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Delay Time Estimation Using Hilbert Transform and New Extrapolation Procedure

Yosuke TSUCHIYA* and Yasushi MIKI**

We present a new method for estimating the delay time of a minimum-phase system with delay. It is based on the Hilbert transform relationship between the log magnitude and the phase of a minimum-phase system. An extrapolation procedure for the frequency characteristics is required to describe a discrete causal system which corresponds to the actual continuous system. The algorithm for the extrapolation is improved to stabilize the frequency characteristics, and bi-directional extrapolation procedure is presented.

Key Words: delay time, Hilbert transform, extrapolation, causal system

1. Introduction

To measure the acoustic impedance of a material, the distance between its surface and an observation point must be measured accurately. In the case of a grass land or a gravel layer, however, it is not easy to determine the reflecting surface visually. A new method presented here is based on the fact that a reflection system is minimum-phase, and uses the Hilbert transform relationship between the log magnitude and phase of frequency characteristics of the system. Using this method, an acoustical boundary can be determined acoustically by estimating the delay time of a sound reflection system.

A basic principle of the delay time estimation using Hilbert transform has been proposed by one of the authors.[1),2)] The main subject of the present paper is how to give the total frequency characteristics of a discrete causal system from a finite number of measurement data. It has been shown that the transformation matrix which transforms the lower half of frequency characteristics of a discrete causal system to the higher half exists uniquely, just as Hilbert transform transforms the real part of the frequency characteristics to the imaginary part thereof. However , this transformation is too sensitive to an additive non-causal noise for a practical use. Hence, we have introduced an extrapolation technique.[2)]

We, first, improve the extrapolation algorithm proposed before in order to stabilize the final result, and further, we present a new bi-directional extrapolation procedure for the band-limited measurement data.

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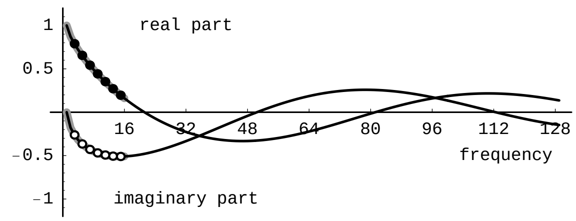


Fig. 1 Real part and imaginary part of the frequency characteristics of a causal system. These are the Hilbert transform pair in a continuous system.
Circles: frequency characteristics obtained by measurement.

2. Delay time estimation

2.1 Extrapolation procedure for the discrete causal system

In a real system, frequency characteristics are expanded to infinite frequency range, but in a actual measurement, only a finite number of data within a finite frequency range are obtainable, as shown in circles in **Fig. 1**. The band-limited system in general does not satisfy causality. But we can describe a discrete causal system by modifying the frequency characteristics obtained by measurement using extrapolation technique.

Let

$$Z(k) = X(k) + jY(k) \quad (k = 0, 1, ..., K-1) \qquad (1)$$

be the frequency characteristics, where $Z(k)$ ($k = 0, 1, ..., K/4-1$) are given by measurement, and the complex conjugate relations are assumed for negative frequencies, i.e.,

$$Z(K-k) = Z^*(k) \qquad (2)$$

Frequency characteristics $Z(k)$ ($k = K/4, K/4+1, ..., K/2$) are to be extrapolated. The procedure is as follows:

1. Set

$$X(k) = x_k \quad (k = K/4,\ K/4+1,\ ...,\ K/2) \qquad (3)$$

2. Take the DHT(Discrete Hilbert Transform) of $X(k)$: $H[X(k)]$
3. Define the squared error:

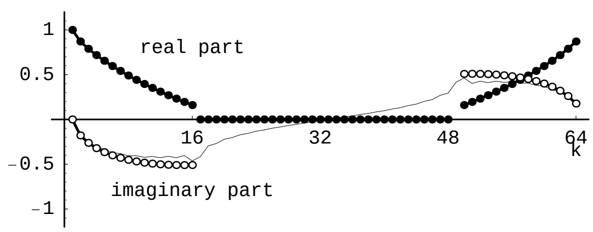
$$E = \sum_{k=0}^{K/4-1} (H[X(k)] - Y(k))^2 + \sum_{k=K/4}^{K/2} W^2(k) \quad (4)$$

