Scheduling of Order-picking with Replenishment
in a Warehouse Environment

Jie GONG*, Hirofumi TAMURA#, Toshimitsu HIGASHI+ and Jun OTA*

In this paper, given the orders and stock in a warehouse, we present a metaheuristic-based scheduler that aims to solve a scheduling problem of order-picking with replenishment. Our objective is to minimize the maximum operation time of order-picking carts. The problem is described with respect to the carts to replenish and pick up products separately within the same period. Sub-problems in this research are (1) how to reduce delays among carts and (2) how to reduce the carts’ travel distance. Several operators aiming to generate transitions in the schedule are implemented. After that, a local search procedure is used to solve the routing problem and reduce the delays among carts. The experiment results show that the metaheuristic can make significant improvements with respect to minimizing the operation time.

Key words: warehouse management, order-picking, replenishing, metaheuristics, scheduling

1. INTRODUCTION

1.1 Background

Improvements in warehouse management in recent years are being attributed to gains in logistics. Logistics improvements lead to increased efficiency in warehouse operations and to more reliable customer service. The main operations for warehouse include receiving, storing, order-picking, and shipping. One study has shown that order-picking accounts for as much as 55% of the total warehouse operating cost [1].

Pallets and products are picked from the storage locations by pickers. With common low-level order-picking systems, pickers are humans employing carts.

Nowadays, order-picking is the costliest activity in a warehouse. It is divided into a reserve area and a picking area (forward area). Because of the shortage products during intensive order-picking, replenishment is very important [2]. The replenishment activity becomes more frequent and crucial during a busy picking period. More attention is currently being given to such problems as timing and replenishment.

1.2 Literature Survey

Warehouse design has traditionally been divided into tactical or operational levels [3]. On a tactical level, replenishment is mainly discussed in the field of inventory theory. Many studies have been made in this field [4]-[6]. Such studies have dealt with a multi-retailer problem, multi-item problems, and the limited-capacity problem of warehouses. Several studies on layout design have also been conducted [7]. Another issue is storage assignment, which involves the number of products to be placed in forward and reserve areas. The decisions concerning the problems described are commonly called the forward-reserve problem [8].

In this problem, the regular replenishment quantity from the reserve to the forward area is concerned.

On the operational level, the picking problem has been addressed as a routing method problem, which aims to sequence the picking items to ensure a good route [9][10]. Such a picking problem can be represented in the form of the Traveling Salesman Problem (TSP) and solved with adequate algorithms [11]. In these studies, one important assumption is that the stock (product quantity in the storage location) is unlimited and there is no need to replenish the product supplies. The replenishment problem on the operational level has been discussed in [12]. The warehouse environment discussed in the paper is somewhat specific, whereas the replenishment area and the picking area are completely separate. Therefore, it is difficult to apply the method directly to a general warehouse environment.

In previous studies, the replenishment problem was mainly addressed on a tactical level, such as the forward-reserve problem. On the operational level, only a few studies have been conducted with regard to order-picking with replenishment. Thus, in this

Fig. 1 A model of order-picking in a warehouse
Table 1 An example of the order list by shops

<table>
<thead>
<tr>
<th>Shop</th>
<th>Product 1</th>
<th>Product 2</th>
<th>Product 3</th>
<th>...</th>
<th>Product n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shop A</td>
<td>10</td>
<td>2</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>Shop B</td>
<td>40</td>
<td>35</td>
<td>15</td>
<td>...</td>
<td>20</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Shop M</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>...</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 2 An example of the stock sheet

<table>
<thead>
<tr>
<th>Location</th>
<th>Product</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AA</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>AB</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>AC</td>
<td>80</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>n</td>
<td>CY</td>
<td>20</td>
</tr>
</tbody>
</table>

research, our objective is to model and present a search-based metaheuristic to solve the problems related to order-picking with replenishment on the operational level.

The problem of order-picking with replenishment differs from typical order-picking problems. Both of the replenishment operation and the picking operation are made in the common working area. Both the replenishment and picking operations occur in a common work area. A primary constraint is that the picking cart operation cannot be completed until the products that are in short supply are replenished.

