Trans. of the Society of Instrument and Control Engineers
Vol.E-6, No.1, 26/35 (2007)

Gain Switching Observer for Systems with Time-varying Transmission Delay

Yukinori NAKAMURA*, Kiminao KOGISO*, Naotoshi ADACHI** and Kenji SUGIMOTO*

Control systems in which signals are transmitted over a network have been attracting great attention recently. In such systems, transmission delay varies irregularly due to congestion of the network. This results in control performance degradation. In order to solve this problem, this paper proposes an observer design method for the systems with time-varying transmission delay. Using a time-stamp of each received packet, the method switches observer gains to estimate the present state of the plant. Then, the observer design is reduced to stabilization of a switched estimation error system. Linear matrix inequalities (LMIs) make it possible to design the observer multiple gains so that the stability of the error system is guaranteed even if the gains are switched randomly. Furthermore, the performance of the gain switching observer is improved by specifying decay rate of a Lyapunov function for the error system. Finally, numerical and experimental examples illustrate the effectiveness of the proposed method.

Key Words: gain switching observer, transmission delay, jitter, linear matrix inequality

1. Introduction

Studies on networked control systems have been attracting great attention recently,[1)~3)] where a communication network is used to exchange signals for control among control system components (sensors, controllers, actuators, etc.) By using network, we have various advantages such as lower cost to install an instrument in a factory and easier maintenance of the instrument. Furthermore, using infrastructure such as the Internet, fast communication is achieved easily. However, when packets are sent over the public network, the transmission delay varies irregularly depending on congestion of the network. The variation of delay is called jitter, which causes control performance degradation.[4)]

For systems with delay, a state prediction method such as the Smith predictor[5)] is discussed in the literature. When the full state can not be measured, an observer is used to estimate the state from observation signals transmitted over a network. One study[6)] proposes a robust H_∞ observer that can estimate the state of discrete-time systems with delay. In the network architecture field, buffer memory[7)] is used to reduce the variation of delay. In the memory, received signals are saved within a certain period and sorted in the order of transmission. This procedure makes the delay length constant.

However, methods in[6)] cannot be applied to the state estimation for systems with time-varying transmission delay. The variation of delay is a phenomenon peculiar to networked control systems. By using the buffer memory, signals are stored for a period of maximum delay length. However, the method is unsuitable for feedback control, where real time process is essential.

In the research field of network communication, it is known that the delay length is measurable via time-stamp[8)]; information as to when the data were sent. The scenario is the following: First, the sender includes the time-stamp to every packet[9)] (see **Fig. 1**). The receiver calculates the difference between the time-stamp and the signal's arrival time. Then, delay length is obtained. This means that delay length is known at the instance of receiving signals. In this paper, we design a gain switching observer using the time-stamp information under random delay. According to the transmission delay of observation signals, the observer switches gain and estimates the current state of the plant. The design of the observer is reduced to the stabilization problem for estimation error systems with arbitrary switching.

This paper is organized as follows: In Section 2, we formulate the estimation problem. In Section 3, we describe our design method for the observer and prove that estimation error converges to zero. Section 4 shows the design method to specify the decay rate of the Lyapunov

* Graduate School of Information Science, Nara Institute of Science and Technology, 8916-5, Takayama, Ikoma, Nara
** Department of Systems Management Engineering, Faculty of Engineering, Kansai University, 3-3-35, Yamate, Suita, Osaka

TR 0001/07/E-601–0026 © 2007 SICE

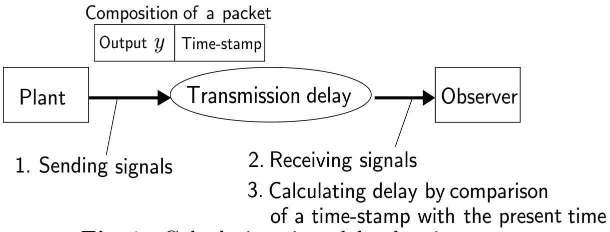


Fig. 1 Calculating time delay by time-stamp

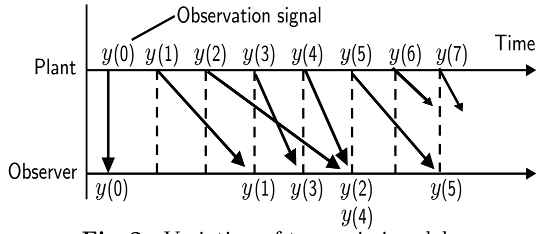


Fig. 2 Variation of transmission delay

function. Sections 5 and 6 illustrate numerical examples and experimental results. Finally, this paper concludes in Section 7.

2. Problem Formulation

2.1 Problem statement

Consider a linear time-invariant discrete-time plant

$$x(t+1) = \bar{A}x(t) + \bar{B}u(t), \tag{1a}$$

$$y(t) = \bar{C}x(t), \quad t = 0,\ 1,\ 2,\ \cdots, \tag{1b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^p$ denote state, input, and output vectors respectively. We assume that $(\bar{C},\ \bar{A})$ is observable. Our problem is observer-based estimation for (1). Observation signal $y(t)$ at time t is transmitted to the observer. Here, we assume the following conditions for transmission delay $h(t)$ at time t.

(A1) $h(0) = 0$ for simplicity.

(A2) $h(t)$ varies randomly, but is known at the instant of receiving data via time-stamp information.

(A3) $h \in \mathcal{Z}^+_{h_d} \equiv \{0,\ 1,\ 2,\ \cdots,\ h_d\}$, where $\mathcal{Z}^+_{h_d}$ is the set of nonnegative integers not larger than h_d and h_d is constant maximal time delay.

Note. Since we use the time-stamp, the delay $h(t)$ is known at the instance of receiving a packet. Note that the delay length is not known a priori.

Note. To measure $h(t)$, we assume the use of a particular protocol in this paper.

Note. **(A3)** implicitly indicates that no packet loss is allowed in this network.

Under these assumptions, delay $h(t)$ varies randomly. Here, we denote transmission delay of observed signal $y(t)$ by $d(t)$. Note that $d(t)$ is different from $h(t)$, because the packet observed at t was sent at $t - d(t)$ while the packet sent at t will be observed at $t + h(t)$.

Fig. 2 and **Table 1** show signal arrival sequence and variation of delay, respectively. * means a case not receiving signals and hence d is indefinite. For example, $h(3) = 1$ at time $t = 3$. Then, the observer receives signal $y(1)$ with 2 steps delay. Therefore, $d(3) = 2$. When $y(2)$ and $y(4)$ are received at time $t = 5$, then $d(5) = 1$

Table 1 Time delay $h(t)$ and $d(t)$

Sample time t	0	1	2	3	4	5
$y(t)$ received at the observer	$y(0)$	*	*	$y(1)$	$y(3)$	$y(2),\ y(4)$
Delay $h(t)$	0	2	3	1	1	2
Delay $d(t)$	0	*	*	2	1	1, 3

and 3. In this paper, we consider delay $d(t)$, rather than $h(t)$. The next section shows the rule of selecting signals for estimation in cases without receiving signals and with receiving multiple signals simultaneously.

