Special issue on International Workshop on Networked Sensing System

Trans. of the Society of Instrument and Control Engineers
Vol.E-S-1, No.1, 21/26 (2006)

A Bayesian Theory of Cooperative Calibration and Synchronization in Sensor Networks†

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This paper proposes a method for calibrating networked sensors from local duplicated measurements in couples of sensors among them. In the Bayesian framework, we show the system and procedure for the optimum collaborative calibration/synchronization is composed by: 1) sensor-wise maintenance of the offset values and a corresponding column of inverse of the estimation error covariance matrix (confidence matrix), 2) incremental updates of the confidence matrix elements for the coupled sensors with the confidence of the duplicated measurement, and 3-1) centralized computation for inverting the confidence matrix into the estimation error covariance matrix and sensor-wise computation for obtaining updated estimates of the offset values, or 3-2) global iterative computation of the updated estimates among all the sensors.

Key Words: synchronization, calibration, sensor network, Bayes estimate

1. Introduction

Sensor networks consist of numerous nodes being capable of sensing, computation, and communication. They cooperate each other to combine individual sensor measurements into a high-level sensing result relying on the consistency among their internal clocks and references. In large-scale sensor networks, however, the consistency-keeping is a very difficult task since sensor nodes are usually inaccessible because of their deployed, dynamic, and ad hoc natures.

This paper deals with the method for the calibration and synchronization of networked sensors [3),4),6)] from duplicated measurements among them [7),8)]. It is well known that the information for calibration/synchronization is obtained only when two or more sensors detect the same object and compare the results each other. But in usual sensor networks, those events occur only locally and are not shared by all the sensors. Examples include: detecting the same signal waveform with individual time stamps, measuring the same quantity with each internal references, eventually encountered mobile location sensors at the same place and time, etc. How should we design the correcting scheme of their internal clocks or references for the steady growth of global consistency? How should we combine the sensor-to-sensor corrections into the consistent ones among all the sensors? How should we propagate the established results among a sensor group to another group of sensors established differently?

2. Theory

2.1 Problem and notation

We consider the synchronization problem. Extension to general calibration cases will be straightforward.

At an event, a couple of sensors detect the difference between each offset time. Let the estimates of the offset time of each sensor after the kth event be $(\Delta_1^k, \Delta_2^k, \cdots, \Delta_n^k)$. Let the estimation error covariance matrix of them be $\Sigma_k = [\ \sigma_{ij}^k\ ]$. At the $(k+1)$th event between the sensors i and j, the difference t_{ij} of their offset times was observed with the error variance σ^2. Then, obtain the new estimates of the offset time $(\Delta_1^{k+1}, \Delta_2^{k+1}, \cdots, \Delta_n^{k+1})$ and their estimation error covariance matrix $\Sigma_{k+1} = [\ \sigma_{ij}^{k+1}\ ]$ or its equivalence.

2.2 Successive Bayesian estimate [10)]

Let the all measurements until the kth event be $\boldsymbol{Y}^k$. Let the measurement at the $(k+1)$th event be $\boldsymbol{y}^{k+1}$. Let the variable to be estimated be $\boldsymbol{x}$. Then, by using the Bayes formula and the assumption that the measurement noises are independent, it follows that

$$p(\boldsymbol{x}|\boldsymbol{Y}^{k+1}) = p(\boldsymbol{x}|\boldsymbol{y}^{k+1}, \boldsymbol{Y}^k) = \frac{p(\boldsymbol{y}^{k+1}|\boldsymbol{x})p(\boldsymbol{x}|\boldsymbol{Y}^k)}{\int p(\boldsymbol{y}^{k+1}|\boldsymbol{x})p(\boldsymbol{x}|\boldsymbol{Y}^k)d\boldsymbol{x}},$$

where $p(\cdot|\cdot)$ denotes the conditional probability density function (pdf). This equation provides us the Bayes esti-

