x(t)$ in the optimization (5) can be considered as a parameter in a convex programming problem. This kind of optimization is called Multi-parametric Quadratic Programming(MpQP). Furthermore, in the MpQP we can have explicit solutions including the parameter and apply them in order to reduce the on-line computation burden [2)].

5. On-line Reference Management

This section derives a reference management rule from the explicit solutions to the MpQP problem (5) in order to construct the on-line type reference governor. From here, the parameter x is described for $x(t)$.

5.1 Explicit Solution in State

The explicit solutions including the state parameter are obtained by a so-called Karush-Kuhn-Tacker optimality condition (KKT condition) [1)]. The condition is as follows,

$$Hz + G'\lambda = 0, \tag{7}$$

$$\lambda_i(G^iz - N^i - S^ix) = 0, \quad i = 1, \cdots, q, \tag{8}$$

$$\lambda \ge 0,$$

$$Gz \le N + Sx, \tag{9}$$

using Lagrange multipliers $\lambda \in \Re^q$, where the subscript i

is the number of a row component and q is the number of inequalities in the MpQP (5). Now that $H \succ 0$, from (7),

$$z = -H^{-1}G'\lambda \tag{10}$$

holds. Substituting (10) for (8), we obtain the following complementary condition for each row,

$$\lambda^i(-G^iH^{-1}G'\lambda - N^i - S^ix) = 0,\ i = 1,\cdots,q. \tag{11}$$

Here, solve the MpQP with $x = x_0 \in O_{-T^*}(\bar{w})$ once. Among the inequality constraints in (9), active Lagrange multipliers are denoted as $\tilde{\lambda} > 0$ and inactive ones are as $\check{\lambda} = \mathbf{0}$. From (9), matrices $\tilde{G},\ \tilde{S},\ \tilde{N}$ are uniquely defined which satisfy $\tilde{G}z^* = \tilde{S}\tilde{x} + \tilde{N}$. With this, the complementary condition (11) explicitly includes the state parameter,

$$-\tilde{G}H^{-1}\tilde{G}'\tilde{\lambda} - \tilde{N} - \tilde{S}x = 0, \tag{12}$$

where if $\tilde{G}$ is row full rank, then $(\tilde{G}H^{-1}\tilde{G}')^{-1}$ exists. In this case, from (12) $\tilde{\lambda}$ is written as

$$\tilde{\lambda} = -(\tilde{G}H^{-1}\tilde{G}')^{-1}(\tilde{N} + \tilde{S}x) \tag{13}$$

where if $\tilde{G}$ is not row full rank, we require another procedure because of degeneracy [2]. In this paper, we exclude such a case and assume that $\tilde{G}$ is row full rank.

On the other hand, (10) is also written below using $\tilde{\lambda}$,

$$z = -H^{-1}\tilde{G}'\tilde{\lambda}. \tag{14}$$

After eliminating $\tilde{\lambda}$ from (13) and (14), the optimizer $z(x)$ to the MpQP forms in explicit state parameter x,

$$z(x) = H^{-1}\tilde{G}'(\tilde{G}H^{-1}\tilde{G}')^{-1}(\tilde{N} + \tilde{S}x). \tag{15}$$

The explicit solution (15) is available over the parameter region, in which all states provide the same matrices $\tilde{G},\ \tilde{S},\ \tilde{N}$, and the region is named as a critical region $CR_0^{T^*}$. Now that z in (14) must satisfy the inequality constraint in the MpQP(5), we have

$$GH^{-1}\tilde{G}'(\tilde{G}H^{-1}\tilde{G}')^{-1}(\tilde{N} + \tilde{S}x) \le N + Sx, \tag{16}$$

and since $\lambda \ge 0$ holds, we have

$$-(\tilde{G}H^{-1}\tilde{G}')^{-1}(\tilde{N} + \tilde{S}x) \ge 0. \tag{17}$$

Therefore, rearranging (16) and (17), the critical region $CR_0^{T^*}$ is written in a polytope below,