Note. From the definition of $d(t)$, assumptions **(A1)**-**(A3)** hold for delay $d(t)$, similarly as for $h(t)$.

2.2 Augmented system

In this section, we formulate the system with the transmission delay. Consider an augmented system with h_d delay operators, z^{-1}, serially connected to the plant (1) as illustrated in **Fig. 3**. Output $y^{\#}_d(t) \in \mathbb{R}^p, d \in \mathcal{Z}^+_{h_d} \backslash 0$ from the delay z^{-1} means observation signal $y(t-d)$. In the case of delay d, the observer detects $y^{\#}_d(t)$. $\mathcal{Z}^+_{h_d} \backslash 0$ is the set excluding element zero. Here, we define the state $\tilde{x} \in \mathbb{R}^{n \times p h_d}$ of the augmented system as

$$\tilde{x} = \begin{bmatrix} x' & y^{\#\prime}_1 & \cdots & y^{\#\ \prime}_{h_d} \end{bmatrix}',$$

where $'$ denotes transposition. Then, the augmented system is described by

$$\tilde{x}(t+1) = A\tilde{x}(t) + Bu(t), \tag{2a}$$

$$\tilde{y}(t) = C_{d(t)}\tilde{x}(t), \quad d(t) \in \mathcal{Z}^+_{h_d} \backslash 0, \quad t = 0,\ 1,\ 2,\ \cdots, \tag{2b}$$

$$A = \begin{bmatrix} \bar{A} & \mathbf{0}_{n\times p} & \cdots & \cdots & \cdots & \mathbf{0}_{n\times p} \\ \bar{C} & \mathbf{0}_{p\times p} & \cdots & \cdots & \cdots & \mathbf{0}_{p\times p} \\ \mathbf{0}_{p\times n} & I_p & \ddots & & & \vdots \\ \vdots & \mathbf{0}_{p\times p} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0}_{p\times n} & \mathbf{0}_{p\times p} & \cdots & \mathbf{0}_{p\times p} & I_p & \mathbf{0}_{p\times p} \end{bmatrix},$$

$$B = \begin{bmatrix} \bar{B}' & \mathbf{0}'_{p\times m} & \cdots & \mathbf{0}'_{p\times m} \end{bmatrix}',$$

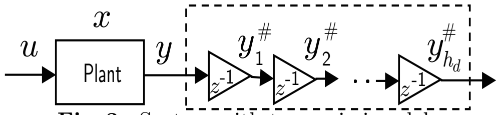


Fig. 3 System with transmission delay

$$C_d = \begin{cases} \begin{bmatrix} \bar{C} & \mathbf{0}_{p\times ph_d} \end{bmatrix} & \text{if } d = 0 \\ \begin{bmatrix} \mathbf{0}_{p\times(n+pd-p)} & I_p & \mathbf{0}_{p\times(ph_d-pd)} \end{bmatrix} & \text{if } d \in \mathcal{Z}^+_{h_d}\backslash 0, \end{cases}$$

with identity matrix $I_p \in \mathbb{R}^{p\times p}$ and zero matrix $\mathbf{0}_{k\times q} \in \mathbb{R}^{k\times q}$ respectively. Note that C_d in (2b) is time-varying according to transmission delay d of signal $\tilde{y}(t) = y_d^\# = y(t-d)$ received at time t. In the next section, we consider the design of an observer for (2). Since the augmented system (2b) includes state x of plant (1), the state estimation for (2b) induces that for (1).

3. Gain Switching Observer

3.1 Structure of switching observer

Our gain switching observer consists of the augmented systems and multiple observer gains $F_d, d \in \mathcal{Z}^+_{h_d}$ as shown in **Fig. 4**. This observer selects F_d among those gains if it recognizes the transmission delay d of the received signal $y_d^\#$. We define estimate as

$$\hat{\tilde{x}} = \begin{bmatrix} \hat{x}' & \hat{y}_1^{\#'} & \cdots & \hat{y}_{h_d}^{\#'} \end{bmatrix}',$$

where $\hat{\tilde{x}} \in \mathbb{R}^{n+ph_d}$, $\hat{x} \in \mathbb{R}^n$, and $\hat{y}_d^\# \in \mathbb{R}^p$ are the estimations of $\tilde{x}$, x, and $y_d^\#$ respectively. Our gain switching observer for system (2) is given by

$$\hat{\tilde{x}}(t+1) = A\hat{\tilde{x}}(t) + Bu(t) + F_{d(t)}(\tilde{y}(t) - C_{d(t)}\hat{\tilde{x}}(t)), \quad (3)$$

$$d(t) \in \mathcal{Z}^+_{h_d}, \quad t = 0,\ 1,\ 2,\ \cdots.$$

3.2 Rule of selecting signals for state estimation

In this section, we explain our state estimation method. Under the assumption **(A1)**,

(B1) Output signal $y(0)$ arrives at the observer at instance $t = 0$.

From **(A2)**-**(A3)**, the following events **(B2)**-**(B4)** occur.

(B2) The observer does not receive any signals for period T_α $(0 < T_\alpha < h_d)$ (see **Fig. 5**(a)).

(B3) The observer receives one or multiple signals at the same sampling time $0 < t = t_\beta$ (see Fig. 5(b)).

(B4) A signal $y(t_1)$ arrives at the observer later than another signal $y(t_2)$, $t_1 < t_2$ (see Fig. 5(c)).

In all of the cases **(B1)**-**(B4)**, our estimation method is as follows.

(C) At sampling time $t = t_\gamma$, the observer employs the newest signal among the received signals for $0 \le t \le t_\gamma$.

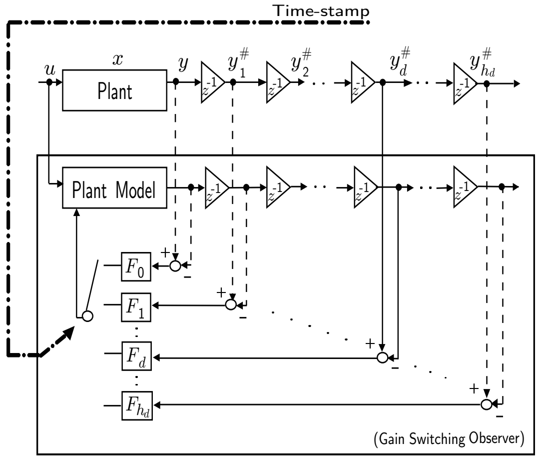


Fig. 4 Augmented system and gain switching observer

Note. If the observer receives signal $y(t_2)$ earlier than $y(t_1)$, $t_1 < t_2$ (see Fig. 5(c)), then the observer does not use $y(t_1)$ for estimation.