† Presented at INSS2004 (2004.6)

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(Received November 16, 2004)
(Revised April 9, 2005)

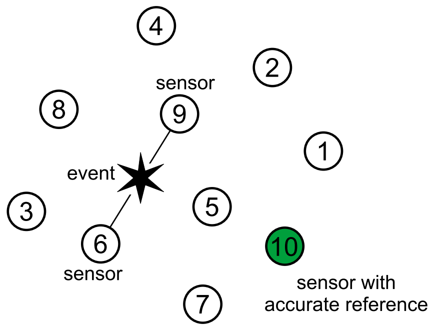


Fig. 1 Situation of the problem. Most sensors in a network are uncalibrated/unsynchronized although some may be accurate. The inconsistency is detected only when two sensors of them detect eventually the same event.

mates of $\boldsymbol{x}$ based on the whole measurements $\boldsymbol{Y}^{k+1}$ until the $(k+1)$th event ($k=0$ indicates the initial state before the first event) and also the updating formula from the pdf $p(\boldsymbol{x}|\boldsymbol{Y}^k)$ until the kth event to the pdf $p(\boldsymbol{x}|\boldsymbol{Y}^{k+1})$ including the $(k+1)$th event.

For the normal distribution case when the observation equation is expressed as $\boldsymbol{y}=B\boldsymbol{x}+\boldsymbol{n}$, the updated pdf is also the normal distribution, hence is characterized by the average vector and the covariance matrix. The exponent of the product $p(\boldsymbol{y}^{k+1}|\boldsymbol{x})p(\boldsymbol{x}|\boldsymbol{Y}^k)$ is expressed as

$$(\boldsymbol{y}-B\boldsymbol{x})^T\Sigma_{\boldsymbol{n}}^{-1}(\boldsymbol{y}-B\boldsymbol{x})+(\boldsymbol{x}-\boldsymbol{x}^k)^T\Sigma_k^{-1}(\boldsymbol{x}-\boldsymbol{x}^k)$$
$$\equiv(\boldsymbol{x}-\boldsymbol{x}^{k+1})^T\Sigma_{k+1}^{-1}(\boldsymbol{x}-\boldsymbol{x}^{k+1}),$$

where $\Sigma_{\boldsymbol{n}}$ is the covariance matrix of the observation noise. Comparing both sides of the above equation, the updating equations for the average vector and the covariance matrix of $\boldsymbol{x}$ is thus expressed, respectively, as

$$\Sigma_{k+1}^{-1}=B^T\Sigma_{\boldsymbol{n}}^{-1}B+\Sigma_k^{-1} \tag{1}$$
$$\boldsymbol{x}^{k+1}=\boldsymbol{x}^k+\Sigma_{k+1}B^T\Sigma_{\boldsymbol{n}}^{-1}(\boldsymbol{y}-B\boldsymbol{x}^k). \tag{2}$$

2.3 Successive estimate of offset times: direct solution

For the synchronization problem, actual forms of the pdf are expressed as

$$p(\boldsymbol{y}^{k+1}|\boldsymbol{x})=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\{-\frac{1}{2}\frac{((\Delta_i-\Delta_j)-t_{ij})^2}{\sigma^2}\}$$
$$p(\boldsymbol{x}|\boldsymbol{Y}^k)=\frac{1}{|2\pi\Sigma^k|^{1/2}}\exp\{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{x}^k)^T\Sigma_k^{-1}(\boldsymbol{x}-\boldsymbol{x}^k)\}.$$