1.3 A Warehouse Model

A warehouse model is shown in Fig. 1. The storage locations hold initial quantities of products (stock). Each location has its own location number, as shown in the figure. The items to be picked from given locations are obtained from the orders sent by shops. The items in short supply are transported by carts from an Automated Storage and Retrieval System (AS/RS) to replenish the storage sites. The items selected from a corresponding order are picked up and deposited at the shed for loading into trucks and delivery to the customers. Order-picking schedules are made by clustering tasks and by assigning them to carts assignments.

The rest of this paper is organized as follows: Section 2 offers an overview of the problem of order-picking with replenishment; Section 3 is a discussion of the methodology of the research; Section 4 is a presentation of the experiment results; and Section 5 is the conclusion.

2. Problem Statement

An overview of the warehouse picking problem is given first. The products are stored in stationary storage locations throughout the warehouse (Fig. 1). Each location holds quantities of unique types of products and has a unit quantity called a case. The shop order and the stock for every product serve as inputs to the picking problem, as shown in Tables 1 and 2, respectively. For one storage location, there is a capacity that is represented by $CAP_p$. If the quantity of products exceeds the capacity, the unloading time of the exceeding part is assumed to be $N_{un}$ times more than normal.

The orders placed on the warehouse should be picked up by the carts from the storage locations and transferred to the shed. A cart will start from the shed and pick up the products at the various locations. If the quantity reaches the capacity of the cart, represented as $CAP_p$, the cart will return to the shed and will unload the products. This is referred to as a trip and assigned to carts. Figure 2 shows an example of the trip. In this case, the trip connects four picking locations.

As the products are picked up, the stock decreases, and it is necessary to replenish them at the storage sites. In the pickup process, the products in shortfall are uploaded on the carts from the AS/RS and replenished to the sites. When the cart is empty, it returns to the AS/RS and uploads products for the next replenishment round. This circuitous route is called a replenishment trip. The carts undertake multiple pickup trips that are started and completed at a shed.

We then determine the cart sequence for pickup and replenishment, which is the output as shown in Fig. 3. Here, a schedule composed of two carts $(C_1$ and $C_2$) is shown. The numbers without parentheses correspond to storage locations, while the numbers in parentheses are the numbers of items to be picked or replenished at the corresponding locations. The start of trip is denoted by “s” or “r.” Here, “s” represents the shed and “r” represents AS/RS. Here, we define the action as the element of one pickup/replenishment operation, which contains the cart activity information. The information includes the cart ID, its location ID, and the type of action: replenishment, pickup, or load quantity.

Carts are scheduled to complete the order-picking and replenishment operations as soon as possible. However, because of the existence of picking and replenishment tasks in the same period, several delays occur and increase the difficulty of solving the problem. There are several types of delays experienced by carts.

1) Loading Queue Delay ($D_{LQ}$): This type of delay occurs when carts must load and unload products at the same site and at the same time. These interactions are not only variable but also unpredictable and they can cause a cart to diverge from its base time significantly. If the two carts have to do the same operations (replenishment or picking) at the same location in the same period, the carts operate in the order in which they arrive, that is, first in,
first out. The waiting time is called the loading queue delay time.

2) Replenishment Wait Delay ($D_{w}$): This type of delay occurs when a picking cart waits for the replenishment at the sites in short until the replenishment cart finishes its task. The waiting time is referred to as the replenishment delay.

3) Punishment Loading Delay ($D_{l}$): This type of delay occurs when the number of products after replenishment exceeds the capacity at the storage location. The load or pick time for the products that exceeds the capacity takes $N_{w}$ ($>1$) times than the standard load or pick time. This type of delay is referred to as a punishment loading delay.

4) Collision Avoidance Delay ($D_{c}$): This type of delay occurs when a cart requires extra time to avoid a collision with another cart.

As the problem is stated in this paper, the density of carts is relatively low and because mutual avoidance ability of cart drivers is so high, the degree of delay is not so large even two carts go through each other. Therefore, the fourth collision avoidance delay remains small compared to the total operation time and other delays. Thus, we ignore the collision avoidance delays and focus on the other three types of delays.