When no or multiple signals arrive at step t, then, from Fig. 2 and Table 1, delay $d(t)$ is indefinite or multiple values, respectively. However, from the rule **(C)**, delay d is unique. For example, the observer does not receive signals at time $t = 0$, but uses the newest signal $y(0)$, $d(0) = 0$ (see **Fig. 6**). At time $t = 1$, the delay of signal $y(0)$ changes from zero to one step. Then, the observer switches to gain F_1. In case of simultaneous arrival of $y(1)$ and $y(2)$, the newest signal is $y(2)$ with delay $d(3) = 1$.

3.3 Design of the gain switching observer

Since this observer (3) switches gain according to transmission delay of observation signals, the estimation error system is also switched. To guarantee the stability of the error system for arbitrary switching, we design the gain F_d. We define estimation error $\tilde{e} \in \mathbb{R}^{n+ph_d}$ as

$$\tilde{e} = \tilde{x} - \hat{\tilde{x}}. \quad (4)$$

From (2)-(4), we obtain

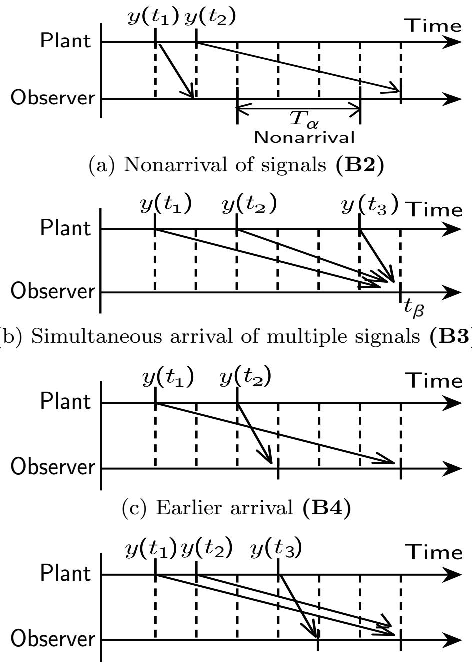
$$\tilde{e}(t+1) = (A - F_{d(t)}C_{d(t)})\tilde{e}(t), \quad (5)$$

$$d(t) \in \mathcal{Z}^+_{h_d},\ t = 0,\ 1,\ 2,\ \cdots.$$

Note that $F_{d(t)}$ and $C_{d(t)}$ in (5) are time-varying. We design the observer using common Lyapunov solution so that system (5) is stable for arbitrary switching. Now consider $h_d + 1$ Lyapunov inequalities for a single P

$$(A-F_dC_d)'P(A-F_dC_d)-P < 0,\ P > 0,\ d \in \mathcal{Z}^+_{h_d}. \quad (6)$$

From (6) and the Schur complement, we obtain the matrix inequalities



(a) Nonarrival of signals **(B2)**

(b) Simultaneous arrival of multiple signals **(B3)**

(c) Earlier arrival **(B4)**

(d) Earlier arrival and simultaneous arrival

Fig. 5 Various types of transmission delay

$$\begin{bmatrix} -P & P(A-F_dC_d) \\ (A-F_dC_d)'P & -P \end{bmatrix} < 0,\ d \in \mathcal{Z}^+_{h_d}.$$

Let $X_d = PF_d,\ d \in \mathcal{Z}^+_{h_d}$. Then, these matrix inequalities are expressed as $h_d + 1$ LMIs

$$\begin{bmatrix} -P & PA - X_dC_d \\ A'P - C_d'X_d' & -P \end{bmatrix} < 0,\ d \in \mathcal{Z}^+_{h_d}. \quad (7)$$

If there exist $P > 0$ and $X_d, d \in \mathcal{Z}^+_{h_d}$, we design gain $F_d,\ d \in \mathcal{Z}^+_{h_d}$ as follows.

$$F_d = P^{-1}X_d,\ d \in \mathcal{Z}^+_{h_d}. \quad (8)$$

Now, let us check the stability of error system (5) with gain $F_d,\ d \in \mathcal{Z}^+_{h_d}$ designed by the above method. From assumptions **(A1)-(A3)**, event **(B1)** occurs at $t = 0$ and **(B2)**, **(B3)**, or **(B4)** happens for any $t > 0$. Therefore, we have the following lemma.

Lemma 1. Assume that there exists $P > 0$ satisfying LMI (7) and that the events **(B1)-(B4)** occur. Then, the positive definite function $V(t) = \tilde{e}'(t)P\tilde{e}(t)$ satisfies

$$\Delta V(t) \equiv V(t+1) - V(t) < 0, \quad t = 0,\ 1,\ 2,\ \cdots.$$

Proof. The proof of this lemma is given in the Appendix. ∎

From **Lemma 1**, we obtain the following result.

Theorem 1. Assume that there exist $P > 0$ and $X_d,\ d \in \mathcal{Z}^+_{h_d}$ such that LMIs (7) hold for the augmented system (2). If the observer (3) is designed by taking

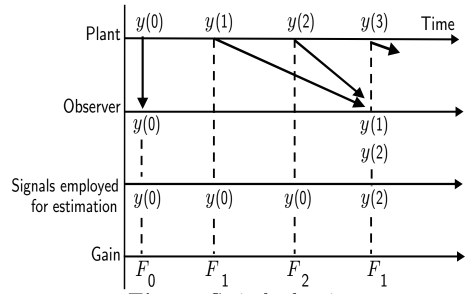


Fig. 6 Switched gains

$F_d = P^{-1}X_d,\ d \in \mathcal{Z}^+_{h_d}$, then the origin $\tilde{e} = 0$ is asymptotically stable.

Proof. We consider a candidate of Lyapunov function $V(t) = \tilde{e}'(t)P\tilde{e}(t)$. Under assumptions **(A1)-(A3)**, we always have one of the events **(B1)-(B4)**. From **Lemma 1**, in the cases of those events, $\Delta V < 0$ holds. From Lyapunov's stability theorem, the origin $\tilde{e} = 0$ is asymptotically stable. ∎

From **Theorem 1**, we can ensure estimation error $\tilde{e}$ converges to zero by the proposed observer. Therefore, system (2) is estimated successfully.

4. Performance Improvement of Switching Observer

To improve performance of our observer, we design gain F_d so that the estimation error (Lyapunov function) is decreased in a specified rate. Let us denote candidate of Lyapunov function $V(t)$ by $\tilde{e}'(t)P\tilde{e}(t),\ P > 0$. We design gain $F_d,\ d \in \mathcal{Z}^+_{h_d}$ satisfying the following inequality

$$V(t+1) < r^2V(t), \quad 0 < r \le 1, \quad (9)$$

where r is decay rate of $V(t)$. From the definition of $V(t)$, (9) is described as

$$(A - F_dC_d)'r^{-1}P(A - F_dC_d) - rP < 0,\ P > 0, \quad (10)$$
$$d \in \mathcal{Z}^+_{h_d}, \quad 0 < r \le 1.$$

From the Schur complement, (10) is equivalent to

$$\begin{bmatrix} -rP & P(A-F_dC_d) \\ (A-F_dC_d)'P & -rP \end{bmatrix} < 0, \quad (11)$$
$$d \in \mathcal{Z}^+_{h_d}, \quad 0 < r \le 1.$$

Now let $Y_d = PF_d,\ d \in \mathcal{Z}^+_{h_d}$. Then, (11) is reduced to LMIs for P and Y_d

$$\begin{bmatrix} -rP & PA - Y_dC_d \\ A'P - C_d'Y_d' & -rP \end{bmatrix} < 0, \quad (12)$$
$$d \in \mathcal{Z}^+_{h_d}, \quad 0 < r \le 1.$$

If there exist $P > 0$ and Y_d satisfying (12), gain $F_d,\ d \in \mathcal{Z}^+_{h_d}$ is calculated as follows:

$$F_d = P^{-1} Y_d,\ d \in \mathcal{Z}^+_{h_d}. \qquad (13)$$

Thus, the next theorem holds.