$$CR_0^{T^*} = \{\, x \in \Re^n \mid M_{cr_0}x \le m_{cr_0}\,\}, \tag{18}$$

where the matrix $M_{cr_0}^{T^*}$ and the vector $m_{cr_0}^{T^*}$ are below;

$$M_{cr_0}^{T^*} = \begin{bmatrix} GH^{-1}\tilde{G}'(\tilde{G}H^{-1}\tilde{G}')^{-1}\tilde{S} - S \\ (\tilde{G}H^{-1}\tilde{G}')^{-1}\tilde{S} \end{bmatrix},$$

$$m_{cr_0}^{T^*} = \begin{bmatrix} W - GH^{-1}\tilde{G}'(\tilde{G}H^{-1}\tilde{G}')^{-1}\tilde{N} \\ -(\tilde{G}H^{-1}\tilde{G}')^{-1}\tilde{N} \end{bmatrix}.$$

Therefore, in summary, if the measured state is in the critical region $CR_0^{T^*}$, we can obtain the optimal solution to the MpQP from (15). Using this, we can construct the reference management rule.

5.2 Reference Management Rule

From (15) and $z = \hat{r}^{T^*} + H^{-1}f(x)$ in section **4.2**, z is eliminated to have the following piecewise affine function of the parameter x,

$$\hat{r}^{T^*} = F_0^{T^*}x + g_0^{T^*} \quad \forall x \in CR_0^{T^*}, \tag{19}$$

where the matrices $F_0^{T^*} \in \Re^{p_1T^* \times n}$ and the vectors $g_0^{T^*} \in \Re^{p_1T^*}$ are below:

$$F_0^{T^*} = H^{-1}\{\tilde{G}(\tilde{G}H^{-1}\tilde{G}')^{-1}\tilde{S} - Q_{12}PQ_{11}\},$$
$$g_0^{T^*} = H^{-1}\{\tilde{G}(\tilde{G}H^{-1}\tilde{G}')^{-1}\tilde{N} + (Q_{12}P + Q)\hat{\bar{w}}\}.$$

The reference management rule (19) consists of the matrices $\tilde{G},\ \tilde{S},\ \tilde{N}$, calculated in the case of $x = x_0$, and it holds over the critical region $CR_0^{T^*}$. However, since only one rule (19) is insufficient, we have to calculate the other regions and management rules, respectively by using $x(1) \in O_{-(T^*-1)}(\bar{w})$, $x(2) \in O_{-(T^*-2)(\bar{w})}$, $x(3) \in O_{-(T^*-3)(\bar{w})}$, and so on. Furthermore, if we consider the case of the existence of the modeling errors and noises, it is preferable to calculate them such that all regions $O_{-T^*}(\bar{w})$ are laid with the critical regions, in case the different transition of the state occurs between ideal and real cases.

If there exist some spaces inside $O_{-T^*}(\bar{w})\backslash O_{-(T^*-1)}(\bar{w})$ except for the known $CR_0^{T^*}$, we can calculate another management rule using the same T^*. The corresponding region is denoted as $CR_1^{T^*}$ and is obtained from a certain state $\tilde{x} \in O_{-T^*}(\bar{w})\backslash(O_{-(T^*-1)}(\bar{w}) \cup CR_0^{T^*}))$ and the MpQP with T^*, in the similar way of section **5.1**. This procedure is repeated until

$$O_{-(T^*-1)}(\bar{w}) \cup (\bigcup_{i=0}^{l_{T^*}} CR_i^{T^*}) = O_{-T^*}(\bar{w}) \tag{20}$$

holds. As a result, we can have all management rules $F_i^{T^*}, g_i^{T^*}$ and the corresponding critical regions $CR_i^{T^*}$, $i = 0, 1, \cdots, l_{T^*}$ that provide the optimal managed reference signal.