$$W(k) = \frac{1}{M}[(X(k) - X(k-1)) + j(H[X(k)] - H[X(k-1)])] \qquad (5)$$

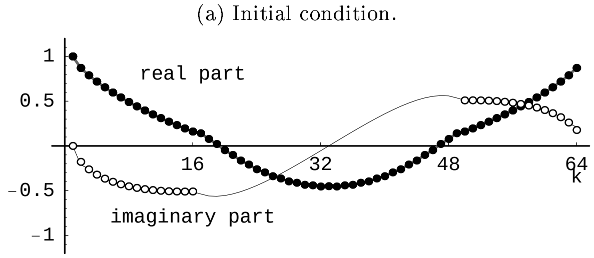
4. Find minimum E with respect to the variables x_k $(k = K/4,\ K/4+1,\ ...,\ K/2)$.

2.2 Results of extrapolation

An example of extrapolation is shown in **Fig. 2**. The points at $k = 1$ through 16 show the frequency characteristics obtained by measurement, and points $k = 49$ through 64 show the complex conjugates. The real part of the frequency characteristics between $k = 17$ and 48 are set to zero as initial values. The imaginary part of the frequency characteristics calculated from the DHT of the real part (thin line), shows a difference from the imaginary part obtained by measurement (open circles). It is due to the fact that the measured characteristics does not satisfy causality. **Figure 2** (b) shows the final result, and a good agreement is achieved between thin line and open circles.



(a) Initial condition.



(b) Final result of extrapolation.

Fig. 2 Procedure for extrapolation. (a) Real part (filled circles) and imaginary part (open circles) of the frequency characteristics are given in the lower frequency range. (b) The thin line shows the DHT of the real part of the total frequency characteristics after extrapolation.

2.3 Stabilization of extrapolation procedure

The second term in Eq. (4) is an additional term to stabilize the extrapolation against possible non-causal noise. Without addition of this term causes considerable variations in extrapolated values (open circles in **Fig. 3**).

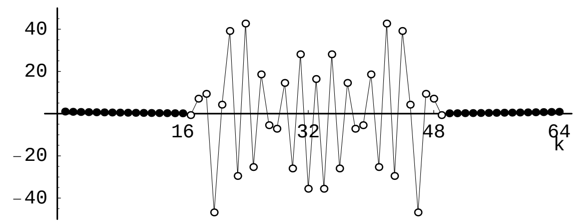


Fig. 3 Result of extrapolation without stabilizing process. Open circles show the frequency characteristics obtained by extrapolation from the measured data (filled circles).

2.4 Estimation of the delay time

By the extrapolation procedure described above, the frequency characteristics are modified to satisfy the causality in the discrete system. Here follows the procedure for estimating the delay time.

1. Take the complex logarithm of $Z(k)$:

$$\tilde{Z}(k) = \tilde{X}(k) + j\,\tilde{Y}(k) \qquad (6)$$

$$\tilde{X}(k) = \log|Z(k)|\,,\ \ \tilde{Y}(k) = \arg Z(k) \qquad (7)$$

2. Take the DHT of the log magnitude: $H[\tilde{X}(k)]$
3. Define the squared error:

$$E' = \sum_{k=0}^{K'} (H[\tilde{X}(k)] - \tilde{Y}(k) - k\Delta)^2 \qquad (8)$$

where K' with respect to Δ, which gives an estimate of delay.

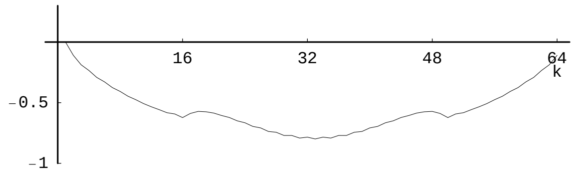
4. Find minimum E' with respect to Δ, which gives an estimate of delay.

