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Mass Measurement Using a System Containing an On-Off Relay with Dead Zone

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A mass measurement system using the self-excited oscillation of a relay feedback system was proposed. The system contains an on-off relay with dead zone and switches force acting on the object in relation to the position. An analytical study showed that the mass of the object is determined from the time interval measurement of the on-state and off-state periods. An apparatus was developed for experimental study. It uses a voice coil motor for generating force, and a pair of photo interrupters for detecting the switching positions. The effects of system parameters on measurement accuracy were studied experimentally. Under the tuned conditions, the measurement errors were within 0.2%. The effects of damping on measurement accuracy were also studied experimentally.

Keywords: Mass measurement, Space engineering, Relay feedback, Self-excited vibration, Nonlinear control

1. Introduction

Laboratory environment for conducting weightless or low-gravity experiments will be available due to the development of space technology. Such experiments will have applications to material technology, biology, and other fields of science and engineering. Mass measurement under weightless conditions will be necessary to perform these experiments. Several measurement systems have been already proposed[1)-9)]; the reference[10)] gives a review on them. The authors have developed mass-measurement systems that use a dynamic vibration absorber as a device both for mass measurement and vibration control[5),6)]. It has been experimentally shown that they perform measurements with a percent-order[5)] or higher[6)] accuracy. However, the reduction of weight is a very critical requirement to space equipment so that measurement systems fit for lightening should be considered.

In this paper, a mass measurement system with simpler mechanism is proposed, which is characterized by using relay feedback. The relay feedback method was successfully applied to auto-tuned PID controllers for process control[11)]. The authors have applied it to a measurement system, which contains an on-off relay with *hysteresis*[12)]. Since the *velocity* of the object is fed back in the system, it is equipped with a velocity sensor, which is usually heavy and costly. In contrast, the mass measurement system proposed in this paper contains an on-off relay with *dead zone* and feeds back the *position* of the measurement object. It can be realized with a compact and low-cost sensor such as photo interrupter.

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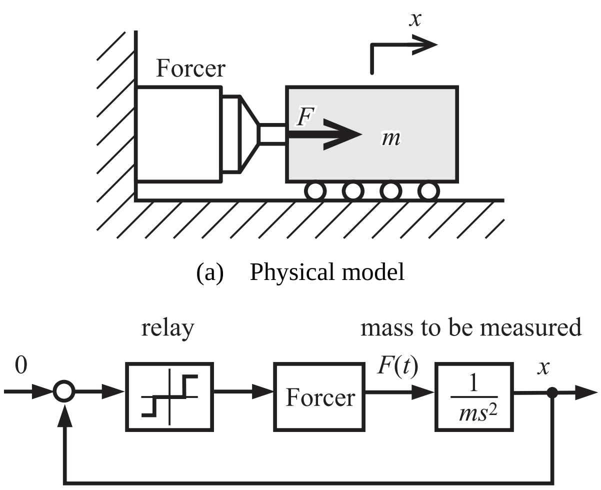
2. Principles of Measurement

The proposed mass measurement system is made up of four elements:

(1) force generator (forcer) to move a measurement object,
(2) sensor to detect the displacement of the object,
(3) controller to produce switching signals,
(4) amplifier to drive the forcer.

Fig.1a shows a physical model for studying the principles of the proposed mass measurement system, and Fig.1b shows a block diagram of its control system. The operation of the control system is shown in **Fig.2**. The force $F(t)$ produced by the forcer is switched in relation to the



(a) Physical model

(b) Closed-loop system including a relay element

Fig.1 Measurement system

displacement x of the object to satisfy

$$\left.\begin{aligned} F(t) &= -F_0 \quad \text{when} \quad X_0 \le x, \\ F(t) &= 0 \quad\;\;\;\, \text{when} \quad -X_0 < x < X_0, \\ F(t) &= +F_0 \quad \text{when} \quad x \le -X_0. \end{aligned}\right\} \tag{1}$$

When the generator is controlled in these ways, a periodic oscillation is excited as shown in Fig.2.