Remark 14. The partition number of the critical regions l_{T^*} corresponds to combination of the active inequalities in (9). It is known that the upper bound of the partition number is 2^q, where q is the number of inequalities in the MpQP [2]. Therefore, it is possible to check (20) by finite calculation. Additionally, a more effective partitioning method of $CR_i^{T^*}$, $i = 0, 1, \cdots, l_{T^*}$ is reported in the literature 12).

Secondly, we set the region $CR_0^{T^*-1}$ that includes one-step transmitted state $x(1) = Ax_0 + B\hat{r}^{T^*}(1)$ as the base, and then, calculate $CR_i^{T^*-1}$ and $F_i^{T^*-1}$, $g_i^{T^*-1}$, $i = 1, 2, \cdots, l_{T^*-1}$ on T^*-1. By repeating the calculation about CR_0^k, $k = T^*-2, \cdots, 2, 1$, the region $O_{-T^*}(\bar{w})$ is perfectly partitioned, and we have the explicit solution $\hat{r}_i^k = F_i^k x + g_i^k, \forall x \in CR_i^k$, $k = 1, 2, \cdots, T^*$, $i = 0, 1, \cdots, l_k$.

Consequently, the following reference governor can be obtained, which is based on the measured state $x(t)$,

$$r(t) = \begin{cases} \hat{r}_i^k(1) & \text{if } x(t) \in CR_i^k, \quad k = 1, \cdots, T^*, \\ & \qquad\qquad\qquad i = 0, \cdots, l_k, \\ \bar{w} & \text{if } x(t) \in O_\infty(\bar{w}). \end{cases} \tag{21}$$

Remark 15. If the modeled dynamics Σ has no errors, it is sufficient to calculate the critical region CR_0^k and the corresponding management rule F_0^k, g_0^k, $k = 1, 2, \cdots, T^*$ under $i = 0$.

We can show the following lemma about the obtained reference governor (21).

Lemma 3. **Assumptions 1, 2**, and **3** hold. Then, the optimal solution $\hat{r}^{T^*}$ to (4) for the constant reference $\bar{w} \in W$ is coincident with managed signals of the reference governor (21) over the span $[0, \ T^*-1]$, where T^* is given by (3).

Proof. We consider the case when **Assumptions 1, 2**, and **3** hold. For the constant reference $\bar{w} \in W$, **Lemma 2** shows that the reference management problem (4) is always feasible. Next, the MpQP (5) is equivalent to the problem (4) and the solution that satisfies the KKT condition is optimal. Therefore, since the initial state x_0 is common, the reference management rule (21) derived from the KKT condition corresponding to the MpQP, provides the optimal solution to the problem (4).

Therefore, under **Assumptions 1, 2**, and **3** we can show the following main result about the closed-loop system equipped with the reference governor.

Theorem 1. For the constant reference $\bar{w} \in W$, the closed-loop system Σ with the reference governor that consists of (21) can guarantee fulfilling the constraint condition (2).

Proof. From **Lemma 3**, the signal managed by the rule (21) is the optimal solution to the reference management problem and achieves the terminal condition. In this case, from **Lemmas 1, 2** it is shown that the constraint fulfillment for infinite steps can be achieved.

Remark 16. From **Theorem 1**, since the reference governor optimally manages the reference in the sense of the objective (4a), the equipped closed-loop system has good tracking performance about the output z_1.

The reference management rules (21) have been proposed under the assumption of no modeling errors and noises. However, since they have the structure to reflect the current state of the system in managing the reference, it is hoped that they are a practical approach even though there exist the modeling errors and noises. Then, in the next section, we will validate the effectiveness of the proposed reference governor using a practical plant.

6. Experimental Validation

In this section, using a real position servomechanism in **Fig. 4**, we make experimental evaluations of the proposed control approach. The utilized digital controller through this experiment is a personal computer, RT-Linux v3.1 Intel Pentium 3 733MHz 256MB.