The problem is presented as follows. Let:

- $C = \{c_1, c_2, \ldots, c_k\}$: a set of carts
- $A_i = \{a_{i1}, a_{i2}, \ldots, a_{i|} \} (i \in C)$: a set of actions of the cart $i$
- $k$: total number of carts
- $n_i$: total number of actions assigned to cart $i$ ($i \in C$)
- $\mu_i$: operation time of cart $i$ with delay time ($i \in C$)
- $\max_{i \in C} (\mu_i)$: total makespan

Objective function: $M \rightarrow \min$.

3. Research Methodology

3.1 Approach

Scheduling is defined as the allocation of carts to a collection of tasks. The transition from one solution to another can improve the efficiency of finding a better solution. In most scheduling problems, tasks have been determined beforehand and never change in the scheduling process. In the field of split delivery vehicle routing problems (SDVRP), the concept of task splitting is introduced for deriving solutions [11]. Similarly, in the paper, we propose several operators introducing task splitting during the search process.

In many warehouses, orders required for a specific day are just fixed several minutes before the warehouse opens. Then the planned/scheduled tasks must be finished in several minutes. Thus, maximum computation time $t_{\text{max}}$ is an important condition.

The metaheuristic based on Simulated Annealing (SA) is described in this section. First, we introduce the Simulated Annealing structure, and then, a general heuristic to generate an initial schedule is introduced in paragraph 3.3. Operators to move action from cart to cart are described in detail in paragraph 3.4. Local Search as a procedure will be proposed in paragraph 3.5; it aims to minimize the routing cost and reduce the delay caused by interaction between carts.

3.2 SA Scheduler Structure

Simulated Annealing is a popular metaheuristic that has found extensive use for solving complex combinatorial optimization problems [13]. In SA, the search begins at an initial temperature $T_{\text{init}}$ and proceeds until the stopping conditions are satisfied. We used one stopping condition when the current computation time reached the maximal computation time $t_{\text{max}}$. The flowchart of the SA scheduler is shown in detail in Fig. 4.

The algorithms about initial schedule generation, operator selection, and local search will be discussed in detail in paragraphs 3.3, 3.4, and 3.5, respectively.

3.3 Initial Schedule Generation

The initial schedule is generated by a strategy that is widely practiced due to its ease of implementation. The current scheduler generates trips by clustering the orders as a modified S-shaped pattern explained in Section 4.1. The products are picked up or replenished based on an S-shaped routing [14], which involves a round trip to all of the shelves, shed, and AS/RS, as shown in Fig.
5. If the quantities to be picked up during the trip will exceed the capacity of cart (CAP) at the next picking location, the cart will stop picking and will return to the shed or AS/RS. At the beginning, all the carts start at the AS/RS to replenish products. The next generated trip will be assigned to the cart that returns first. This is repeated until there are no more unassigned replenishments in the warehouse. At that time, the carts tasked with replenishment will be reassigned to pick-up trips from the last site of replenishment. If there are no more picking trips to be assigned, no further activity is required.

3.4 Operator Selection
The operators in a scheduling problem relocate actions from one cart to another. The three types of operators are defined below: Action Transfer (total transfer), Action Exchange (total action swap), and Action Split (partial transfer).

1) Action Transfer: Action Transfer is a move wherein Aa of Ci is transferred from Ci to another cart Cj. The transfer is done such that a new trip is constructed for Aa at the end of the action sequence of Cj. Here, i, j, and k are randomly selected.

2) Action Exchange: An Action Exchange is a move in which the Aa from Ci is relocated to another Cj. Similarly, the Aa of Cj is relocated to Ci. After this operation, both Cj and Ci will add one new trip containing Aa at the end of the respective action sequences. Here, i, j, k, and l are randomly selected.

3) Action Split: If actions are split, a better balance can be achieved among the carts. The Action Split operator is an extension of the Action Transfer that allows partial relocation of an action from one cart to another, as shown in Fig. 6.

We can estimate the amount to items i* of Aa to be moved from Ci to another cart Cj in order to achieve a better makespan balance between two carts.

\[ i^* = \frac{\mu_i - \mu_j - T_p}{2T_{load}} \] (1)

In (1), \( \mu_i \) and \( \mu_j \) are the makespans of Cj and Ci before the action split operation. \( T_p \) is the increased travel cost of the new trip of a new split action, which is moved to Cj. \( T_{load} \) is the Loading time per one case. Equation (1) is constructed to create a balance between \( \mu_i \) and \( \mu_j \).