Figure 4 shows an example of the estimation of a delay time in a minimum-phase system with delay.

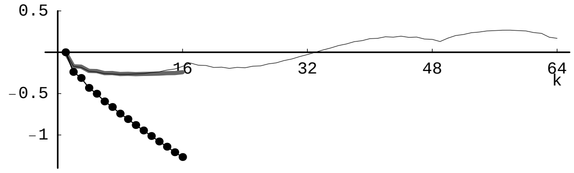
3. Delay time estimation of sound reflected from a boundary of porous material [3),4)]

3.1 Model of the porous material

In former sections, we have used a sound reflection system from the boundary of the porous material with the



(a) Log magnitude.



(b) Phase.

Fig. 4 (a) Log magnitude of the extrapolated frequency characteristics shown in **Fig. 2** (b). (b) DHT of the log magnitude (thin line), phase by measurement (dots), and phase after adjustment (thick line).

acoustic impedance:

$$Z(f) = R(f) + j\,X(f), \tag{9}$$

$$R(f) = \frac{q}{\Omega}\left[1 + 0.070\left(\frac{f}{\sigma_e}\right)^{-0.632}\right], \tag{10}$$

$$X(f) = -0.107\frac{q}{\Omega}\left(\frac{f}{\sigma_e}\right)^{-0.632}. \tag{11}$$

where q and Ω are the tortuosity and the porosity of porous material, respectively, and $q = \Omega = 1$ is assumed for simplicity. Furthermore, σ_e is the flow resistivity of the material and it represents a wide variety of acoustic materials, i.e., $\sigma_e = 10\,\text{k}$ (SI units) represents a glass wool, $\sigma_e = 300\,\text{k}$ a grass land, and $\sigma_e = 20000\,\text{k}$ an asphalt pavement.

3.2 Consideration for the estimation error

The estimated result of the delay time is considered. The normal incidence reflection characteristics are shown in **Fig. 5** for the porous material with the flow resistivity $\sigma_e = 300\,\text{k}$.

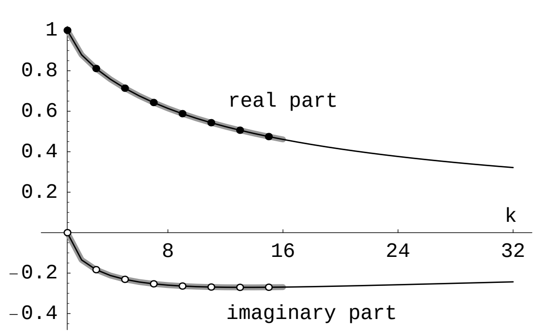


Fig. 5 Reflection characteristics of a porous material ($\sigma_e = 300\,\text{k}$). Circles show the discrete frequency characteristics given in the range 0 - 5 kHz.

Let $K = 64$ in Eq. (1). The frequency characteristics at $k = 1$ through 16 are given (circles in **Fig. 5**), and those at $k = 17$ through 32 are to be extrapolated. When the sampling frequency is assumed to be 20 kHz, the frequency range will be 0 - 5 kHz for $k = 1$ - 16, and 5 - 10 kHz for $k = 17$ - 32.

In order to evaluate the estimation error, following examination are made. Let

$$Z_n(k) = Z(k)e^{-j2\pi kn\Delta/N} \quad (n = 1,\,2,\,...,\,N) \tag{12}$$

be a set of frequency characteristics containing predetermined delay times. Suppose that $N = 64$, and $\Delta = 1/16$. Delays are given in the range $(0,\ 4\,T_s)$, where T_s is the sampling period and $T_s = 50\,\mu s$.

Figure 6 shows the log magnitude estimated for all values of n in Eq. (12). We can see the periodical dependence of the log magnitude on delay, and the period is nearly equal to 2 T_s.