6.1 Description of Constrained System

6.1.1 Plant

The practical plant Σ_p consists of a DC-motor, a gear, a hard shaft and a load in **Fig. 4**. With $x_p = [\theta_L \ \ \dot{\theta}_L]'$, the model of Σ_p can be described by the following state space form,

$$\dot{x}_p(t) = \begin{bmatrix} 0 & 1 \\ 0 & -2\zeta w_n \end{bmatrix} x_p(t) + \begin{bmatrix} 0 \\ w_n^2 \end{bmatrix} u(t),$$
$$z_1(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p(t),$$

where θ_L is a rotate angle of the motor, the control $u = V$ and the output $z_1 = \theta_L$ measured by an encoder. ζ and w_n have the parameters in **Table 1**,

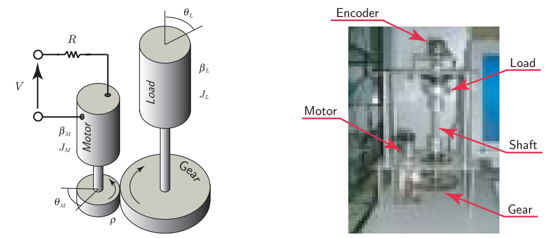
$$\zeta = \frac{R(B_M + \frac{1}{\rho}B_L) + k_T k_b}{2\{Ak_T R(J_M + \frac{1}{\rho}J_L)\}^{1/2}}, \ \ w_n = \left\{ \frac{Ak_T}{R(\rho J_M + J_L)} \right\}^{\frac{1}{2}}.$$

We have identified both parameter values from the experimental step response data of $\theta_L = 90$ [deg] = 1.5708 [rad]. As a result, $\zeta = 0.7$ and $w_n = 7$. Additionally, we have known that their values were different in the other case of not $\theta_L = 90$.

Moreover, Σ_p has a saturation regarding the input volt $z_0 = u$, which is given by

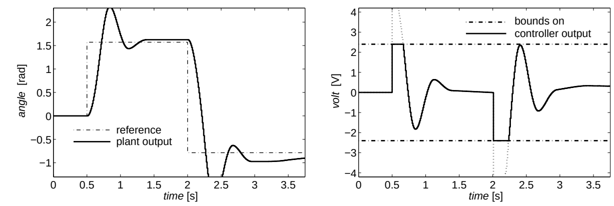
$$-2.4 \le u(t) \le 2.4 \qquad \forall\, t \in \mathbb{Z}^+. \tag{22}$$

To measure the load rotational velocity $\dot{\theta}_L$, we have used an arithmetic average method of 20 position data, measured by sampling time 1.0 millisecond.



(a) Model illustration (b) Model picture

Fig. 4 Position servomechanism



(a) Reference w and controlled output $z_1 = \theta_L$.

(b) Control $z_0 = u$ and the constraint (22). Dotted line is controller output signal before being saturated.

Fig. 5 Step responses of the constrained system without the proposed reference governor.

6. 1. 2 Controller Design

A controller Σ_c has been designed by loop-shaping. The specification is to possess the stability and the tracking performance to a given reference. Σ_c is represented as the following transfer function from an error $r - z_1$ to a control u,

$$K_c\left(1+\frac{1}{T_c s}\right),$$

where $K_c = 3$ and $T_c = 1$. Then, it has been discretized by sampling time 10 milliseconds with zero-order hold, and implemented on the Pentium computer.

6. 1. 3 Control Problem

We consider the tracking control problem under the constraint (22). The reference signal w is the following,

$$w(t) = \begin{cases} 0, & 0 \le t < 0.5 \text{ [s]}, \\ \bar{w}_1, & 0.5 \le t < 2.0 \text{ [s]}, \\ \bar{w}_2, & 2.0 \le t \text{ [s]}, \end{cases} \tag{23}$$

where $\bar{w}_1 = 1.5708$ [rad] and $\bar{w}_2 = -0.7853$ [rad] $= -45$[deg]. The initial states $x_p(0)$ and $x_c(0)$ of Σ_p and Σ_c respectively are all zeros.