We propose the Split_I methodology to implement the above three operations. The algorithm will be explained as follows.

In the Split_I and Non-split_I, we make a decision to select the action exchange or the action transfer by utilizing \( i^* \) at Step 3. On the other hand, in Split_R and Non-split_R, we select the action exchange and the action transfer randomly. We can consider other classes to include the split operation or not. Split_I and Split_R include the split operation, and Non-split_I and Non-split_R do not, and we proceed from Step 1 to Step 3, skipping Step 2. The relationship of the four methods is shown in Table 3.

3.5 Local Search
The proposed operators just described are easy to implement. However, these operators will cause the following two problems.

a) The travel distance for the related carts will be affected and result in longer makespans.

b) Some delays may occur in the unchecked new schedule.

The next two parts deals with “a)” and “b)” respectively.

The first problem is to determine how to change the sequence of the actions and how to cluster the actions into some trips to minimize the traveling cost.

After the routing problem has been solved, the next step is to minimize the delays caused by the interactions between carts.

Trip Exchange is a simple process that aims to minimize the delays by exchanging the trip sequence inside the corresponding cart. The Trip Exchange is different according to the type of delay.

1) Determine which trip causes loading queue delays and exchange this trip with the last trip.

2) Determine which replenishment trip to remedy a product shortfall produces a delay (causing replenishing waiting delay) and exchange that trip with the first trip.

4. SIMULATION AND RESULTS
4.1 The Reference Scheduler Here, for a comparison, we introduce the reference scheduler that is currently employed by the industrial partner in this research [15]. The strategy is selected
due to the following two reasons: (a) there have been no former algorithms in the literature that deal with the same problem statement in this paper, and (b) the problem is still solved in real warehouses with a specific rule-based algorithm. Therefore, the proposed algorithm is modeled as the reference scheduler in order to evaluate the proposed algorithm quantitatively. The reference scheduler is similar to the heuristic of the Initial Schedule mentioned in Section 3. The carts based on the S-shaped routing will add products to the trip until the capacity of the trip is reached. The remaining products will be picked up on the next trip. When order-picking starts, all carts will start from the AS/RS to unload replenishments. The picking tasks will not start until all replenishing tasks are finished. The advantage of this scheduler is that the Replenishment Waiting Delay will not occur because stock products are sufficient after replenishing.

4.2 Simulation Results

First, we evaluate the performance of the metaheuristic-based SA scheduler with different operators in computer experiments. The maximal computation time $T_{max}$ is set to be 3 minutes under the environment of a Pentium4 1.2GHz with 1GB RAM. Table 4 shows the simulation environment. The input orders are assigned with respect to shops that represent the regions to which products are delivered. A comprehensive set of data instances is used to evaluate the schedulers. The basic data instances are 3 data as follows:

- Instance 1: 720 cases/shop distributed in 75 locations
- Instance 2: 720 cases/shop distributed in 45 locations
- Instance 3: 720 cases/shop distributed in 30 locations

These data are created based on the orders of real warehouses. The number of shops is set at 2 to 6. Restriction in generating trips is as follows:

- For the picking tasks, trips must consist of products from the same shop.
- For the replenishment tasks, no restriction exists as to the products in the same trip.

Figure 7 shows the average values of makespan (in seconds) with input of Instance 1 to 3. All four sets of operators showed better performances than the reference scheduler. If we compare Split-I with the reference scheduler, a significant improvement (21.9% - 45.3%) is achieved. Both Split_R and Split_I show better performance than Non-Split_R and Non-Split_I. Comparing with Non-Split, Split can achieve, on average, a 2.4% improvement. Thus, the Action Split operator is proved to be efficient to balance the cart’s makespans. Both Non-Split_I and Split_I show better results than Non-Split_R and Split_R. Especially, Split_I achieved the best result, i.e., a 1.9% improvement, compared with nonSplit_R. It is a feasible to determine the operator by calculating the $i^*$.