Since

$$B\equiv\begin{bmatrix} 0 & \cdots & 1 & \cdots & -1 & \cdots & 0 \end{bmatrix},$$

the updating equations are expressed as

$$\Sigma_{k+1}^{-1}=\begin{bmatrix} \lambda_{11}^k & \cdots & \lambda_{1i}^k & \cdots & \lambda_{1j}^k & \cdots & \lambda_{1n}^k \\ \vdots & & \vdots & & \vdots & & \vdots \\ \lambda_{i1}^k & \cdots & \lambda_{ii}^k+\lambda & \cdots & \lambda_{ij}^k-\lambda & \cdots & \lambda_{in}^k \\ \vdots & & \vdots & & \vdots & & \vdots \\ \lambda_{j1}^k & \cdots & \lambda_{ji}^k-\lambda & \cdots & \lambda_{jj}^k+\lambda & \cdots & \lambda_{jn}^k \\ \vdots & & \vdots & & \vdots & & \vdots \\ \lambda_{n1}^k & \cdots & \lambda_{ni}^k & \cdots & \lambda_{nj}^k & \cdots & \lambda_{nn}^k \end{bmatrix}$$

and

$$\begin{bmatrix} \Delta_1^{k+1} \\ \vdots \\ \Delta_i^{k+1} \\ \vdots \\ \Delta_j^{k+1} \\ \vdots \\ \Delta_n^{k+1} \end{bmatrix}=\begin{bmatrix} \Delta_1^k \\ \vdots \\ \Delta_i^k \\ \vdots \\ \Delta_j^k \\ \vdots \\ \Delta_n^k \end{bmatrix}+\Sigma_{k+1}\begin{bmatrix} 0 \\ \vdots \\ \lambda(t_{ij}-\Delta_i^k+\Delta_j^k) \\ \vdots \\ -\lambda(t_{ij}-\Delta_i^k+\Delta_j^k) \\ \vdots \\ 0 \end{bmatrix} \tag{3}$$

where $\lambda\equiv1/\sigma^2$ and λ_{ml}^k is the (m,l)th element of Σ_k^{-1}.

The above equations lead to the following procedures for the optimum synchronization. Each sensor indicated by l maintains a corresponding column or row of Σ_k and Σ_k^{-1}, i.e., $\{\sigma_{l1}^k,\sigma_{l2}^k,\cdots,\sigma_{ln}^k\}$ and $\{\lambda_{l1}^k,\lambda_{l2}^k,\cdots,\lambda_{ln}^k\}$, respectively. At the synchronizing event, the update of $\{\lambda_{l1}^k,\lambda_{l2}^k,\cdots,\lambda_{ln}^k\}$ begins for the coupled sensors $l=i,j$ with

$$\begin{aligned} &\lambda_{im}^{k+1}=\lambda_{im}^k \qquad (m=1,2\cdots,n\neq i,j) \\ &\lambda_{ii}^{k+1}=\lambda_{ii}^k+\lambda, \quad \lambda_{jj}^{k+1}=\lambda_{ii}^k+\lambda \\ &\lambda_{ij}^{k+1}=\lambda_{ij}^k-\lambda, \quad \lambda_{ji}^{k+1}=\lambda_{ji}^k-\lambda, \end{aligned} \tag{4}$$

and for the sensor $l\neq i,j$, remains unchanged as

$$\lambda_{lm}^{k+1}=\lambda_{lm}^k \qquad (m=1,2,\cdots,n) \tag{5}$$

which result in the updated matrix Σ_{k+1}^{-1}. Then its inverse, the estimation error covariance matrix Σ_{k+1}, is taken to obtain $\{\sigma_{l1}^{k+1},\sigma_{l2}^{k+1},\cdots,\sigma_{ln}^{k+1}\}$ for all sensors. By using them, each sensor obtains the update of the offset-time estimates such that

$$\Delta_l^{k+1}=\Delta_l^k+\lambda(\sigma_{li}^{k+1}-\sigma_{lj}^{k+1})(t_{ij}-\Delta_i^k+\Delta_j^k). \tag{6}$$

Clearly, an essential part of the procedure, the matrix inversion, must be performed in a centralized manner. All the elements of Σ_{k+1}^{-1} must be gathered somewhere to take its inverse and the elements of it must be redistributed to the corresponding sensors.