Without the proposed reference governor, then, we have obtained the result of the responses $z_1 = \theta_L$ and $z_0 = u$ in **Fig. 5**. From **Fig. 5**(a), the practical closed-loop system exhibits the very fast response under the reference (23), but also inadmissible voltage inputs for the first reference change at $t = 0.5$ [s] and the second at $t = 2.0$ [s], as shown in **Fig. 5**(b). It is especially noteworthy that because of the saturation, the dotted line in **Fig. 5**(b) is not supplied to the DC-motor. It is just the value for the controller Σ_c to have calculated.

Table 1 Physical parameters of Σ_p

Symbol	Unit	Meaning
A	—	Amplifier gain
ρ	—	Gear ratio
R	Ω	Resistance of armature
k_T	[Nm/A]	Motor electrical constant
k_b	[V/A]	Back electromotive force constant
J_M	[Kgm^2]	Motor inertia
J_L	[Nm/(rad/s)]	Motor viscous friction coefficient
B_M	[Kgm^2]	Load inertia
B_L	[Nm/(rad/s)]	Load viscous friction coefficient

6. 2 Simulation Result

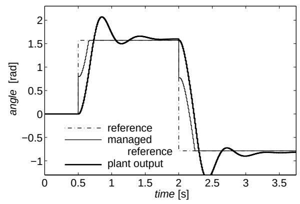
We have made a simulation for the given reference (23). The result is shown in **Fig. 6**. From **Fig. 6**(a), the reference w in a chain line is managed and modified into a thin continuous line r, and then, the plant output $z_1 = \theta_L$ is plotted by thick continuous line. We can see that it has good tracking performance to the reference specified in (23). From **Fig. 6**(b), the control $z_0 = u$ does not violate the constraint condition (22), but partially becomes equal to the limitation value ± 2.4 [V], which is drawn with a chain line. Apparently, the constraint (22) is satisfactorily fulfilled. These results show the effectiveness of the proposed method in the simulation level.

Here, when the reference governor is constructed, we have utilized the shortest steps T_1^* and T_2^* such that $x(0) = [x_p(0)' \ x_c(0)']' \in O_{-T_1^*}(\bar{w}_1)$ and $(I-A)^{-1}B\bar{w}_1 \in O_{-T_2^*}(\bar{w}_2)$ hold, where they are respectively $T_1^* = 15$ and $T_2^* = 23$. Furthermore, we set the partition number as $l_k = 0, \ \forall k \in \{0, \cdots, T_1^*\}$ and $l_k = 0, \ \forall k \in \{0, \cdots, T_2^*\}$ about $\bar{w}_1$ and $\bar{w}_2$, respectively.

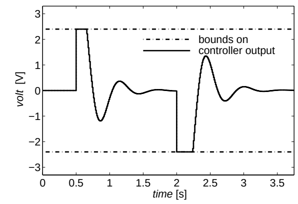
6. 3 Experimental Result

We have made experiments for the reference (23). The results are shown in **Fig. 7**. Comparing the time response in **Fig. 7**(a) with the one in **Fig. 5**(a), we can see the overshoots of the controlled output z_1 where successfully reduced and good tracking performance was achieved. From **Fig. 7**(b) the constraint condition (22) is also satisfactorily fulfilled. This experimental result has no conservatism for the constraint fulfillment at all and is far better than we have seen in any other studies [10),13)].

Moreover, a remarkable point in the experimental result is the achievement of the constraint condition for the reference $\bar{w}_2$, as is shown in **Fig. 7**(b). The plant model parameters obtained by the time response data in the case of $\bar{w}_1$ were used in designing the reference governor. This

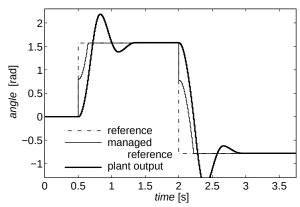


(a) Reference w, managed reference r and controlled output $z_1 = \theta_L$.