Then, we evaluate the improvement of 3 instances by increasing the number of shops. The trend is for the makespans of the reference scheduler to increase in a linear manner as the number of items increases. Split-I can achieve more improvement by increasing the number of shops. It is possible that, as the picking intensity increases, the items tend to concentrate on several specific locations and loading que delay will increase. Under this condition, the proposed metaheuristic is more efficient to reduce the delays and results of minimizing the makespans. The improvement of 3 instances does not show much difference when the number of shops is low (e.g., 2 or 3). However, when the number of shops increases (e.g., when the number of shops is 5 or

<table>
<thead>
<tr>
<th>Table 4 Simulation environment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
</tr>
<tr>
<td>Cart speed</td>
</tr>
<tr>
<td>Loading time per one case ($T_{load}$)</td>
</tr>
<tr>
<td>$N_{ps}$</td>
</tr>
<tr>
<td>Average stock</td>
</tr>
<tr>
<td>Cart number</td>
</tr>
<tr>
<td>Location size</td>
</tr>
<tr>
<td>Number of locations</td>
</tr>
<tr>
<td>Location capacity (CAP1)</td>
</tr>
<tr>
<td>Cart capacity (CAP2)</td>
</tr>
<tr>
<td>Loading time at shed or AS/RS</td>
</tr>
<tr>
<td>$T_{max}$</td>
</tr>
<tr>
<td>$A$</td>
</tr>
</tbody>
</table>

37
6), the low-density instance (Instance 1) shows better improvement than the high-density instance (Instance 3).

In a multi-shop picking environment, orders from different shops with a few items each will be combined into one picking instance to improve the picking efficiency. When the number of shops is not large, combining their orders into one picking event will not have a significant effect. On the other hand, as the number of shops increases, combination of the order can produce a high efficiency.

The obtained results of Split-I method has varied even with the same condition because it is a kind of a random-search method. In order to see the influence of the variance, simulations are made for Split-I with 30 times when the number of shops is 2 and in the case of Instance 1, in which the difference between the result of Split-I and that of the reference scheduler is smallest. As a result, the standard score of the value for the reference scheduler has the standard score of -13.6, which means that the result of Split-I and that of the reference scheduler is significantly different.

The reason can be concluded from the comparison of three delays, total $D_{lq}$, total $D_{pl}$, and total $D_{rw}$ in the reference scheduler and Split-I. This is the result when the number of shops is 2 and in the case of Instance 1 (Figure 8). In the reference scheduler, although there are no $D_{lq}$ in the reference scheduler because of first replenishment next picking policy, many $D_{pl}$ and $D_{rw}$ make the makespan longer. The reasoning behind this improvement can be applied to the following items.

- The proposed algorithm can produce a good schedule with shorter makespans.
- Local Search has succeeded in restraining $D_{lq}$ and $D_{pl}$ through Trip Exchange.

For other instances and for other number of shops, almost the same tendency can be obtained with the above-mentioned discussions.

5. CONCLUSION

In this paper, the problem of ordering-picking with replenishment has been addressed.

This research aims to generate the agents’ scheduling and make the agents finish the tasks as soon as possible, that is, minimizing makespans. An efficient search-based metaheuristic is proposed because it is known to give optimal or near-optimal results.

In the metaheuristic, the transition from one solution to a new solution is a serious problem, especially in a complex system. Thus, we presented operators that aim to implement the transition from one schedule to another by way of relocating (moving) the actions between agents. Three operators, Action Transfer, Action Exchange, and Action Split, are identified. All operators are involved in the transfer of action between agents. The split and swap conditions are two determinations with which we are concerned. According to the two conditions, we conclude that there are four sets of operators for the transition of a schedule.

Instances are arranged according to varying the number of shops. Many experiments have been conducted in different instances and in a class-based storage assignment condition. Experiment results proved that a kind of SA scheduler, Split-I had a little improvement than three other scheduler (Split-R, Non-Split-I, Non-Split-R). By comparison with the reference scheduler, the Split-I SA scheduler was efficient enough to balance the makespans between agents. Experimental results revealed that the Split-I SA scheduler can achieve a significant improvement (averaged about 31%) in comparison to the reference scheduler. The trend is for the average improvement increase as the number of shops increases. When the pickup intensity increases, so does the volume of stock in one location, and, as a result, the greater the possibility of a long delay. Under this condition, the proposed metaheuristic is more efficient to reduce the delays and result of minimizing the makespans.

References


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