Theorem 2. If there exist $P > 0$ and $Y_d,\ d \in \mathcal{Z}^+_{h_d}$ such that LMIs (12) hold, then the origin $\tilde{e} = 0$ is asymptotically stable and (9) holds by the switching law (13).

Proof. As stated above, if there exist $P > 0$ and $Y_d,\ d \in \mathcal{Z}^+_{h_d}$ such that LMIs (12) hold for the augmented system (2), then (9) holds. From $0 < r \le 1$ and $P > 0$, we obtain

$$V(t+1) < r^2 V(t) \le V(t). \qquad (14)$$

From (14), $V(t+1) - V(t) < 0$, that is, Lyapunov inequality (6) holds. This means that there exist $P > 0$ and $X_d,\ d \in \mathcal{Z}^+_{h_d}$ such that LMIs (7) hold. Thus, from **Theorem 1**, the origin $\tilde{e} = 0$ is asymptotically stable. ∎

5. Numerical Simulation

In order to verify the effectiveness of the proposed method, the authors have performed a numerical simulation. Consider a continuous-time model of the DC motor adopted in [10]. The state equation of the DC motor is given by

$$\dot{x}_c(t_c) = \begin{bmatrix} 0 & 1 \\ 0 & -9.8 \end{bmatrix} x_c(t_c) + \begin{bmatrix} 0 \\ 49 \end{bmatrix} u_c(t_c), \qquad (15a)$$

$$y_c(t_c) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_c,\quad x_c = \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}', \qquad (15b)$$

where $\theta, \dot{\theta}$, and u are functions of time t_c [sec] and mean a DC motor's angle, angular velocity, and input voltage to the motor, respectively. In this numerical example, we estimate the state $x = [\theta, \dot{\theta}]'$ based on output signal $y = \theta$. Let $\hat{\theta}$ and $\hat{\dot{\theta}}$ be estimations of θ and $\dot{\theta}$. System (15) is discretized into a ZHO equivalent one with the sampling period 10 [ms]. Input is chosen as $u_c = 3\cos(1000 t_c)$. Initial state $\tilde{x}(0)$ of (2) is zero vector. Estimate $\hat{\tilde{x}}(0)$ is chosen as

$$\begin{aligned} \hat{\tilde{x}}(0) &= \begin{bmatrix} \hat{\theta}(0) & \hat{\dot{\theta}}(0) & \hat{y}_1^{\#'}(0) & \cdots & \hat{y}_{h_d}^{\#'}(0) \end{bmatrix}' \\ &= \begin{bmatrix} 0.1 & 1.0 & 0 & \cdots & 0 \end{bmatrix}'. \end{aligned}$$

Fig. 7 shows a sample sequence of the time delay from uniform distribution with maximum 90 [ms] ($h_d = 9$). First, we have designed the switching observer with $r = 1$ stated in Section III. The time response of state and estimate is given in **Fig. 8**. In these figures, solid and dashed lines respectively mean the real state $\theta,\ \dot{\theta}$ and estimated state $\hat{\theta},\ \hat{\dot{\theta}}$. In both figures, it turns out that the estimation converges to the true state as time passes. We can see

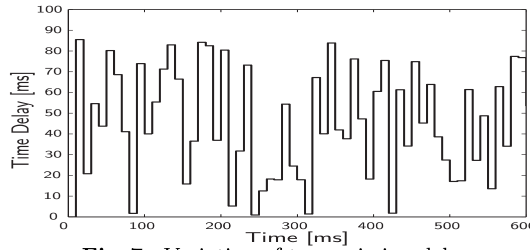
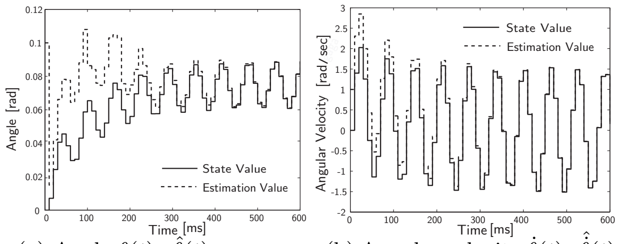
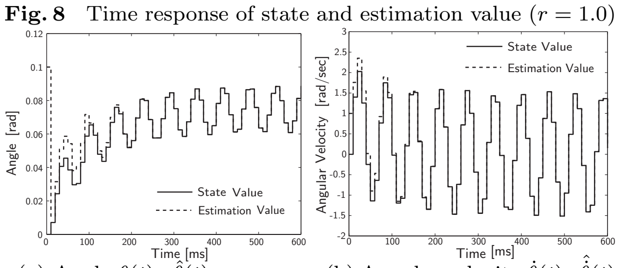


Fig. 7 Variation of transmission delay



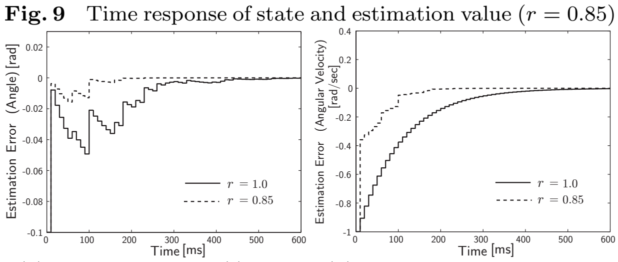
(a) Angle $\theta(t),\ \hat{\theta}(t)$ (b) Angular velocity $\dot{\theta}(t),\ \hat{\dot{\theta}}(t)$

Fig. 8 Time response of state and estimation value ($r = 1.0$)



(a) Angle $\theta(t),\ \hat{\theta}(t)$ (b) Angular velocity $\dot{\theta}(t),\ \hat{\dot{\theta}}(t)$

Fig. 9 Time response of state and estimation value ($r = 0.85$)



(a) Error of angle $\tilde{e}_1(t)$ (b) Error of angular velocity $\tilde{e}_2(t)$

Fig. 10 Comparison of estimation errors ($r = 1.0,\ 0.85$)

that the state is successfully estimated by the proposed observer.