2.4 Successive estimate of offset times: iterative solution

The matrix inversion can be omitted if we use an iterative procedure as follows. Since $\boldsymbol{x}^{k+1}$ is the minimum of

$$J \equiv (\boldsymbol{x}-\boldsymbol{x}^k)^T \Sigma_k^{-1} (\boldsymbol{x}-\boldsymbol{x}^k) + (\boldsymbol{y}-B\boldsymbol{x})^T \Sigma_{\boldsymbol{n}}^{-1} (\boldsymbol{y}-B\boldsymbol{x}),$$

we can apply the steepest descent method

$$\begin{aligned}\boldsymbol{x}^{(i+1)} &= \boldsymbol{x}^{(i)} - \alpha\{\Sigma_k^{-1}(\boldsymbol{x}^{(i)}-\boldsymbol{x}^k) - B^T\Sigma_{\boldsymbol{n}}^{-1}(\boldsymbol{y}-B\boldsymbol{x}^{(i)})\} \\ &= \boldsymbol{x}^{(i)} - \alpha\{\Sigma_{k+1}^{-1}(\boldsymbol{x}^{(i)}-\boldsymbol{x}^k) - B^T\Sigma_{\boldsymbol{n}}^{-1}(\boldsymbol{y}-B\boldsymbol{x}^k)\},\end{aligned}$$

where α is a positive constant and (i) is the number of iteration. The calculation can be performed parallel and distributedly by all sensors. Namely, for the sensor $l \neq i, j$

$$\Delta_l^{(i+1)} = \Delta_l^{(i)} - \alpha \sum_{m=1}^{n} \lambda_{lm}^{k+1} (\Delta_m^{(i)} - \Delta_m^k) \tag{7}$$

and for the sensors $l = i, j$,

$$\begin{aligned}\Delta_i^{(i+1)} &= \Delta_i^{(i)} - \alpha\{\sum_{m=1}^{n} \lambda_{im}^{k+1} (\Delta_m^{(i)} - \Delta_m^k) \\ &\qquad - \lambda(t_{ij} - \Delta_i^k + \Delta_j^k)\}\end{aligned} \tag{8}$$

$$\begin{aligned}\Delta_j^{(i+1)} &= \Delta_j^{(i)} - \alpha\{\sum_{m=1}^{n} \lambda_{jm}^{k+1} (\Delta_m^{(i)} - \Delta_m^k) \\ &\qquad + \lambda(t_{ij} - \Delta_i^k + \Delta_j^k)\}.\end{aligned} \tag{9}$$

To ensure rapid convergence and stability, α should be varied according to

$$\alpha^{-1} \propto \sum_{i=1}^{n} \lambda_{ii}.$$

The information to be transferred among sensors at each iteration is only the transient values of estimates $(\Delta_1^{(i)}, \Delta_2^{(i)}, \cdots, \Delta_n^{(i)})$ and the gain α of the iteration.

3. Numerical simulation

3.1 Initial setting

The offset times of almost all sensors are large and independent each other. This leads when $k = 0$ to

$$\sigma_{ll}^0 = \sigma_{rand}^2, \quad \sigma_{lm}^0 = 0 \quad (m = 1, 2, \cdots, n-1) \tag{10}$$

where σ_{rand}^2 is the estimated variance of the offset time. When the offset times can have an unknown systematic bias, it should be included by setting

$$\sigma_{ll}^0 = \sigma_{rand}^2 + \sigma_{bias}^2, \quad \sigma_{lm}^0 = \sigma_{bias}^2, \tag{11}$$

where σ_{bias}^2 is the estimated variance of the bias. Actually, the inverse of them must be distributed as

$$\lambda_{ll}^0 = \frac{1}{\sigma_{rand}^2} - \frac{\sigma_{bias}^2}{\sigma_{offset}^2(\sigma_{rand}^2 + n\sigma_{bias}^2)} \tag{12}$$