The periods during which $F(t) = \pm F_0$ and $F(t) = 0$ are designated as T_1 and T_2, respectively. The mass is assumed to pass the threshold positions $\pm X_0$ at velocities of $\pm V_0$. When $F(t) = F_0$, the equation of motion is given by

$$m\ddot{x} = F_0. \tag{2}$$

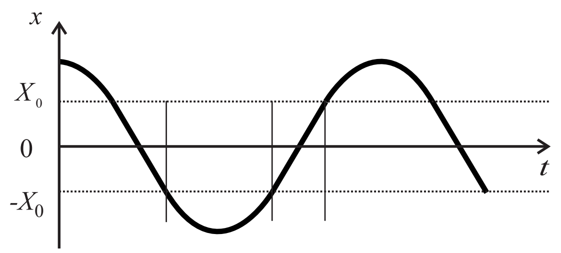
Solving eq.(2) with $\dot{x}(0) = -V_0$ leads to

$$2V_0 = \frac{F_0}{m} \cdot \frac{T_1}{2}. \tag{3}$$

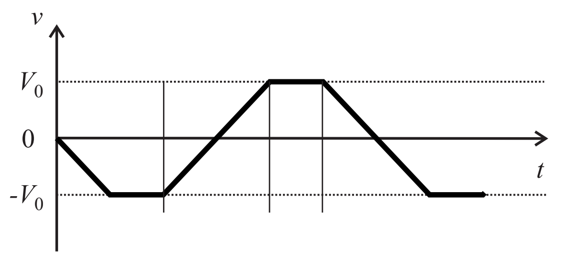
When $F(t) = 0$, the equation of motion is given by

$$m\ddot{x} = 0. \tag{4}$$

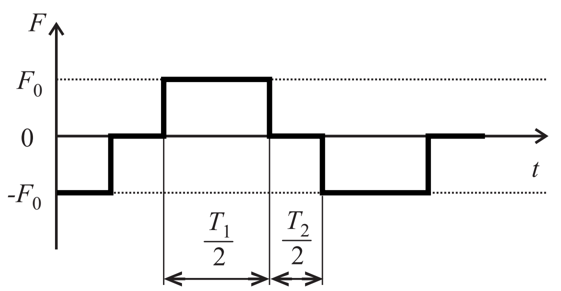
Solving eq.(4) with $\dot{x}(t) = V_0$ $(0 \le t \le T_2/2)$ and $x(0) = -X_0$ leads to



(a) Displacement of the mass



(b) Velocity of the mass



(b) Force acting on the mass

Fig.2 Operation of control system

$$2X_0 = V_0 \cdot \frac{T_2}{2}. \tag{5}$$

Combining eq.(3) with eq.(5) leads to

$$m = \frac{F_0}{16X_0} T_1 T_2. \tag{6}$$

Therefore, the mass of measurement object is determined from the time interval measurement of T_1 and T_2. It is to be noted that mass is determined independently of the velocity V_0.

3. Experiment

3.1 Experimental apparatus

Fig.3 show a schematic drawing of an apparatus developed for experimental study on the proposed mass measurement system. Its photograph is shown by **Photo 1**. **Fig.4** shows an outline of control and measurement system.

The apparatus has two voice coil motors (VCM's) whose maximum output force is 9.8N. One of them is used as a forcer, which is driven by a current-controlled power amplifier. The other is used for velocity monitoring or generating disturbances such as damping force to study their effects on measurement accuracy. The movers of the VCM's are connected with a mechanical coupling to move together. A plate spring is also connected to them for keeping their position at a middle point without control.

A pair of photo interrupters is used to detect the switching positions $\pm X_0$. The value of X_0 is adjusted by changing the width of a target plate that is attached to the connected movers. It is to be mentioned that the cost

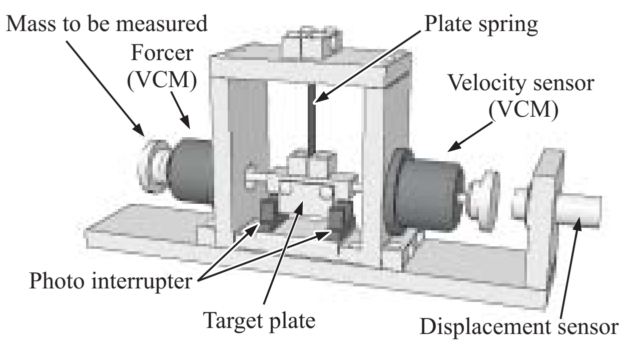


Fig.3 Schematic diagram of experimental apparatus

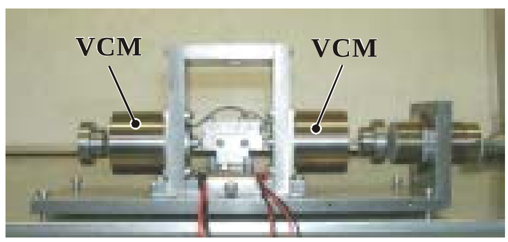


Photo 1 Photograph of the experimental apparatus

and weight of photo detector are much less than those of velocity sensors such as voice coil motor[12] and Doppler-type sensors[8), 9)].