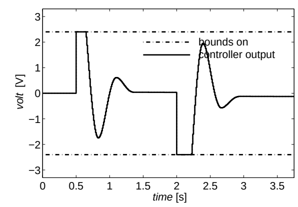


(b) Control $z_0 = u$ and the constraint (22).

Fig. 6 Simulation results of step responses of the constrained system with the proposed reference governor.

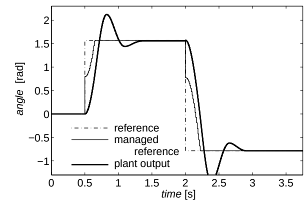


(a) Reference w, managed reference r and controlled output $z_1 = \theta_L$.

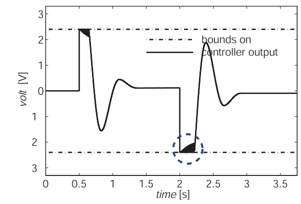


(b) Control $z_0 = u$ and the constraint (22).

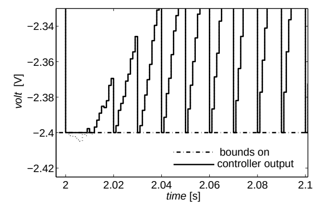
Fig. 7 Experimental results of step responses of the constrained system with the proposed reference governor.



(a) Reference w, managed reference r and controlled output $z_1 = \theta_L$.



(b) Control $z_0 = u$ and the constraint (22).



(c) A scaled up graph of **Fig. 8**(b) regarding the circle-marked region.

Fig. 8 Experimental results to validate whether or not the constraint are fulfilled under the condition that sampling times of the constrained closed-loop systems and the proposed reference governor are different.

means that we might expect an appearance of constraint violation caused by the modeling errors and noises under the different reference $\bar{w}_2$. Indeed, we obtained not only the constraint satisfaction but also good tracking performance. This implies that the designed reference governor is practicable even if there exist modeling errors and noises.

Remark 17. Through the experimental validation, we set the sampling period in the reference governor and the control system as 10 [ms]. When it had been set as 1.0 [ms], the control experiment could not be performed because the size of the C-Language program in which the reference management rule is written is too big to execute on-line. Therefore, we can guess that the utilized control equipment could have a practical sampling period between 1.0 [ms] and 10 [ms].

6.4 Consideration of Implementing Data Size

When one uses the proposed reference governor, the finally obtained management rule needs to be actually implemented in the computer. However, it may take too much computation cost or size to implement and to execute the rule in real time. To reduce the size of their data, therefore, we may perform the control with different sampling times between the constrained system and the proposed reference governor.

Let us show what responses we obtain between sampling instants of reference management, using the same control apparatus. For the goal, we have discretized the same controller Σ_c with 1.0 millisecond by zero-order hold and have implemented it, where the reference governor manages the reference signals with respect to each 10 millisecond. Then, the responses are shown in **Fig. 8**, which is plotted by 1.0 millisecond.

Fig. 8(a) is almost the same as **Fig. 7(a)**. In **Fig. 8**(b), the controller output z_0 has oscillations in the duration of operating the reference governor. To scrutinize the response, **Fig. 8**(c) is shown. It is a graph to be scaled up regarding the circle-marked region in **Fig. 8**(b). This illustrates that although the constraints are certainly fulfilled at each sampling instant of the reference governor, in the interval the control violates the constraints or leaves the conservatism.

This sufficient investigation does not appear anywhere in the literatures, so it is a very interesting and meaningful result for future works of developing the reference governor and other control approaches for the constrained system. From the engineering viewpoint of the reference governor's original object of fulfilling the constraints, there is room for improvement because we could have actually checked the possibility of the constraint violations.

7. Conclusion

In this paper, we have proposed the reference governor approach to systems with state and/or control constraints. The reference management rule has been obtained from the explicit solution to the reference management problem (4) through the corresponding MpQP (5), which explicitly considers not only constraint fulfillment but also tracking performance. Since the rule is in the form of a piecewise state affine function, the optimal managed signal subject to the specified constraint for each time has been decided on-line, based on the measured state of Σ. Furthermore, the effectiveness of the proposed method is shown by simulation. In the experimental evaluation we also have performed a demonstration of the effectiveness and practicability. Moreover, we have checked that there is room for improvement regarding constraint fulfillment from the viewpoint of the original object of a reference governor.