Fig. 9 shows the state and estimate via switching observer with decay rate $r = 0.85 < 1$. In the figure, solid and dashed lines represent the state and estimate for the plant. From Fig. 9, the estimate tracks the state. Thus, in the case of $r = 0.85$, the switching observer can estimate the state. We denote the estimation error of θ and $\dot{\theta}$ by $\tilde{e}_1 = \theta - \hat{\theta}$ and $\tilde{e}_2 = \dot{\theta} - \hat{\dot{\theta}}$. **Fig. 10** shows the estimation error in the case of $r = 1.0$ and 0.85. Solid and dashed lines are the error in $r = 1$ and $r = 0.85$, respectively. From this figure, we can see that the estimation error in $r = 0.85$ converges to zero faster than the case of $r = 1.0$. Therefore, by setting the decay rate corresponding to specifying domain of pole assignment of the switching observer, the estimation performance is improved. These results show that, for the systems with a time-varying delay system, the proposed observer can estimate the state successfully.

6. Experiment

To verify the performance of the observer, an experiment of estimation for the inverted pendulum was conducted. Transmission delay was generated via emulator and observation signals were recorded from the plant stabilized by the controller. After the recording of signals, the observer estimated the state offline. **Fig. 11** shows the delay generated by network emulator NIST Net [11]. The continuous-time state equation of the inverted pendulum is given by

$$\dot{x}_c(t_c) = A_c x_c(t_c) + B_c u_c(t_c),$$
$$y_c(t_c) = C_c x_c(t_c),$$

$$A_c = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m^2 l^2 g}{F} & -\frac{(J+ml^2)(c_c a^2 R + K_m^2 K_g^2)}{F a^2 R} & \frac{mlc_p}{F} \\ 0 & \frac{(M+m)mlg}{F} & \frac{ml(c_c a^2 R + K_m^2 K_g^2)}{F a^2 R} & -\frac{(M+m)c_p}{F} \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 0 \\ \frac{(J+ml^2)K_m K_g}{FaR} \\ -\frac{mlK_m K_g}{FaR} \end{bmatrix},$$

$$C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad F = (M+m)J + Mml^2,$$

$$x_c = [z,\ \theta,\ \dot{z},\ \dot{\theta}]',$$

where z, θ, $\dot{z}$, $\dot{\theta}$, and u are position of the cart, angle of the pendulum, velocity of the cart, angular velocity of the pendulum, and input voltage, respectively. **Table 2** shows the value of the parameters of the pendulum. In this experiment, the observer estimates the state x via observation signal $y = [z, \theta]'$. The pendulum is discretized into a ZOH equivalent system with sampling period 10 [ms]. The maximum of delay is 110 [ms] ($h_d = 11$). The decay rate is chosen as $r = 1$.

Fig. 12 shows the time response of the estimate and state of the pendulum. Solid and dashed lines represent the estimate and state. Though we could not measure the velocity and angular velocity in the experiment, difference approximation is represented in Fig. 12(c)-(d). In Fig. 12(a)-(b), the position z and angle θ are successfully estimated. Since there exists modeling error for the plant,

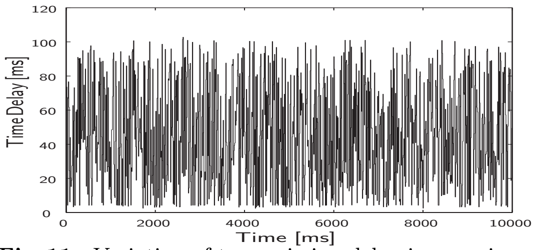
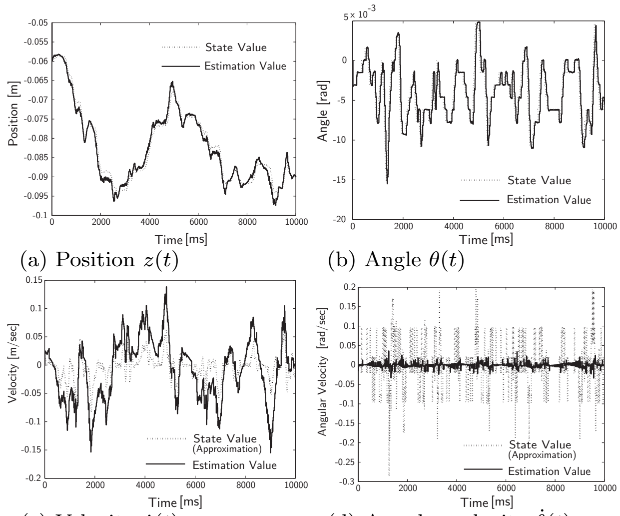


Fig. 11 Variation of transmission delay in experiment

Table 2 Parameters of an inverted pendulum

K_m	torque constant	0.00767 V/rad $\sec^{-1}$
R	armature resistance	2.6 Ω
K_g	gear ratio	3.7
a	radius of pinion	0.00635 m
M	mass of cart	0.895 kg
$2l$	length of pendulum	0.628 m
m	mass of pendulum	0.215 kg
g	gravity acceleration	9.8 m/$\sec^2$
J	inertia moment of pendulum	0.0076 N m $\sec^2$
c_p	viscous friction of rotation axis for pendulum	8.4779 N m sec
c_c	viscous friction of cart	0.001384 N sec/m



(a) Position $z(t)$ (b) Angle $\theta(t)$

(c) Velocity $\dot{z}(t)$ (d) Angular velocity $\dot{\theta}(t)$

Fig. 12 State estimation of an inverted pendulum

it can be verified that our observer is robust against model error from the result. In Fig. 12(c)-(d), there remains an error between the approximation and estimate. This is considered to be calculation error of the approximation. Thus, state estimation of z and θ via the switching observer is accomplished under random delay.

7. Conclusion

In this paper, we have proposed a gain switching observer for systems with time varying delay. By means of solving linear matrix inequalities, the observer is designed so that the origin of the error system is asymptotically stable. Furthermore, we have also proposed a design method in order that the performance of the estimation is improved in terms of the decay rate of the Lyapunov function. Numerical examples and an experiment have shown the effectiveness of the switching observer. Future works will involve design of the switching observer-based controller and performance evaluation of the controller.