Weights for changing the mass to be measured are attached to one end of the connected movers. At the other end, a sensor target for an eddy-current displacement sensor is fixed. The sensor is used for monitoring the position of the movers.

The control algorithm described by eq.(1) is implemented with a DSP-based digital controller. The controller sends command signal (0, $\pm I_0$) to the power amplifier through a D/A converter. The electromagnetic force is switched to the corresponding values (0, $\pm F_0$). The periods T_1 and T_2 are measured with a digital oscilloscope.

3.2 Experimental results

(1) Identification of VCM's Parameter

The electromagnetic force generated by a VCM is given by

$$F = K_i i \quad [\mathrm{N}], \tag{7}$$

where

i : current in the coil [A],

K_i : current-force coefficient [N/A].

When the velocity of the mover is v [m/s], back electromotive force e is induced which is given by

$$e = K_b v \quad [\mathrm{V}], \tag{8}$$

where

K_b : back electromotive force coefficient [Vs/m].

Ideally, the following is satisfied.

$$K_i = K_b . \tag{9}$$

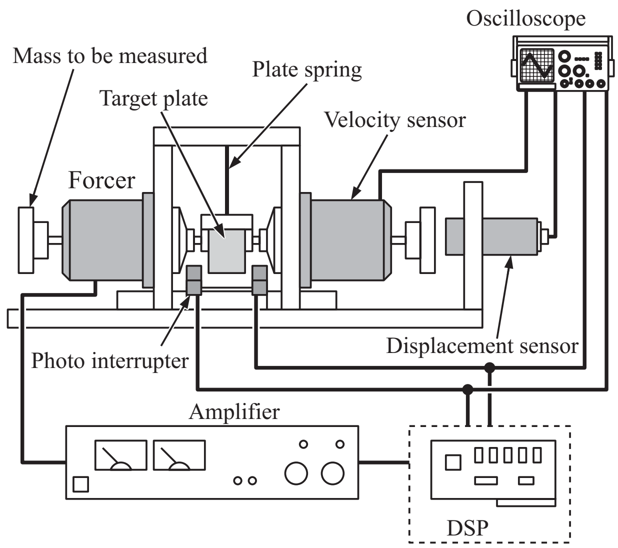
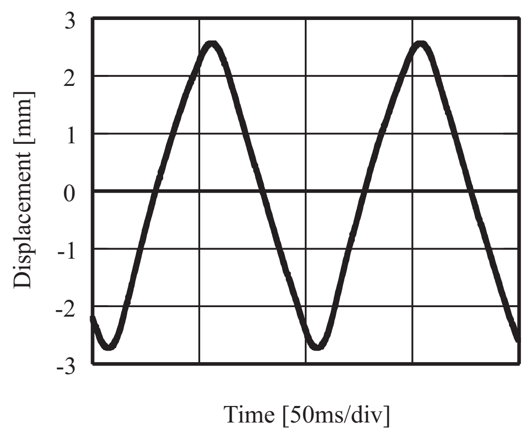


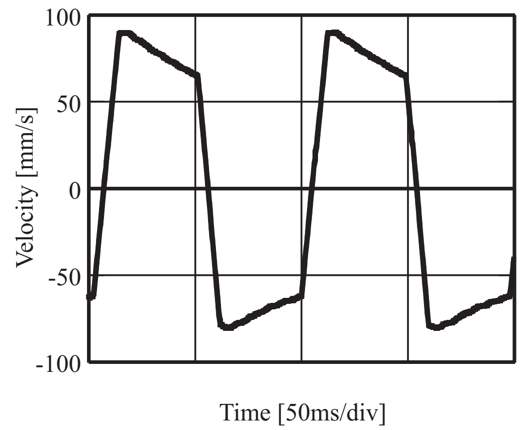
Fig.4 Outline of control and measurement system

Thus, K_b is identified in this work.