$$\lambda_{lm}^0 = -\frac{\sigma_{bias}^2}{\sigma_{rand}^2(\sigma_{rand}^2 + n\sigma_{bias}^2)} \tag{13}$$

which can be calculated independently by each sensor. In addition to the above uncalibrated and unsynchronized sensors $1, 2, \cdots, n-1$, we mixed an accurate one with a reliable standard clock as

$$\sigma_{nn}^0 \ll \sigma_{rand}^2, \quad \sigma_{nm}^0 = 0. \tag{14}$$

To each sensor l, a true offset time Δ_i was generated randomly according to the assumed variance of it. All initial estimates of the offset time are

$$\Delta_1^0 = \Delta_2^0 = \cdots = \Delta_n^0 = 0. \tag{15}$$

3.2 Time difference measurement

For each measurement, sensors i and j are randomly selected. The measurement was generated by adding a random noise with a variance σ^2 to the true difference $\Delta_i - \Delta_j$. The estimates of the offset time were updated according to the prescribed equations.

3.3 Results

Fig.2 shows an example of result. The sensors are given large and random offset times initially. According to the

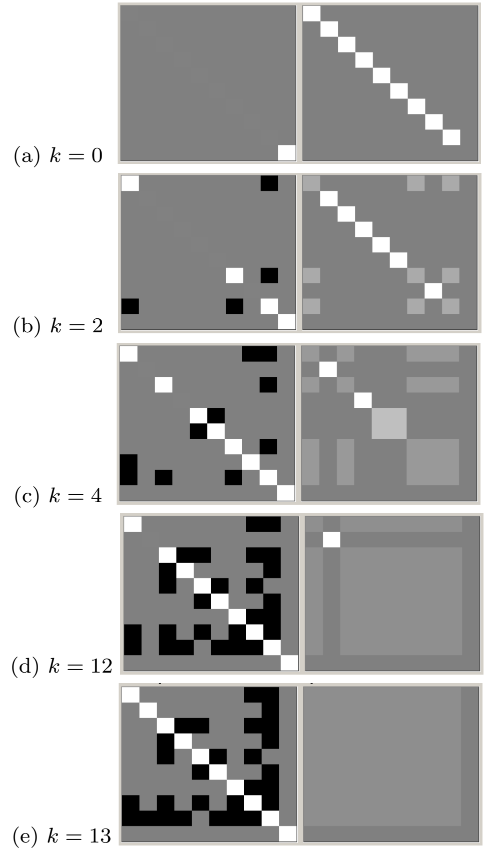


Fig. 3 Brightness view of the estimation error covariance matrix of the experiment in Fig.2. The brightness (middle gray means zero) of small squares indecates the value of the matrix element. The left and right images are Σ_k^{-1} and Σ_k, respectively. Initially ($k = 0$), the estimation errors are large and independent. Until $k = 4$, the sensors (1,3,7,8,9) and (5,6) are coupled together and grouped independently. The elements corresponding to each group are filled uniformly. This shows they are synchronized.