References

1) L. A. Montestruque and P. Antsaklis: Stability of model-based networked control systems with time-varying transmission times, IEEE Transactions on Automatic Control, **49**-9, 1562/1572 (2004)
2) S. H. Low, F. Paganini, and J. C. Doyle: Internet congestion control, IEEE Control Systems Magazine, **22**-2, 28/43 (2002)
3) G. C. Walsh, O. Beldiman, L. G. Bushnell: Error encoding algorithms for networked control systems, Automatica, **38**-2, 261/267 (2002)
4) T. Shiotsuki: A study on a time-delay of network and control of force/tactile communication systems, Systems, Control and Information, **45**-12, 695/702 (2001) (in Japanese)
5) O. J. M. Smith: A controller to overcome dead time, ISA J., **6**, 28/33 (1956)
6) Z. Wang, B. Huang, and H. Unbehauen: Robust H_∞ observer design of linear state delayed systems with parametric uncertainty: the discrete-time case, Automatica, **35**-6, 1161/1167 (1999)
7) K. Sunada: Realized VoIP with communication infrastructure technologies, IPSJ Magazine, **42**-2, 136/140 (2001) (in Japanese)
8) M. Santifiler: TCP/IP and NFS Internetworking in a UNIX environment, Addison-Wesley Toppan (1993) (in Japanese)
9) H. Miyahara, Y. Oie: Computer network, Kyoritsu Shuppan (1999) (in Japanese)
10) K. Kogiso and K. Hirata: A reference governor in a piecewise state affine function, Proc. 42nd IEEE Conf. Decision and Control, 1747/1752 (2003)
11) http://snad.ncsl.nist.gov/nistnet/

Appendix A. Proof of lemma 1

A. 1 the case of (B1)

When **(B1)** occurs, the observer switches to gain $F_{d(0)} = F_0$ from $d(0) = 0$ (**Fig. A. 1**). Then, from (3) and (4), we obtain the following equation.

$$\begin{aligned}\Delta V(0) &= (A\tilde{x}(0) - A\hat{\tilde{x}}(0) - F_0(C_0\tilde{x}(0) - C_0\hat{\tilde{x}}(0)))'P \\ &\quad (A\tilde{x}(0) - A\hat{\tilde{x}}(0) - F_0(C_0\tilde{x}(0) - C_0\hat{\tilde{x}}(0))) \\ &\quad - (\tilde{x}(0) - \hat{\tilde{x}}(0))'P(\tilde{x}(0) - \hat{\tilde{x}}(0)) \\ &= \tilde{e}(0)'((A - F_0C_0)'P(A - F_0C_0) - P)\tilde{e}(0).\end{aligned}$$

Since $P > 0$ and F_0 satisfy LMIs (7), the next inequality holds.

$$(A - F_0C_0)'P(A - F_0C_0) - P < 0.$$

Therefore, for any $\tilde{e}(0)$, we obtain

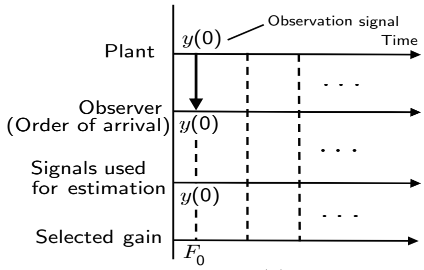


Fig. A.1 Time delay $d(0) = 0$ at $t = 0$

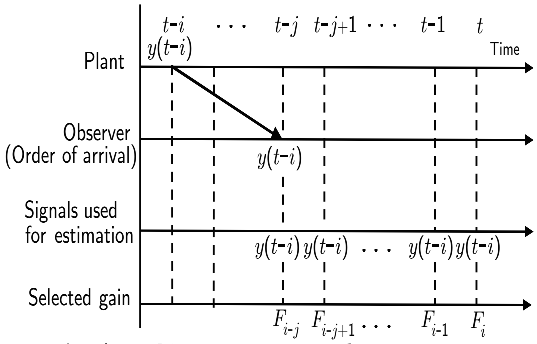


Fig. A.2 Not receiving signals at some time

$$\Delta V(0) < 0.$$

A. 2 the case of (B2)

We consider event **(B2)**. Assume that, at $t-j+1,\ t-j+2,\ \cdots,\ t$, the observer does not receive signals and $y(t-i)$ is the latest signal (see **Fig. A. 2**). At $t-j$, signal $y(t-i)$ arrives at the observer. Then, from **(C)**, $y(t-i)$ is used for estimation at $t-j,\ t-j+1,\ \cdots,\ t$. Time delay of $y(t-i)$ at $t-j$ is

$$d(t-j) = i - j, \quad d \in \mathcal{Z}^+_{h_d}. \tag{A. 1}$$

Since signal $y(t-i)$ arrives at the observer before time t, the next inequality

$$1 \le j,$$

holds. From the definition of i, j and assumption **(A2)**, we obtain

$$j \le i \le h_d.$$

Thus, the following inequality

$$1 \le j \le i \le h_d,$$

is satisfied.

An interval between time $t-i$ and t is i steps. Thus, transmission delay of signal $y(t-i)$ at t is $d(t) = i$. Then, from **(C)**, the observer is expressed as

$$\hat{\tilde{x}}(t+1) = A\hat{\tilde{x}}(t) + Bu(t) + F_i(y(t-i) - C_i\hat{\tilde{x}}(t))$$

$$= A\hat{\tilde{x}}(t) + Bu(t) + F_i(\tilde{y}(t-j) - C_i\hat{\tilde{x}}(t)), \tag{A. 2}$$

$$1 \le j \le i \le h_d, \quad t = 1,\ 2,\ \cdots.$$

Note that, from the augmented system (2), $y(t-i) = \tilde{y}(t-j)$ holds. Using $(A.2)$, we have the following equation:

$$\tilde{e}(t+1) = A\tilde{x}(t) + Bu(t) - (A\hat{\tilde{x}}(t) + Bu(t) + F_i(\tilde{y}(t-j) - C_i\hat{\tilde{x}}(t))). \tag{A. 3}$$

From the definition of system (2),

$$\tilde{y}(t-j) = C_{i-j}\tilde{x}(t-j), \tag{A. 4}$$

is satisfied. Then, from (A. 3) and (A. 4), we obtain

$$\tilde{e}(t+1) = A\tilde{x}(t) - A\hat{\tilde{x}}(t) - F_i(C_{i-j}\tilde{x}(t-j) - C_i\hat{\tilde{x}}(t)). \tag{A. 5}$$

From the definition of A, B, and C_d, the following equations hold.

$$C_{k-1} = C_k A, \tag{A. 6}$$

$$C_k B = 0, \tag{A. 7}$$

$$k \in \mathcal{Z}_{h_d}^{+}\backslash 0.$$

Here, using the equation (A. 6), we have

$$C_{i-j} = C_i A^j. \tag{A. 8}$$

In addition, from (A. 7) and (A. 8), we obtain the following equation.

$$C_i \sum_{l=1}^{j} A^{l-1}Bu(t-l) = \sum_{l=1}^{j} C_{i+1-l}Bu(t-l) = 0. \tag{A. 9}$$

From (A. 2), (A. 8) and (A. 9), we have

$$\begin{aligned}
\tilde{e}(t+1) &= A\tilde{x}(t) - A\hat{\tilde{x}}(t) - F_i(C_iA^j\tilde{x}(t-j) \\
&\quad + C_i\sum_{l=1}^{j} A^{l-1}Bu(t-l) - C_i\hat{\tilde{x}}(t)) \\
&= A\tilde{x}(t) - A\hat{\tilde{x}}(t) - F_iC_i(A^{j-1}(A\tilde{x}(t-j) \\
&\quad + Bu(t-j)) + \sum_{l=1}^{j-1} A^{l-1}Bu(t-l) - \hat{\tilde{x}}(t)) \\
&= A\tilde{x}(t) - A\hat{\tilde{x}}(t) - F_iC_i(A^{j-1}\tilde{x}(t-j+1) \\
&\quad + \sum_{l=1}^{j-1} A^{l-1}Bu(t-l) - \hat{\tilde{x}}(t)).
\end{aligned}$$