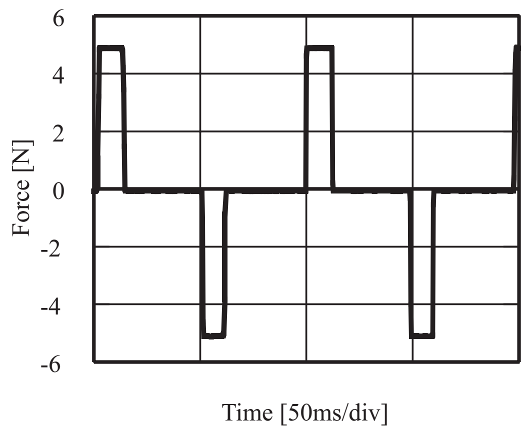
In identifying, one of the VCM's is driven sinusoidally and voltage induced in the coil of the other is measured. The velocity of the movers is detected with an external Doppler-type sensor (not shown in Fig.3). Then, their ratio is calculated. The identified values are



(a) Displacement



(b) Velocity



(c) Force

Fig.5 Operation of control system when $X_0 = 1.5$ mm and $F_0 = 4.88$ N

K_b =4.32 [Vs/m] (VCM for force generation),

K_b =4.38 [Vs/m] (VCM for velocity monitoring).

In the following experiments, the amplitude of the command signal I_0 is determined based on this identified value.

(2) Observation of the relay feedback system's operation

Fig.5 shows the applied force, velocity and displacement of the movers when $F_0 = 4.88[\mathrm{N}]$ and $X_0 = 1.5[\mathrm{mm}]$. It is observed that velocity decreases when $F_0 = 0$ ($|x| \le X_0$). This is mainly because of friction in the VCM's.

(3) Selection of parameters:

The effects of the parameters X_0 and F_0 on measurement accuracy are experimentally studied. **Fig.6** shows measurement results for 13 samples from 356.6g to 418.2g when X_0 is fixed to 1mm and F_0 is set as

(a) $F_0 = 1.95\mathrm{N}$, (b) $F_0 = 4.88\mathrm{N}$, (c) $F_0 = 7.81\mathrm{N}$.

In this figure, the estimated value m_e versus the actual one m_r is plotted where the latter is determined with a high accuracy balance with a resolution of 1mg. The linearity is best in the case of (c). However, there are percent-order discrepancies between the three values estimated according to eq.(6). Moreover, the estimated values are smaller than the actual one by about 10%. Thinkable reasons for these errors are

(1) Effects of friction as seen in Fig.5,

(2) Identification error of K_i ,

(3) Error in setting X_0 .

There are two approaches to reducing measurement error. One is to remove each of the error factors individually. The other is to calibrate the measurement system based on trial measurements. In this work, the latter is applied mainly because it is difficult to reduce mechanical frictions with the developed apparatus.

Fig.7 shows measurement results when F_0 is fixed to 4.88N and X_0 is set as

(a) $X_0 = 0.5\mathrm{mm}$, (b) $X_0 = 1.0\mathrm{mm}$, (c) $X_0 = 1.5\mathrm{mm}$

The ratio of T_1 to T_2 becomes

(a) $T_1 : T_2 \cong 4:1$, (b) $T_1 : T_2 \cong 1:1$, (c) $T_1 : T_2 \cong 1:3$.

The linearity is best in the case of (b).

These results indicate that F_0 should be large and the ratio of T_1 to T_2 should be one to one for accurate measurement. Thereby, another measurement was carried out when F_0 and X_0 were set as $F_0 = 9.37\mathrm{N}$ and $X_0 = 1.25\mathrm{mm}$ that made the ratio of T_1 to T_2 about one to one. The obtained data are fitted to a straight line by the least-squares method as

$$m_r = 5.58 + 1.016 m_e \quad [\mathrm{g}] \tag{10}$$

(4) Measurement errors

Another measurement was again carried out to check the effectiveness of the calibration based on eq.(7). **Fig.8** compares the calibrated value m_c with the actual value m_r. It demonstrates that m_c agrees with m_r well. **Fig.9** presents the relative error given by

$$e = \frac{m_c - m_r}{m_r} . \tag{11}$$

The maximum and the average of their absolute values are 0.2% and 0.1%, respectively.