References

1) M. S. Bazaraa, H. D. Sherali, and C. M. Shetty. *Nonlinear programming — theory and algorithms (2nd ed.)*, John Wiley & Sons, 1993.
2) A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos. "The explicit linear quadratic regulator for constrained systems", *Automatica*, **38(1)**, 3/20, 2002.
3) A. Bemporad and E. Mosca. "Fulfilling hard constraints in uncertain linear systems by reference managing", *Automatica*, **34(4)**, 451/461, 1998.
4) A. Casavola, E. Mosca, and D. Angeli. "Robust command governors for constrained linear systems", *IEEE Trans. on Automatic Control*, **45(11)**, 2071/2077, 2000.
5) E. G. Gilbert. "Linear control systems with pointwise-in-time constraints: What do we do about them?", *Proceedings of the 1992 American Control Conference*, 2565, 1992.
6) E. G. Gilbert and I. Kolmanovsky. "Nonlinear tracking control in the presence of state and control constraints: a generalized reference governor", *Automatica*, **38(12)**, 2063/2073, 2002.
7) E. G. Gilbert and K. T. Tan. "Linear systems with state and control constraints: The theory and application of maximal output admissible sets", *IEEE Trans. on Automatic Control*, **36(9)**, 1008/1020, 1991.
8) P. Kapasouris and M. Athans and G. Stein. "Design of feedback control systems for stable plants with saturating actuators", *Proceedings of the 27th IEEE Conference of Decision and Control*, 469/479, 1988.
9) K. Kogiso and K. Hirata. "An off-line reference management technique for linear constrained systems with time-varying uncertainties", *Proceedings of the 4th Asian Control Conference*, 77/82, 2002.
10) R. H. Miller, I. Kolmanovsky, E. G. Gilbert and P. D. Washabaugh. "Control of constrained nonlinear systems: A case study", *IEEE Control Systems Magazine*, **20(1)**, 23/32, 2000.
11) T. Sugie and T. Fukui. "Reference shaping for uncertain systems with constraints and its experimental validation", *Proceedings of the 2003 American Control Conference*, 1236/1241, 2003.
12) P. Tøndel, T. A. Johansen and A. Bemporad. "An algorithm for multi-parametric quadratic programming and explicit MPC solutions", *Automatica*, **39(3)**, 489/497, 2003.
13) K. Kogiso, H. Minemura and K. Hirata. "Verification of reference governor control approaches to experimental systems with control constraints", *Trans. of the Japan Society of Mechanical Engineers C*, **69(681)**, 1238/1246, 2003 (in Japanese).
14) T. Sugie, H. Yamamoto. "Reference management for closed-loop systems with state and control constraints", *Trans. of The Society of Instrument and Control Engineers*, **37(9)**, 849/855, 2001 (in Japanese).
15) K. Hirata. "Feasible states parameterizations for off-line reference management", *Proceedings of SICE 1st Annual Conference on Control Systems*, 603/606, 2001 (in Japanese).
16) K. Hirata, K. Kogiso. "An off-line reference management technique for constraint fulfillment", *Trans. of The Institute of Systems, Control and Information Engineers*, **14(11)**, 554/559, 2001 (in Japanese).
17) K. Hirata, M. Fujita. "Analysis of conditions for non-violation of constraints on linear discrete-time systems with exogenous inputs", *Trans. of The Institute of Electrical Engineers of Japan. C*, **118(3)**, 390/384, 1998 (in Japanese).
18) K. Hirata, M. Fujita. "Control of systems with state and control constraints based on connected reference signals", *Trans. of The Institute of Systems, Control and Information Engineers*, **12(10)**, 586/595, 1999 (in Japanese).

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