Iterating the above procedure, we obtain a following equation.

$$\begin{aligned}
\tilde{e}(t+1) &= (A\tilde{x}(t) - A\hat{\tilde{x}}(t) - F_iC_i(\tilde{x}(t) - \hat{\tilde{x}}(t)) \\
&= (A - F_iC_i)\tilde{e}(t).
\end{aligned}$$

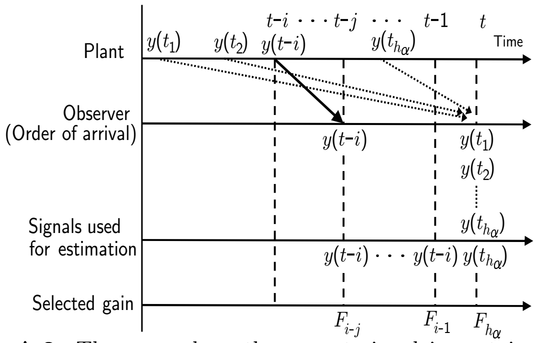


Fig. A.3 The case where the newest signal is contained in signals received at t.

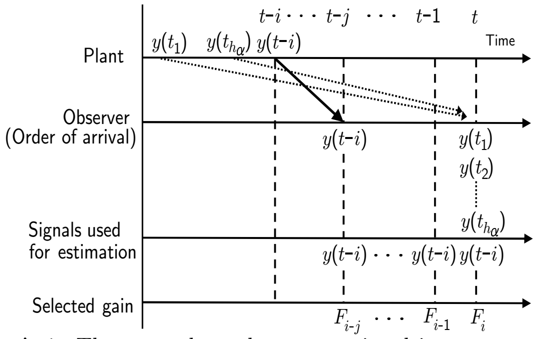


Fig. A.4 The case where the newest signal is not contained in signals received at t.

Thus,

$$V(t+1) = \tilde{e}'(t)(A - F_iC_i)'P(A - F_iC_i)\tilde{e}(t),$$

is satisfied. For $\Delta V(t)$, we obtain

$$\Delta V(t) = \tilde{e}'(t)((A - F_iC_i)'P(A - F_iC_i) - P)\tilde{e}(t).$$

From the assumption, there exists $P > 0$ satisfying

$$(A - F_iC_i)'P(A - F_iC_i) - P < 0.$$

Therefore,

$$\Delta V(t) < 0$$

holds.

A. 3 the case of (B3)

(B3) is divided into two cases.

(B3.1) The observer receives one or multiple signals at time $t > 0$. Furthermore, these signals include the newest signal (**Fig. A. 3**).

(B3.2) The observer receives one or multiple signals at time $t > 0$. Furthermore, the newest signal arrives at the observer before t (**Fig. A. 4**).

First, we consider the case of **(B3.1)**. Suppose that the observer receives α signals at $t > 0$. Time delays of these signals are denoted by $h_1, h_2, \cdots, h_{\alpha-1}$ and h_α, where

$$1 \le \alpha \le h_d + 1, \tag{A. 10}$$

$$0 \le h_\alpha < h_{\alpha-1} < \cdots < h_1 \le h_d, \tag{A. 11}$$

hold. Note that, in $\alpha = 1$, the observer receives one signal at t. Assume that $y(t-i)$ is the newest signal at $t-1$. At $t-j,\ t-j+1,\ \cdots,\ t-1$, the signal $y(t-i)$ is employed for estimation from the rule **(C)**. Then, time delay of $y(t-i)$ at $t-j$ is

$$d(t-j) = i - j. \tag{A. 12}$$

The following equation is satisfied in the same way as the case of **(B2)**,

$$1 \le j \le i \le h_d + 1. \tag{A. 13}$$

Since an interval between time $t-i$ and $t-1$ is $i-1$ steps, time delay of $y(t-i)$ at time $t-1$ is $d(t-1) = i-1$. Then, the observer selects gain F_{i-1}. From (A. 11), at time t, signal $y(t_{h_\alpha}) = \tilde{y}(t) = C_{h_\alpha}\tilde{x}(t)$ is received. Thus, the gain is switched to F_{h_α} from the rule **(C)**. The observer is given by

$$\begin{aligned}\hat{\tilde{x}}(t+1) &= A\hat{\tilde{x}}(t) + Bu(t) + F_{h_\alpha}(y(t_{h_\alpha}) - C_{h_\alpha}\hat{\tilde{x}}(t))\\ &= A\hat{\tilde{x}}(t) + Bu(t) + F_{h_\alpha}C_{h_\alpha}(\tilde{x}(t) - \hat{\tilde{x}}(t)),\end{aligned} \tag{A. 14}$$

$$t = 1,\ 2,\ \cdots.$$

At time $t-1$, the observer employs $y(t-i) = \tilde{y}(t-j) = C_{i-j}\tilde{x}(t-j)$. Then, the estimate $\hat{\tilde{x}}$ is given by

$$\begin{aligned}\hat{\tilde{x}}(t) &= A\hat{\tilde{x}}(t-1) + Bu(t-1) + F_{i-1}(y(t-i)\\ &\quad - C_{i-1}\hat{\tilde{x}}(t-1))\\ &= A\hat{\tilde{x}}(t-1) + Bu(t-1) + F_{i-1}(C_{i-j}\tilde{x}(t-j)\\ &\quad - C_{i-1}\hat{\tilde{x}}(t-1)).\end{aligned} \tag{A. 15}$$

From (A. 14) and (A. 15), we obtain

$$\begin{aligned}\tilde{e}(t+1) &= \tilde{x}(t+1) - \hat{\tilde{x}}(t+1)\\ &= A\tilde{x}(t) - A\hat{\tilde{x}}(t) - F_{h_\alpha}(y(t_{h_\alpha}) - C_{h_\alpha}\hat{\tilde{x}}(t))\\ &= A^2\tilde{x}(t-1) - A(A\hat{\tilde{x}}(t-1) + F_{i-1}(C_{i-j}\tilde{x}(t-j)\\ &\quad - C_{i-1}\hat{\tilde{x}}(t-1))) - F_{h_\alpha}(C_{h_\alpha}\tilde{x}(t) - C_{h_\alpha}(A\hat{\tilde{x}}(t-1)\\ &\quad + Bu(t-1) + F_{i-1}(C_{i-j}\tilde{x}(t-j)\\ &\quad - C_{i-1}\hat{\tilde{x}}(t-1))))\\ &= A^2\tilde{x}(t-1) - A(A\hat{\tilde{x}}(t-1) + F_{i-1}(C_{i-j}\tilde{x}(t-j)\\ &\quad - C_{i-1}\hat{\tilde{x}}(t-1))) - F_{h_\alpha}(C_{h_\alpha}A\tilde{x}(t-1)\\ &\quad - C_{h_\alpha}(A\hat{\tilde{x}}(t-1) + F_{i-1}(C_{i-j}\tilde{x}(t-j)\\ &\quad - C_{i-1}\hat{\tilde{x}}(t-1)))).\end{aligned}$$