It is to be mentioned that the range of mass measured in the experiments is limited. The main reason is to maintain the friction conditions. A promising approach to

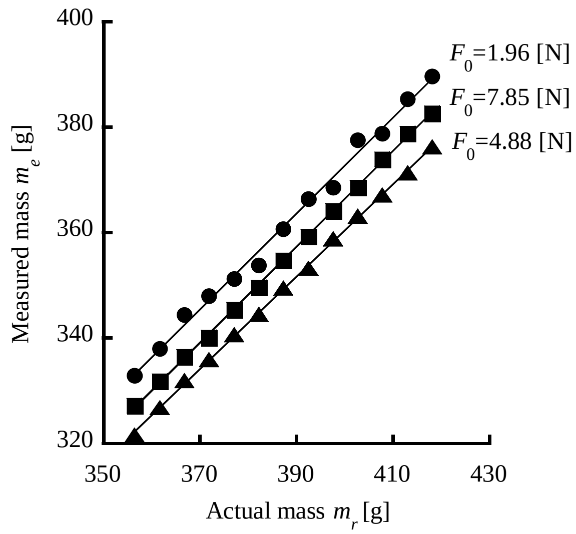


Fig.6 Measurement results when X_0 is fixed to 1mm

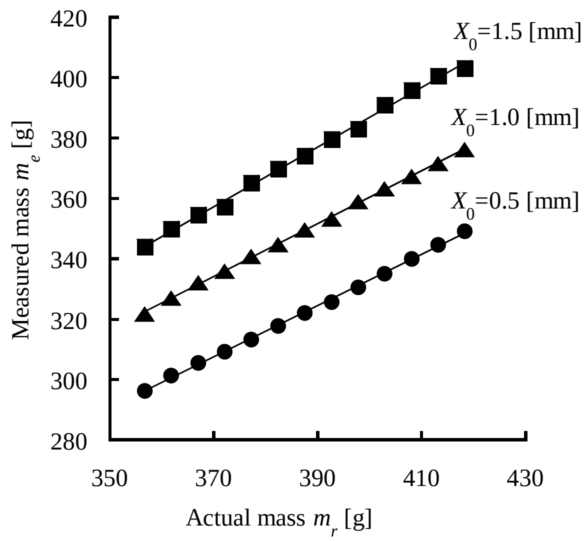


Fig.7 Measurement results when F_0 is fixed to 4.88N.

performing measurement without disturbance caused by mechanical friction is to use noncontact bearings such as air bearing[8), 9)] and magnetic bearing for supporting moving parts.

(5) Effects of damping

Next, the effects of undesirable damping on measurement accuracy are studied. Damping force is generated with the VCM for velocity monitoring as follows. Back electromotive force given by eq.(8) is induced in the coil when the velocity of the movers is v. A resistance R is placed across the terminals of the coil as shown by **Fig.10**. Then current flows in the coil through the resistance, and as a result damping force acts on the movers of the VCM's. When the inductance of coil is neglected, current flowing in the coil is given by

$$i = \frac{K_b}{r+R} v, \tag{12}$$

where

r : resistance of the coil (=2.9 Ω).

From eqs.(7) and (9), damping force f_{damp} is given by

$$f_{damp} = K_i i = \frac{K_b^2}{r+R} v. \tag{13}$$

Therefore, damping coefficient c is approximately given by

$$c = \frac{K_b^2}{r+R}. \tag{14}$$

The resistance R is set as

(a) $R = \infty$ (open circuit) $\Rightarrow$ $c = 0$,
(b) $R = 10.9\Omega$ $\Rightarrow$ $c = 1.40$ Ns/m,
(c) $R = 5.6\Omega$ $\Rightarrow$ $c = 2.27$ Ns/m,
(d) $R = 0.7\Omega$ $\Rightarrow$ $c = 5.35$ Ns/m.