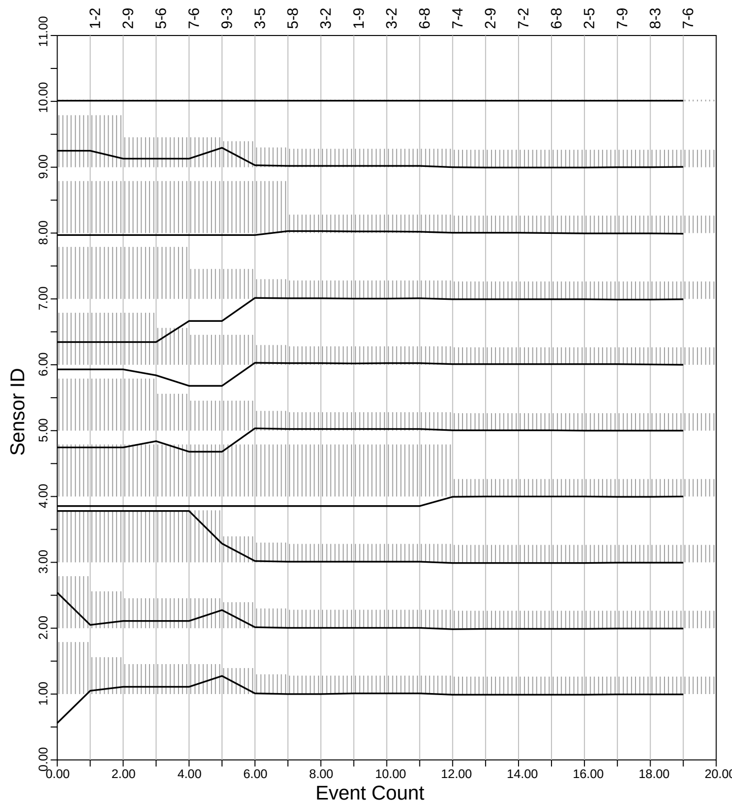


Fig. 2 An example of result. The sensors are given initially large and random offset times shown by the 0th event. According to the increase of the event count, the number of coupled sensors increases and the estimation error variances reduce monotonically for the coupled ones. For the lately coupled sensors, the reduction of error variance is drastic. But after novel sensors exhausted ($k = 12$), they become almost constant. The topmost row of numbers indicate coupled sensors at the event.

increase of the event count, the number of coupled sensors increases and the estimation error variances reduce monotonically for the coupled ones. For the lately coupled sensors, the reduction of error variance is drastic. But after novel sensors exhausted ($k = 12$), they become almost constant. Fig.3 shows brightness-encoded views of the sequence of Σ_k^{-1} (left) and Σ_k (right). Initially, the estimation errors are large and independent. Until $k = 4$, the sensors (1,3,7,8,9) and (5,6) have been coupled together and grouped independently. The elements of Σ_k corresponding to each group are filled uniformly. This shows they are synchronized (fully correlated).

Fig.4 shows another example of result. Except for sensor 10, offset times are assumed to be biased, hence Σ_k is still large after the synchronization from 1 to 9 has completed. At $k = 16$, one sensor encountered the sensor 10 with an accurate clock, then all the offset times including the bias became zero at a time. Fig.5 shows brightness-encoded views of this experiment. At $k = 14$, all the sensors except one finished coupling and the corresponding elements of Σ_k became flat although the values were nonzero. At $k = 16$ shown in Fig.5(e), one of the sensors encountered the sensor 10 with an accurate clock. Then all the nonzero elements of Σ_k decreased at a time to a level of sensor 10, which means all clocks of sensors were adjusted to an accurate one of the sensor 10.

4. Possible generalization

The primary subject of generalization will be the introduction of the handling capability of the temporal increase of ambiguity. The problem involves inaccuracy of clock frequency which grows into the varying offset time in the passage of time. Generalization to this problem is possible by introducing the diffusion process of the conditional pdf between the events, which results in the continuous time Kalman filtering with discrete observations. Another interesting subject will be the extension to the simultaneous calibration of clock and location. This will be possible when two or more RF sources [8] are detected simultaneously by a couple of sensors. Interesting sce-