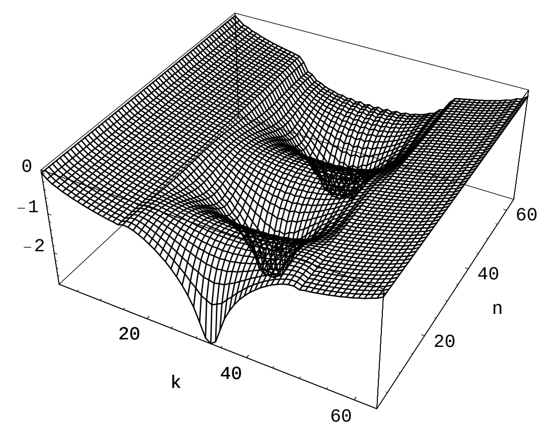


Fig. 6 Log magnitude of the extrapolated frequency characteristics for several values of given delays. Periodical changes are seen with increasing delays.
k: discrete frequency. n: given delays ($\times T_s/16$).

Figure 7 shows the relation between actual and estimated delay. The unit of axes is T_s. The estimation curve for $\sigma_e = 300\,\text{k}$ has a periodical swell with the same period as the log magnitude shown in **Fig. 6**. We can fit a regression line for the data as shown below.

$$y = 0.136254 + 0.938897x \tag{13}$$

The error between the regression line and the estimated result for $\sigma_e = 300\,\text{k}$ is shown in **Fig. 8**. The error lies in the range $(-0.2,\ +0.1)T_s$. Substituting $x = 0$ in Eq. (13), we obtain the time origin $y = 0.13\,T_s = 6.8\,\mu s$. Since the maximum value of the error is $0.2\,T_s$, the regression analysis reduces the error by half.

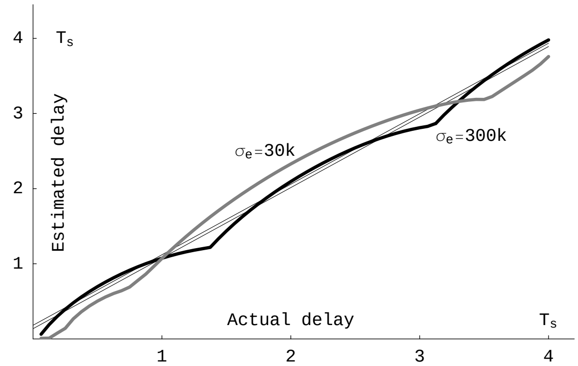


Fig. 7 Relation between actual and estimated delay. The unit of abscissa and ordinate is T_s.
Solid line: $\sigma_e = 300\,\mathrm{k}$. Gray line: $\sigma_e = 30\,\mathrm{k}$.
Thin lines: regression lines obtained from estimated values.

The estimated result for $\sigma_e = 30\,\mathrm{k}$ is also shown in **Fig. 7**, where the estimation error at the time origin is about $9\,\mu s$.

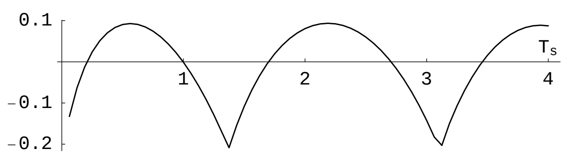


Fig. 8 Difference between the regression line and the estimated delay shown in **Fig. 7** ($\sigma_e = 300\,\mathrm{k}$).

3.3 Comparison with the cross-correlation method

The cross-correlation method is a general technique for estimating delay time. A computational analysis for the similar experiment as described above is shown in **Fig. 9**. A porous model with $\sigma_e = 300\,\mathrm{k}$ is used, and the test signal is assumed to be band limited to 5 kHz. The result shows that the estimation error is about $49\,\mu s$, and is 7 times as large as the error obtained by the proposed method.