The relative errors are shown in **Fig.11** where the same equation (10) is used for calibration in each case. It indicates that the estimated value becomes lighter as the damping force increases. However, such errors can be reduced by renewing the calibration equation eq.(10) in each case. **Table 1** summarizes the re-calibrated results. It demonstrates that measurement accuracy is maintained even in the presence of damping if appropriate calibration

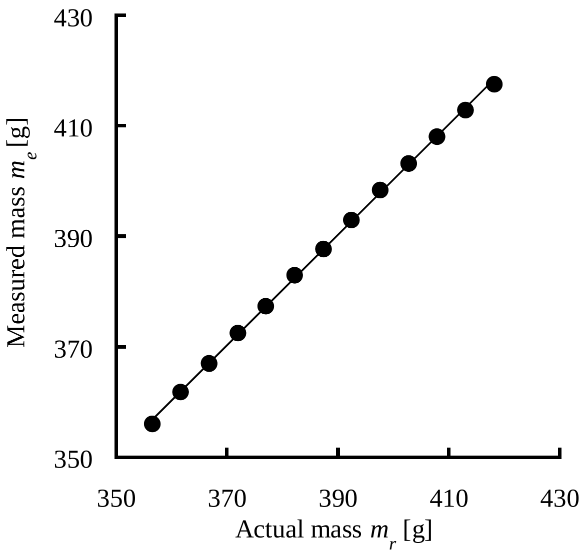


Fig.8 Measurement results after calibration.

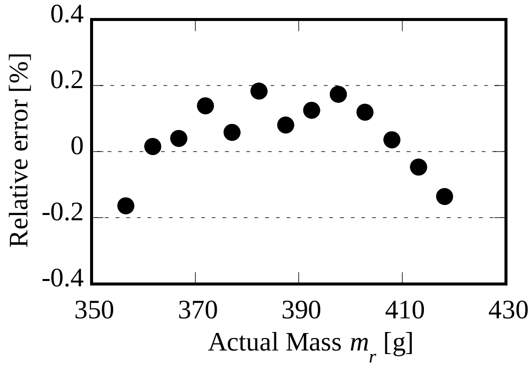


Fig.9 Relative errors of the calibrated values.

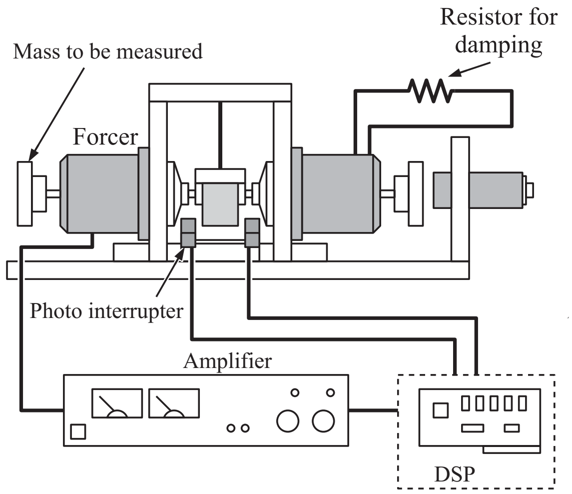


Fig.10 Experimental setup in studying the effects of damping

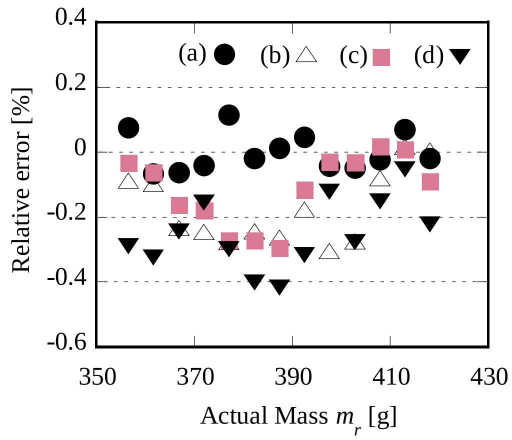


Fig.11 Effects of damping on measurement accuracy

Table 1 Effects of damping on measurement accuracy

c [Ns/m]	T_1 [ms]	T_2 [ms]	maximum error [%]
0	31.1	25.6	0.11
1.40	24.4	32.6	0.15
2.27	21.4	37.2	0.18
5.35	15.8	50.3	0.17

is carried out.

4. Conclusions

A mass measurement system using the self-excited oscillation of a nonlinear system was developed. The system contains an on-off relay with dead zone and switches force acting on the object in relation to the position. The mass of the object is determined from the time interval measurement of the on-state and off-state periods. The effects of system parameters on measurement accuracy were experimentally studied. Under appropriate conditions, the maximum error was within 0.2%. It was also shown that measurement accuracy was maintained even in the presence of undesirable damping.

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