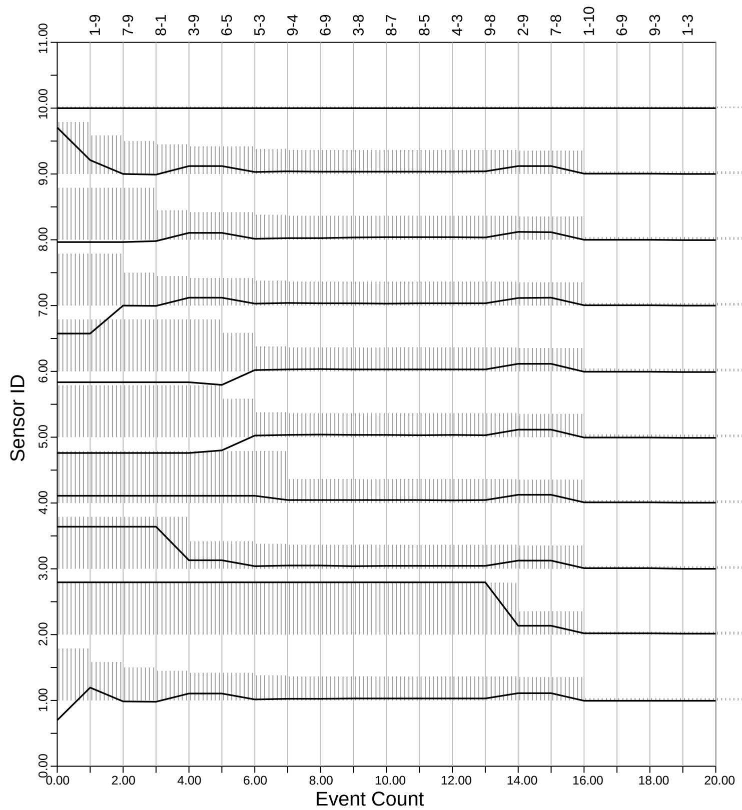


Fig. 4 An example of result. Except for sensor 10, offset times are assumed to be biased, hence Σ_k is still large after the synchronization from 1 to 9 has completed. At $k = 16$, one sensor encountered the sensor 10 with an accurate clock, then all the offset times including the bias became zero at a time.

nario is the use of RF waveform transmitted from RF-ID as the synchronizing event. The ID tag can be used both to assure the duplicated detection of event and to share the physical parameters and models (frequency, location, pose, etc.) of the RF source and the tagged object as those in the Tag-based Vision Systems [11)~14)].

5. Conclusion

This paper proposed a method for calibrating networked sensors from local duplicated measurements in a couple of sensors among them. In the Bayesian framework, we obtained the solution to this problem and showed that:

(1) The solution is expressed as an updating scheme of the estimates $\Delta_1^k, \Delta_2^k, \cdots, \Delta_n^k$ and the inverse of estimation error covariance matrix Σ_k^{-1}.

(2) Each sensor holds $\Delta_1^k, \Delta_2^k, \cdots, \Delta_n^k$ and a corresponding column of Σ_k^{-1}.

(3) At a pair-wise duplicated measurement, the coupled sensors increase the related 2×2 elements of Σ_k^{-1} by the inverse of observation noise variance $1/\sigma^2$ to obtain the update Σ_{k+1}^{-1}.

(4) In the centralized scheme, all the elements of Σ_{k+1}^{-1} is gathered, inverted, and redistributed to the corresponding sensors to obtain the new estimates $\Delta_1^{k+1}, \Delta_2^{k+1}, \cdots, \Delta_n^{k+1}$.

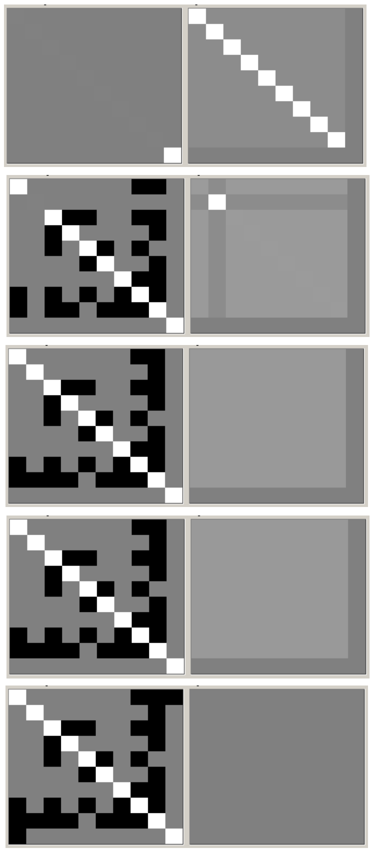
(5) In the decentralized scheme, $\Delta_1^{k+1}, \Delta_2^{k+1}, \cdots, \Delta_n^{k+1}$ are calculated by each sensor using an iteration scheme although the transient estimates must be transfered each other among all the sensors.