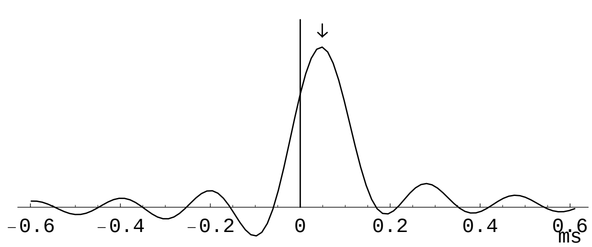


Fig. 9 Determination of the time origin of the impulse response by cross-correlation method.

4. Bi-directional extrapolation procedure

So far we have assumed that the frequency characteristics are available in the low frequency range. However, transmitters used for acoustical measurements, especially in the range of ultrasonic, have generally narrow-band frequency characteristics not only to the higher frequency range but also to the lower frequency range.

4.1 Extrapolation procedure

The bi-directional extrapolation procedure is quite similar to the extrapolation procedure described above. We assumed that the frequency characteristics $Z(k)$ ($k = K/8+1, K/8+2, ..., 3K/8$) are obtained by measurement, and $Z(k)$ ($k = 0, 1, ..., K/8$; $k = 3K/8+1, 3K/8+2, ..., K/2$) are to be extrapolated. Then Eq. (3) in Section 2.1 is rewritten as

$$X(k) = x_k \qquad (k = 0, 1, ..., K/8; \quad k = 3K/8+1, 3K/8+2, ..., K/2) \quad (14)$$

and Eqs. (4-5) as

$$E = \sum_{k=0}^{K/8} W^2 + \sum_{k=K/8+1}^{3K/8} (H[X(k)] - Y(k))^2 + \sum_{k=3K/8+1}^{K/2} W^2(k) \quad (15)$$

$$W(k) = \frac{1}{M}[(X(k) - X(k-1)) + j(H[X(k)] - H[X(k-1)])] \quad (16)$$

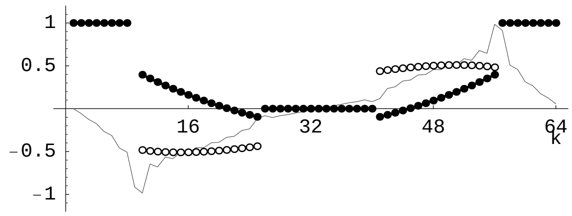
Find the variable x_k ($k = 0, 1, ..., K/8$; $k = 3K/8 + 1, 3K/8+2, ..., K/2$) which becomes the minimum E.

4.2 Result of extrapolation

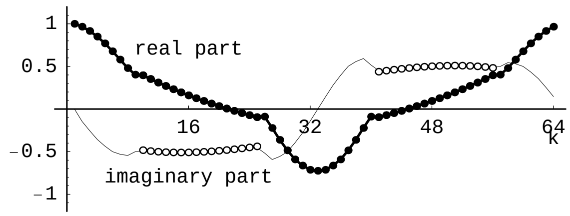
Figure 10 (a) shows the initial condition, where initial values of the real part of frequency characteristics in high frequency range are set to zero and those in low frequency range are set to unity, since the reflection coefficient at frequency zero is assumed to be unity. **Figure 10** (b) shows the result of extrapolated real part (thick line) and the imaginary part (thin line) which is calculated from the DHT of the real part.

4.3 Considerations for the estimation error

Similar considerations are made as described in Section 3.2. We assumed that the frequency characteristics in the range 2.5 - 7.5 kHz are given by measurement and those in the range 0 - 2.5 kHz and 7.5 - 10 kHz are to be extrapolated.



(a) Initial condition.



(b) Final result of extrapolation.

Fig. 10 Procedure for extrapolation. (a) Real part (filled circles) and imaginary part (open circles) of the frequency characteristics are given in the middle frequency range. (b) The thin line shows the DHT of the real part of the total frequency characteristics after extrapolation.