From (A. 8) and (A. 9), the following equation holds.

$$\begin{aligned}\tilde{e}(t+1) &= A^2\tilde{x}(t-1) - A(A\hat{\tilde{x}}(t-1)\\ &\quad + F_{i-1}(C_{i-1}A^{j-1}\tilde{x}(t-j)\\ &\quad + C_{i-1}\sum_{l=2}^{j} A^{l-2}Bu(t-l)\\ &\quad - C_{i-1}\hat{\tilde{x}}(t-1))) - F_{h_\alpha}(C_{h_\alpha}A\tilde{x}(t-1)\\ &\quad - C_{h_\alpha}(A\hat{\tilde{x}}(t-1) + F_{i-1}(C_{i-1}A^{j-1}\tilde{x}(t-j)\\ &\quad + C_{i-1}\sum_{l=2}^{j} A^{l-2}Bu(t-l) - C_{i-1}\hat{\tilde{x}}(t-1))))\\ &= A^2\tilde{x}(t-1) - A(A\hat{\tilde{x}}(t-1) + F_{i-1}C_{i-1}\\ &\quad (A^{j-2}\tilde{x}(t-j+1) + \sum_{l=2}^{j-1} A^{l-2}Bu(t-l)\\ &\quad - \hat{\tilde{x}}(t-1))) - F_{h_\alpha}C_{h_\alpha}(A\tilde{x}(t-1) - (A\hat{\tilde{x}}(t-1)\\ &\quad + F_{i-1}C_{i-1}(A^{j-2}\tilde{x}(t-j+1)\\ &\quad + \sum_{l=2}^{j-1} A^{l-2}Bu(t-l) - \hat{\tilde{x}}(t-1)))).\end{aligned}$$

Iterating the above procedure, we obtain an equation

$$\begin{aligned}\tilde{e}(t+1) &= (A - F_{h_\alpha}C_{h_\alpha})(A - F_iC_i)(\tilde{x}(t-1) - \hat{\tilde{x}}(t-1))\\ &= (A - F_{h_\alpha}C_{h_\alpha})(A - F_{i-1}C_{i-1})\tilde{e}(t-1)\\ &= (A - F_{h_\alpha}C_{h_\alpha})\tilde{e}(t).\end{aligned}$$

The following equation holds.

$$V(t+1) = \tilde{e}'(t)(A - F_{h_\alpha}C_{h_\alpha})'P(A - F_{h_\alpha}C_{h_\alpha})\tilde{e}(t).$$

From the above equation, we have

$$\Delta V(t) = \tilde{e}'(t)((A - F_{h_\alpha}C_{h_\alpha})'P(A - F_{h_\alpha}C_{h_\alpha}) - P)\tilde{e}(t).$$

From the assumption, there exist $P > 0$ and $X_d,\ d \in \mathcal{Z}^+_{h_d}$ satisfying LMIs (7). Then, we obtain

$$(A - F_{h_\alpha}C_{h_\alpha})'P(A - F_{h_\alpha}C_{h_\alpha}) - P < 0.$$

Thus,

$$\Delta V(t) < 0$$

holds.

In the case of **(B3.2)**, suppose that the observer receives α signals at $t > 0$. Time delays of these signals are denoted by $h_1, h_2, \cdots, h_{\alpha-1}$ and h_α. We assume that these delays satisfy (A. 10) and (A. 11). At $t > 0$, the latest signal is $y(t-i)$ which is received at time $t-j$. In this case, the observer uses $y(t-i)$ at $t-j,\ t-j+1,\ \cdots,\ t$ from the rule **(C)**. At time t, delay of $y(t-i)$ is $d(t) = i$. Then, the observer selects F_i. The equation of the observer is given by

$$\begin{aligned}\hat{\tilde{x}}(t+1) &= A\hat{\tilde{x}}(t) + Bu(t) + F_i(y(t-i) - C_i\hat{\tilde{x}}(t))\\ &= A\hat{\tilde{x}}(t) + Bu(t) + F_i(\tilde{y}(t-j) - C_i\hat{\tilde{x}}(t))\end{aligned}$$

$$1 \leq j \leq i \leq h_d, \quad t = 1,\ 2,\ \cdots.$$

This case is the same as **(B2)**. From the same procedure as case **(B2)**, we obtain $\Delta V(t) < 0$.

A. 4 the case of (B4)

From the proof in case **(B3)**, case **(B3)** includes case **(B4)** (see **Fig. A.** 3 and **Fig. A.** 4). $\Delta V(t) < 0$ holds in case **(B4)**.

Thus, in the cases of **(B1)**-**(B4)**, $\Delta V(t) < 0$ holds.

∎

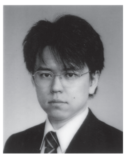
Yukinori Nakamura (Student Member)

He received Bachelor Degree from Kyoto Institute of Technology in 2004. He received Master Degree from Nara Institute of Science and Technology (NAIST) in 2006. He is currently a Ph.D. student at NAIST. His research interest includes networked control systems. He is a member of ISCIE.

Kiminao Kogiso (Member)

Kiminao KOGISO was born in Sapporo, Japan, in 1975. He received the Ph.D. degree in Mechanical Engineering from Osaka University, Japan, in 2004. He worked as a postdoctoral researcher of the 21st Century COE Program in Nara Institute of Science and Technology, from 2004 to 2005. Since 2005, he has been serving as an Assistant Professor in Nara Institute of Science and Technology. He is a member of IEEE, ISCIE, JSME, etc.

Naotoshi Adachi



He received his B. Eng. degree from the Department of Electrical and Electoronic Engineering, Ritsumeikan University, Shiga, Japan, in 1996 and M.Eng. and Dr. Eng. from the Department of Information Systems, Graduate School of Information Science, Nara Institute of Science and Technology, Japan, in 1998 and 2001, respectively. He is currently Assistant Professor at the Faculty of Environmental and Urban Engineering, Kansai University, Suita, Osaka, Japan. His research interests include system modeling and performance analysis of computer and communication systems.

Kenji Sugimoto (Member)

He received a Master's degree and Doctor's degree from Kyoto University in 1982, 1989, respectively. After working with Mitsubishi Electric Corporation, he became Assistant Professor at Kyoto University in 1985. Then he was Associate Professor at Okayama and Nagoya Universities. Since 1999, he has been Professor at Nara Institute of Science and Technology. His current research interests include control theory and system science. He is a member of IEEE, ISCIE, and IEICE.

Reprinted from Trans. of the SICE
Vol. 42 No. 11 1224/1233 2006