(6) For a group of sensors chained by duplicated measurements, all elements of Σ_k become nearly equal, thus estimation errors of the grouped sensors become fully correlated.

(7) Communication needed to achieve the collaborative calibration is described essentially by the backward inversion process of the inverse of estimation error covariance matrix Σ_{k+1}^{-1}.

References

1) L. Doherty et al., "Convex position estimation in wireless sensor networks," Proc. INFOCOM 2001, 2001.
2) J. Elson, L. Girod, and D. Estrin, "Fine-grained network time synchronization using reference broadcasts," submit-



(a) $k=0$

(b) $k=13$

(c) $k=14$

(d) $k=15$

(e) $k=16$

Fig. 5 Brightness view of Σ_k^{-1} (left) and Σ_k (right) for the experiment in Fig.4. At $k=14$, all the sensors except one finished coupling and the corresponding elements of Σ_k became flat although the values were nonzero. At $k=16$, one of the sensors encountered the sensor 10 with an accurate clock. Then all the elements of Σ_k vanished, which means all the sensors were adjusted at a time to the sensor 10.

ted for review, 2002.
3) J. Hightower, C. Valili, G. Borriello, and R. Want, "Design and calibration of the SpotON ad-hoc location sensing system," unpublished, 2001.
4) K. Whitehouse and D. Culler, "Calibration as parameter estimation in sensor networks," Proc. ACM Int. Workshop Wireless Sensor Networks and Applications, Atlanta, 2002.
5) A. Nasipuri and K. Li, "A directionality based location discovery scheme for wireless sensor networks," Proc. ACM Int. Conf. Mobile Computing and Networking, Atlanta, 2002.
6) J. Elson and K. Romer, "Wireless sensor networks: A new regime for time synchronization," Proc. HotNets-1, 2002.
7) V. Bychkovskiy, S. Megerian, D. Estrin, and M. Potkonjak, "A Collaborative Approach to In-Place Sensor Calibration," Proc. 2nd International Workshop on Information Processing in Sensor Networks (IPSN'03).
8) R. L. Moses, D. Krishnamurthy, and R. Patterson, "A self-localization method for wireless sensor networks," EURASIP J. Applied Signal Processing, pp.348-358, 2003.
9) A. Bletsas, "Evaluation of Kalman filtering for network time keeping," IEEE Int. Conf. Pervasive Computing and Communications (PerCom 2003), Dallas-Fort Worth, 2003.
10) A. Jazwinski, *Stochastic Process and Filtering Theory*, Academic Press, New York, chap.6, pp.162-193, 1970.
11) M. Boukraa and S. Ando, "Tag-based vision: Assisting 3-D scene analysis with radio-frequency tags," Proc. 5th Int. Conf Information Fusion, pp.412-418, Annapolis, 2002.
12) M. Boukraa and S. Ando, "Tag-based vision: Assisting 3-D scene analysis with radio-frequency tags," Proc. Int. Conf Image Processing (ICIP 2002), Rochester, Sep. 2002.
13) M. Boukraa and S. Ando, "A radio-frequency tag system for object identification and registration," Proc. 19th Sensor Symp., pp.287-292, Kyoto, May 2002.
14) S. Ando and H. Hontani, "Automatic Visual Searching and Reading of Barcodes in 3-D Scene," Proc. IEEE Int. Vehicle Electronics Conf, 2001, Tottori, pp.49-54, 2001

Reprinted from Trans. of the SICE
Vol. E-S-1 No. 1 21/26 2006.