Figure 11 shows the relation between actual and estimated delay for the bi-directional extrapolation. A regression line fitted for the data is written as

$$y = 0.145643 + 0.998782x \tag{17}$$

The error between the regression line and the estimated result is shown in **Fig. 12**. The error lines in the range $(0.15, +0.05)\,T_s$. Substituting $x = 0$ in Eq. (17), we obtain the time origin $y = 0.14\,T_s = 7.3\,\mu s$. The estimated result for $\sigma_e = 30\,\mathrm{k}$ is also shown in **Fig. 12**, where the estimation error at the time origin is about $8.5\,\mu s$. We find that these estimation errors are comparable with those shown in Section 3.2.

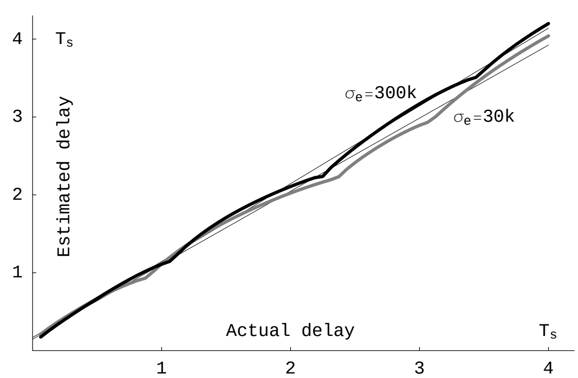


Fig. 11 Relation between actual and estimated delay. The unit of abscissa and ordinate is T_s.
Solid line: $\sigma_e = 300\,\mathrm{k}$. Gray line: $\sigma_e = 30\,\mathrm{k}$.
Thin lines: regression lines obtained from estimated values.

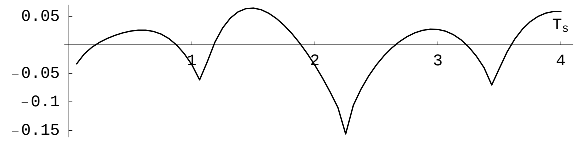


Fig. 12 Difference between the regression line and the estimated delay shown in **Fig. 11** ($\sigma_e = 300\,\mathrm{k}$).

4.4 Comparison with cross-correlation method

A similar discussion described in Section 3.3 is also made. A porous model with $\sigma_e = 300\,\mathrm{k}$ is used, and the test signal is assumed to be band-pass filtered with the frequency range 2.5 - 7.5 kHz. The estimation error is found to be about $27\,\mu s$, and is 4 times as large as the error obtained by the proposed method.

5. Discussions

The amount of the estimation error depends on the flow resistivity of a reflection surface. **Figure 13** shows the error curves for two types of extrapolation procedures described above as a function of the flow resistivity. Generally, the errors decrease with increasing flow resistivity, and two curves show little difference with each other.

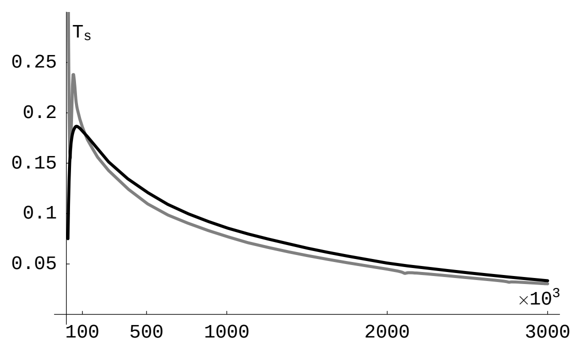


Fig. 13 Estimation errors of the delay time as a function of the flow resistivity.
Gray line: conventional extrapolation.
Solid line: bi-directional extrapolation.

6. Conclusions

The extrapolation procedure for the frequency characteristics is presented with the new stabilizing technique which stabilizes the extrapolation against possible noncausal noise. The delay time estimation is achieved with the accuracy 7 times as high as that by the conventional cross-correlation method. The new extrapolation method, is also presented. This method can be applied to the delay time estimation using a narrow-band test signal, especially in the region of ultrasonic.

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